

$$\det A^t A = \det \begin{pmatrix} \delta & \delta & \delta & 0 \\ 0 & \delta & \delta & \delta \end{pmatrix} = \det \delta I_4.$$

$$\delta = (a^2 + b^2 + c^2 + d^2)$$

$$\det \alpha A = \alpha^4 \det A$$

$$\det A^t A = \begin{cases} \delta^4 \\ \det A \times \det^t A = (\det A)^2 \end{cases}$$

$$(\det A)^2 = \delta^4 \implies \det A = \pm \delta^2$$

$$A = \begin{pmatrix} d & b & c & a \\ -b & a & -d & e \\ -c & d & e & c \\ -d & -c & b & a \end{pmatrix}$$

$$\det A = P(a)$$

$$\det A = \sum_{\sigma \in S_n} \varepsilon(\sigma) \cdot a_{1, \sigma(1)} \cdots a_{n, \sigma(n)}$$

$n = 4$

$P(a)$  est un polynôme,  $\deg P = 4$   
 coefficient dominant = 1.

$$\det A = (a^2 + b^2 + c^2 + d^2)^2$$

$$z) A(a, b, c, d) = \begin{pmatrix} a & & & \\ -b & & & \\ & c & & \\ & & -d & \end{pmatrix} \quad A(a', b', c', d') \\ = \begin{pmatrix} a' & & & \\ -b' & & & \\ & c' & & \\ & & -d' & \end{pmatrix}$$

$$A(a, b, c, d) A(a', b', c', d') = A(a'', b'', c'', d'')$$

$$\det \left( \begin{matrix} \\ \\ \\ \end{matrix} \right) = \det A(a, b, c, d) \times \det A(a', b', c', d') \\ = \det (a'', b'', c'', d'')$$

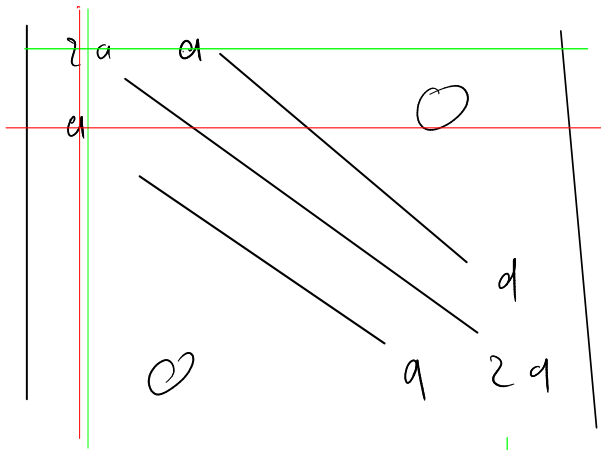
$$S_{ln} = 1 + \dots + h'$$

$$\begin{array}{|ccc|}
 \hline
 s_1 & s_1 & s_1 \\
 \hline
 & s_2 & s_2 \\
 \hline
 & & s_3 \\
 \hline
 & & \\
 \hline
 s_1 & s_2 & s_3 \\
 \hline
 \end{array}
 \begin{array}{|ccc}
 \hline
 s_1 & s_1 & s_1 \\
 \hline
 \vdots & \vdots & \vdots \\
 \hline
 s_{n-2} & s_{n-2} & s_{n-1} \\
 s_{n-2} & s_{n-1} & s_n \\
 s_{n-2} & s_{n-1} & s_n \\
 \hline
 \end{array}
 =
 \begin{array}{|c|}
 \hline
 1 \\
 \hline
 \vdots \\
 \hline
 1 \\
 \hline
 \end{array}
 \begin{array}{|c|}
 \hline
 0 \\
 \hline
 0 \\
 \hline
 * \\
 \hline
 * \\
 \hline
 A \\
 \hline
 \end{array}
 \begin{array}{|c|}
 \hline
 0 \\
 \hline
 \vdots \\
 \hline
 0 \\
 \hline
 n-1 \\
 \hline
 0 \\
 \hline
 n \\
 \hline
 \end{array}$$

$$c_1 \quad c_2 - c_1 \quad c_3 - c_1 \quad \vdots \quad c_n - c_1$$

$$c_k - c_{k-1} \quad c_{n-1} - c_{n-2} \quad c_n - c_{n-1}$$

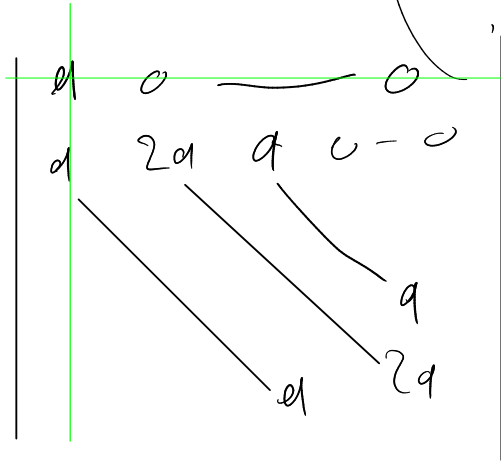
$$\det A = n-1$$

$D_{a,n}$  $n$ 

triangolare

A diagram of a triangular lattice. Nodes are labeled with 'a', 'b', 'c', 'd', and '0'. Edges are labeled with 'a', 'b', 'c', and 'd'.

$$D_{a,n} = 2a D_{a,n-1} - a$$



$$D_{a,n} = 2a D_{a,n-1} - a^2 D_{a,n-2}$$

$$D_{a,n} = u_n'$$

$$u_n = z a u_{n-1} - a^2 u_{n-2}$$

$$z^2 - z a - a^2 = 0$$

we assume  $(r-a)^2$

$$u_n = \lambda a^n + \mu n a^n$$

$$u_2 =$$

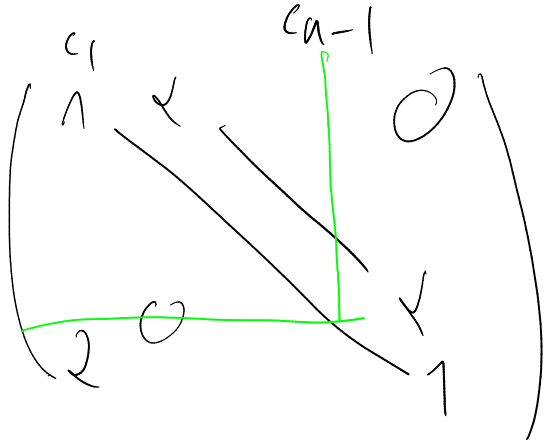
$$u_3 =$$

$$u_n = (n+1) a^n$$

$$A \in GL_n(\mathbb{K}) \iff \det A \neq 0$$

$$\det A \neq 0 \iff \text{rang } A = n$$

$$\sum_n \det M = 0 \implies \text{rang } M \leq n-1$$



A diagram of an  $n \times n$  matrix with a diagonal line from top-left to bottom-right. The top-left element is  $a_1$  and the bottom-right element is  $1$ . A vertical line is drawn from the top row, labeled  $c_{n-1}$ , down to the bottom row, labeled  $x$ . There is a  $0$  in the top-right corner and a  $2$  in the bottom-left corner.

$$M' = \begin{pmatrix} 1 & & & 0 \\ & \ddots & & \\ & & x & \\ & 0 & & 1 \end{pmatrix}$$

$$\det M' = 1$$

$$\text{rang } M' = n-1$$

$$\implies (c_{11} - \dots - c_{n-1}) \text{ libre} \implies \text{rang } M \geq n-1$$

$$\implies \text{rang } M = n-1$$

















