$$\det A^t A = \det \begin{pmatrix} \delta & & & 0 \\ & \delta & & \\ & & \delta & \\ 0 & & & \delta \end{pmatrix} = \det \delta I_4$$

$$\delta = (a^2 + b^2 + c^2 + d^2)$$

$$\det \alpha A = \alpha^4 \det_4 A$$

$$\det A^t A = \begin{cases} \delta^4 \\ \det A \times \det {}^t A = (\det A)^2 \end{cases}$$

$$(\det A)^2 = \delta^4 \implies \det A = \pm \delta^2 ?$$

$$A = \begin{pmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{pmatrix}$$

$\det A = P(a|$

$$\det A = \sum_{\sigma \in S_4} \varepsilon(\sigma) \, a_{1\sigma(1)} \cdots a_{n\sigma(n)} \qquad n = 4$$

$P(a)$ est un polynôme ; $\deg P = 4$

coefficient dominant $= 1$.

$$\det A = (a^2 + b^2 + c^2 + d^2)^2$$

2) $A(a,b,c,d) = \begin{pmatrix} a & & \cdots \\ -b & & \\ -c & & \\ -d & & \end{pmatrix}$ $A(a',b',c',d') = \begin{pmatrix} a' & & \cdots \\ -b' & & \\ -c' & & \\ -d' & & \end{pmatrix}$

$A(a,b,c,d)\,A(a',b',c',d') = A(a'',b'',c'',d'')$

$\det(\qquad) = \det A(a,b,c,d) \times \det A(a',b',c',d')$

$= \det(a'',b'',c'',d'')$

$S_n = 1 + \quad + n'$

$$\begin{vmatrix} s_1 & s_1 & s_1 & \cdots & s_1 & s_1 & s_1 \\ \vdots & s_2 & s_2 & & \vdots & \vdots & \vdots \\ & \vdots & s_3 & & & & \\ & & \vdots & & s_{n-2} & s_{n-2} & \vdots \\ & & & & & & s_{n-1} \\ & & & & s_{n-2} & s_{n-1} & \\ s_1 & s_2 & s_3 & \cdots & s_{n-2} & s_{n-1} & s_n \end{vmatrix} = \begin{vmatrix} 1 & & 0 & & 0 \\ \vdots & \ddots & \vdots & & \vdots \\ & & 0 & & \\ & & k & \ddots & \\ & & * & & \\ & & & n-1 & 0 \\ 1 & & * & * & n \end{vmatrix}$$

A

$c_1 \quad c_2 - c_1 \quad c_3 - c_1 \quad \cdots \quad c_n - c_1$

$c_k - c_{k-1} \quad c_{n-1} - c_{n-2} \quad c_n - c_{n-1}$

$\det A = n-1$

$$D_{a,n} = n\begin{vmatrix} 2a & a & & 0 \\ a & \ddots & \ddots & \\ & \ddots & \ddots & a \\ 0 & & a & 2a \end{vmatrix}$$

trigonale

$$\begin{pmatrix} a & c & & 0 \\ b & \ddots & \ddots & \\ & \ddots & \ddots & c \\ 0 & & b & d \end{pmatrix}$$

$$D_{a,n} = 2a\, D_{a,n-1} - a \begin{vmatrix} a & 0 & \cdots & 0 \\ a & 2a & a & 0 \cdots 0 \\ & a & \ddots & \ddots \\ & & \ddots & 2a & a \\ & & & a & 2a \end{vmatrix}$$

$$D_{a,n} = 2a\, D_{a,n-1} - a^2 D_{a,n-2}$$

1) $a_{1,n} = u_n$

$$u_n = 2a\,u_{n-1} - a^2 u_{n-2}$$

$$r^2 - 2ar + a^2 = 0$$

[illegible] $(r-a)^2$

$$u_n = \lambda a^n + \mu n a^n$$

$u_2 =$

$u_3 =$

$$u_n = (n+1)\,a^n$$

$A \in GL_n(\mathbb{K}) \iff \det A \neq 0$.

$\det A \neq 0 \iff rg\, A = n$.

Si $\det M = 0 \implies rg\, M \leq n-1$

$$M = \begin{pmatrix} 1 & \alpha & & & 0 \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ & & & 1 & \alpha \\ 0 & & & & 1 \end{pmatrix} \quad (c_1, \dots, c_{n-1})$$

$$M' = \begin{pmatrix} 1 & \alpha & & 0 \\ & \ddots & \ddots & \\ & & \ddots & \alpha \\ 0 & & & 1 \end{pmatrix}$$

$\det M' = 1$

$rg\, M' = n-1$

$\implies (c_1, \dots, c_{n-1})$ libre $\implies rg\, M \geq n-1$

$\implies rg\, M = n-1$