ALG 1

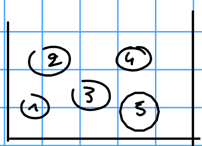
Chap 5 : Dénombrement

Cours 14 (1)

2 - Dénombrement

$E = \bigsqcup_{i=1}^{n} A_i$ et $\operatorname{card} A_i = p$ pour tout i.

$\operatorname{card} E = \sum_{i=1}^{n} \operatorname{card}(A_i) = \sum_{i=1}^{n} p = np$.



1^er tirage : 5 possibilités

2ème tirage : 5 possibilités

5×5

$(\ , \ , \ , \ , \ )$ $\quad 20^5$.

20 poss. 20 poss 20 20 20

A T E S U U I $\quad 26^7$

26 26 ... 26

S U R _ _ $\quad$ (26, 26)

SUR $\quad 3 \times 26^2$

26 _ S U R _ 26

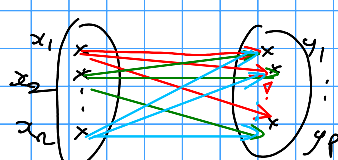
26 _ 26 _ S U R

$\{1 .. p\} \to E$ $\qquad$ $(x_1 .. x_p)$

$i \mapsto x_i$.

$f : E = \{x_1 .. x_n\} \to F$

$x_i \mapsto$



p poss.

p poss

⋮

$\underbrace{p \times p \cdots \times p}_{n \text{ fois}} = p^n = \operatorname{card} F^{\operatorname{card} E}$ poss.

3. Arrangements.

$0! = 1$ $\qquad\qquad$ $5! = 5\times4\times3\times2\times1$

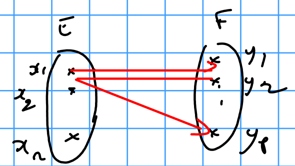
$(n+1)! = (n+1)\times n!$

$= \cdots = (n+1)\, n\times(n-1)\cdots\times 4\times3\times2\times1.$

$$\frac{5!}{3!} = \frac{5\times4\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{3}\times\cancel{2}\times\cancel{1}} = 5\times4.$$

$$\frac{n!}{(n-p)!} = \frac{n\times(n-1)\times(n-2)\times\cdots(n-p+1)\cancel{(n-p)}\cancel{(n-p-1)}\cdots\cancel{3\times2\times1}}{\cancel{(n-p)}\cancel{(n-p-1)}\cdots\cancel{3\times2\times1}}$$

$$\begin{array}{ccccc} (& , & , & , &) \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 20 & 19 & 18 & 17 & 16 \end{array} \qquad \begin{array}{ccccccc} \_ & \_ & \_ & \_ & \_ & \_ & \_ \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 26 & 25 & 24 & 23 & 22 & 21 & 20 \end{array}$$



x_1 : p poss

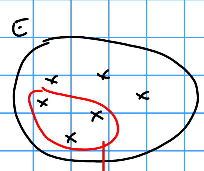
x_2 : $p-1$ poss

$\vdots$

x_n : $p-(n-1) = p-n+1$ poss.

$$p\times(p-1)\times\cdots(p-n+1) = \frac{p!}{(p-n)!}$$

4. Combinaisons $\qquad\qquad$ Cours 14 (2)



3-combinaison de E

• $\underset{A}{\_}\ \underset{A}{\_}\ \underset{S}{\_}\ \underset{N}{\_}\ \underset{A}{\_}\ \underset{N}{\_}$

3 : A $\longrightarrow$ $\binom{6}{3}$ possibilités

2 : N $\longrightarrow$ $\binom{3}{2}$ possibilités

1 : S $\longrightarrow$ 1 possibilité

$$\binom{6}{3}\times\binom{3}{2}\times1 = \frac{6!}{3!\,\cancel{3!}}\times\frac{\cancel{3!}}{2!\,1!} = \frac{6!}{3!\,2!} = \frac{6\times5\times\cancel{4}^{\,2}\times\cancel{3}\times\cancel{2}}{\cancel{3}\times\cancel{2}\times\cancel{2}} = 60.$$

$$\text{NASAAN} \longrightarrow \begin{array}{cccccc} \underline{N_4} & \underline{A_1} & \underline{S_6} & \underline{A_3} & \underline{A_2} & \underline{N_5} \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 6 & 5 & 4 & 3 & 2 & 1 \end{array}$$

$A_1, A_2, A_3, N_4, N_5, S_6.$

$6!$ possibilités

$\text{NASAAN} \longrightarrow \underline{N_4}\ \underline{A_2}\ \underline{S_6}\ \underline{A_3}\ \underline{A_1}\ \underline{N_5}$

$$3!\,2! \longrightarrow \frac{6!}{3!\,2!}$$

ΠOO

ΠO_1O_2

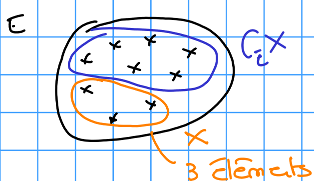
ΠO_2O_1

$$\binom{n}{p} = \frac{n!}{p!\,(n-p)!} = \frac{n\,(n-1)!}{p\times(p-1)!\,(n-p)!} = \frac{n}{p}\binom{n-1}{p-1}$$

$$\binom{n-1}{p-1} = \frac{(n-1)!}{(p-1)!\,((n-1)-(p-1))!} = \frac{(n-1)!}{(p-1)!\,(n-p)!}$$

$$p\binom{n}{p} = n\binom{n-1}{p-1}.$$

$$\mathcal{P}_2(\llbracket 1,3\rrbracket) = \mathcal{P}_2(\{1,2,3\}) = \{\{1,2\},\{1,3\},\{2,3\}\}$$



$\mathrm{card}\, C_E A = \mathrm{card}\, E - \mathrm{card}\, A.$

$$\varphi\circ\psi(X) = \varphi(\psi(X)) = \varphi(\underbrace{CX}_{Y}) = CY = CCX = X \qquad \varphi\circ\psi = id$$

$$\psi\circ\varphi(X) = X \quad , \quad \psi\circ\varphi = id.$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$(a+b)^n = \sum_{p=0}^{n}\binom{n}{p} a^p b^{n-p}$$

$$(a+b)^2 = \sum_{p=0}^{2}\binom{2}{p} a^p b^{2-p} = \binom{2}{0}a^0 b^{2-0} + \binom{2}{1}a^1 b^{2-1} + \binom{2}{2}a^2 b^{2-2} = b^2 + 2ab + a^2.$$

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.$$