EQUA. DIFF

Chap 4 - Equations aux dérivées partielles

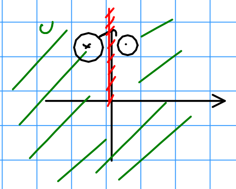
Cours 14 (1)

Equations différentielles ordinaires si $n=1$ (fonction à une variable)

1. Ordre 1.

Ex3. $f(x,y) = \begin{cases} 0 & \text{si } y<0 \\ 0 & \text{si } y\geq 0 \text{ et } x<0 \\ y^3 & \text{si } y \geq 0 \text{ et } x>0 \end{cases}$

$U = \mathbb{R}^2 \setminus \{(0,y),\ y \in \mathbb{R}_+\}$



$\frac{\partial f}{\partial x}(x,y) = 0$ $\qquad f(1,1) = 1$

$f(-1,1) = 0$

$\frac{\partial f}{\partial x}(x,y) = 0$, $\quad \frac{\partial f}{\partial y}(x,y) = 0$ $\quad (x,y) \mapsto h(x)$ avec $h \in \mathcal{C}^1$

$\frac{\partial f}{\partial x}(x,y) = xy$ $\qquad (x,y) \mapsto \frac{x^2}{2}y$ $\qquad \int \lambda x = \lambda \frac{x^2}{2} + \underline{\underline{C}}$

$S = \left\{ (x,y) \mapsto h(y) + \frac{x^2}{2}y,\ h \in \mathcal{C}^1 \right\}$

$g(x,y) = f(x,y)e^{-xy}$

$\frac{\partial g}{\partial y}(x,y) = \frac{\partial f}{\partial y}(x,y)e^{-xy} - x f(x,y) e^{-xy}$

$= e^{-xy}\left(\frac{\partial f}{\partial y}(x,y) - x f(x,y) \right).$

f est solution ssi $\frac{\partial g}{\partial y}(x,y) = 0$ ssi $\exists h\ \mathcal{C}^1$ tq $g(x,y) = h(x)$

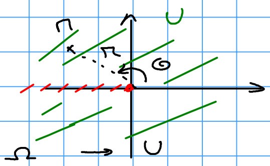
ssi $f(x,y) = h(x)\, e^{xy}$

$D_{e_1'} f(x,y) = df_{(a,b)}(x,y) = a\frac{\partial f}{\partial x}(x,y) + b\frac{\partial f}{\partial y}(x,y)$

$\frac{\partial \tilde{f}}{\partial v_1} = 0$

1. Ordre 1.

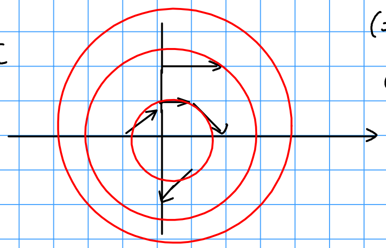
3. EDP à coefficients non constants.



$r > 0$

$\theta \in]-\pi, \pi[$

$(r,\theta) \mapsto (r\cos\theta, r\sin\theta)$



$(x,y) \mapsto (y,-x)$

$(0,1) \mapsto (1,0)$

$(1,-1) \to (-1,-1)$

$$y\frac{\partial f}{\partial x}(x,y) - x\frac{\partial f}{\partial y}(x,y) = 0$$

φ est surjective.

$y = r\sin\theta$, $x = r\cos\theta$

2. EDP d'ordre 2

$\varphi(x) = f(x, y_0)$ $\qquad \varphi''(x) = \frac{\partial^2 f}{\partial x^2}(x, y_0) = 0$

$\varphi(x) = h_1(y_0)x + h_2(y_0)$

$f(x,y) = h_1(y)x + h_2(y)$, $\quad h_1, h_2 \; \mathcal{C}^2$

$\frac{\partial f}{\partial x}(x,y) = h_1(y)$ $\qquad \frac{\partial^2 f}{\partial x^2}(x,y) = 0$.