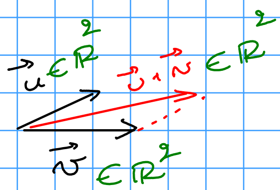
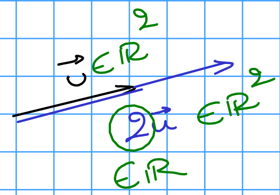
I- Structure d'espace vectoriel

1 - Définition

$(\mathbb{R},+,\times)$, $(\mathbb{C},+,\times)$

vecteurs | de $\mathbb{R}^2$
| du plan





$\lambda \vec{u}$

$$\underset{\in\mathbb{R}}{\lambda} \underset{\in\mathbb{R}^2}{\begin{pmatrix}1\\2\end{pmatrix}} = \underbrace{\begin{pmatrix}\lambda\\2\lambda\end{pmatrix}}_{\in\mathbb{R}^2}$$

$(A,+,\times)$

- $2\left(\begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}1\\2\end{pmatrix}\right) = 2\begin{pmatrix}1\\1\end{pmatrix} + 2\begin{pmatrix}1\\2\end{pmatrix} = \dots$

- $(2+\lambda)\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}2+\lambda\\(2+\lambda)3\end{pmatrix}$
  $= 2\begin{pmatrix}1\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\3\end{pmatrix}$.

- $1 \cdot \begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$

- $2 \times 3 \times \begin{pmatrix}1\\1\end{pmatrix}$
  $= 2 \times \left(3 \times \begin{pmatrix}1\\1\end{pmatrix}\right)$
  $= 2\begin{pmatrix}3\\3\end{pmatrix}$
  $= \begin{pmatrix}6\\6\end{pmatrix}$.

$\underset{\in\mathbb{R}}{0}$ $\neq$ et $\underset{\in\mathbb{R}^2}{\begin{pmatrix}0\\0\end{pmatrix}} = 0_{\mathbb{R}^2}$ $\qquad 2\begin{pmatrix}1\\1\end{pmatrix}$

$\lambda_1, \dots \lambda_n \in \mathbb{K}$, $\vec{x}_1, \dots \vec{x}_n \in E$

$$\underbrace{\underbrace{\lambda_1 \cdot \vec{x}_1}_{\in E} + \underbrace{\lambda_2 \cdot \vec{x}_2}_{\in E}}_{\in E} + \underbrace{\lambda_3 \cdot \vec{x}_3}_{\in E} + \dots + \lambda_n \cdot \vec{x}_n \in E.$$

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2 - Premiers exemples

$\mathbb{R}^2$ $\quad \begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}-1\\2\end{pmatrix}=\begin{pmatrix}0\\3\end{pmatrix}$, $\quad 2\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}2\\4\end{pmatrix}$.

$(1,1)+(-1,2)=(0,3)$.

$$\begin{aligned}P_1-2P_2 &= 1+X+X^2-2(X-X^2)\\ &= 1+1X+1X^2-2X+2X^2\\ &= 1-1X+3X^2.\end{aligned}$$

$$n \text{ lignes} \downarrow \underbrace{\begin{pmatrix} * & * & \cdots & * \\ \vdots & & & \vdots \\ * & \cdots & & * \end{pmatrix}}_{\longleftrightarrow \ m \text{ colonnes}}$$

$\vec{x}\in E=E_1\times\cdots\times E_n \quad : \quad \vec{x}=(\underset{\in E_1}{\vec{x}_1},\dots,\underset{\in E_n}{\vec{x}_n})$

$\mathbb{R}^2\times\mathbb{R}^3 \quad : \vec{x}=(\vec{x}_1, \vec{x}_2) = \left(\underbrace{\begin{pmatrix}1\\0\end{pmatrix}}_{\in\mathbb{R}^2},\underbrace{\begin{pmatrix}2\\1\\0\end{pmatrix}}_{\in\mathbb{R}^3}\right)\in\mathbb{R}^2\times\mathbb{R}^3 \qquad \vec{0}_{\mathbb{R}^2\times\mathbb{R}^3}=\left(\underset{\vec{0}_{\mathbb{R}^2}}{\begin{pmatrix}0\\0\end{pmatrix}},\underset{\vec{0}_{\mathbb{R}^3}}{\begin{pmatrix}0\\0\\0\end{pmatrix}}\right).$

$$2\cdot\vec{x}=\left(2\cdot\begin{pmatrix}1\\0\end{pmatrix},2\cdot\begin{pmatrix}2\\1\\0\end{pmatrix}\right)=\left(\begin{pmatrix}2\\0\end{pmatrix},\begin{pmatrix}4\\2\\0\end{pmatrix}\right)$$

$$\underbrace{\left(\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}2\\1\\0\end{pmatrix}\right)}_{\vec{x}}+\underbrace{\left(\begin{pmatrix}0\\1\end{pmatrix},\begin{pmatrix}3\\1\\1\end{pmatrix}\right)}_{\vec{y}}=\left(\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix},\begin{pmatrix}2\\1\\0\end{pmatrix}+\begin{pmatrix}3\\1\\1\end{pmatrix}\right)$$

$$=\left(\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}5\\2\\1\end{pmatrix}\right).$$

$$(f+g)(x) = \underbrace{f(x)}_{\in \mathbb{R}^2} + \underbrace{g(x)}_{\in \mathbb{R}^2} = (1+x, x^2) + (\exp(x), -x)$$

$$= (1+x+\exp(x), x^2 - x).$$

Rappel : on a $f = g$ ssi $\forall x \in \mathbb{R} \quad f(x) = g(x)$.