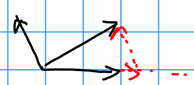
I. Familles de vecteurs

2. Familles libres et liées

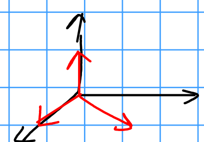
Familles génératrices : $(\vec{x}_1, \dots \vec{x}_n)$, $\underbrace{x = \sum_{i=1}^{n} \lambda_i \vec{x}_i}_{\text{existence de cette décomposition}}$ non forcément uniques

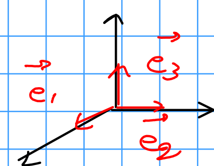
Familles libres : $(\vec{x}_1, \dots \vec{x}_n)$ si $\vec{x}$ se décompose sous la forme $\vec{x} = \sum_{i=1}^{n} \lambda_i \vec{x}_i$, cette décomposition est unique.

uniques



non libre = liée.





- $(1, X, \dots X^n)$ est libre dans $\mathbb{K}_n[X]$:

Soit $(\lambda_0, \lambda_1, \dots \lambda_n) \in \mathbb{K}^{n+1}$ tel que $\lambda_0 \times 1 + \lambda_1 X + \lambda_2 X^2 + \dots + \lambda_n X^n = 0$

$$\lambda_0 \times 1 + \lambda_1 X + \lambda_2 X^2 + \dots + \lambda_n X^n = 0 \times 1 + 0 \times X + 0 \times X^2 + \dots + 0 X^n$$

Donc $\lambda_0 = \lambda_1 = \dots = \lambda_n = 0$.

- $(P_0, P_1, \dots P_n)$ est de degré échelonné

$$\deg P_0 < \deg P_1 < \deg P_2 < \dots < \deg P_n .$$

Exemple : $(1, X, X^2, \dots X^n)$

$(1, X+1, (X+1)^2, \dots (X+1)^n)$

$(X, X^3, X^2(X-1)(X-2), X^6)$

Soit $(\lambda_0, \lambda_1, \dots \lambda_n) \in \mathbb{K}^{n+1}$ tel que

$$\underbrace{\lambda_0 P_0 + \lambda_1 P_1 + \dots + \lambda_{n-1} P_{n-1}}_{d^\circ < d^\circ P_n} + \underbrace{\lambda_n P_n}_{d^\circ P_n} = 0$$

$$P_n = \underbrace{a_0 + a_1 X + \dots +}_{Q_n} a_p X^{\deg P_n}$$

Si $\lambda_n \neq 0$, en notant a_p le coefficient dominant de P_n,

on a $\lambda_0 P_0 + \lambda_1 P_1 + \dots + \lambda_{n-1} P_{n-1} + \lambda_n Q_n + \lambda_n a_p X^{d^\circ P_n} = 0$

donc $\lambda_n a_p = 0$: Absurde.

Donc $\lambda_n = 0$. Donc $\lambda_0 P_0 + \lambda_1 P_1 + \dots + \lambda_{n-1} P_{n-1} = 0$

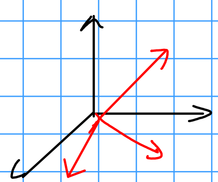
$\lambda_n = 0, \lambda_{n-1} = 0 \dots, \lambda_0 = 0$.

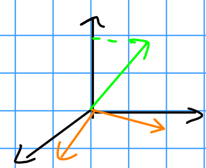
$\underbrace{i}_{\vec{x_2}} = \underbrace{i}_{\lambda \in \mathbb{C}} \times \underbrace{1}_{\vec{x_1}}$ $\qquad$ $i \neq \underbrace{\lambda}_{\in \mathbb{R}} \times 1$

$((2,4), (1,0), (4,4))$

$(4,4) = 1 \times (2,4) + 2 \times (1,0)$ donc la famille est liée.

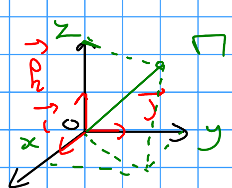
$2\vec{x_1} + 3\vec{x_2} - \vec{x_3} = \vec{0}$ et $(2,3,-1) \neq (0,0,0)$





3_ Bases

Cours 4 (2)



$\vec{OM} = x\vec{i} + y\vec{j} + z\vec{k}$

$\vec{OM}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$z = \underbrace{Re(z)}_{\lambda_1}\underbrace{1}_{\vec{x_1}} + \underbrace{Im(z)}_{\lambda_2} \times \underbrace{i}_{\vec{x_2}}$. $\qquad z \qquad (Re(z), Im(z))$.

$\vec{x} = (x_1, \dots x_n) = x_1 \underbrace{(1, 0, \dots 0)}_{\vec{e_1}} + x_2 \underbrace{(0, 1, 0 \dots 0)}_{\vec{e_2}} + \dots + x_n \underbrace{(0, \dots 0, 1)}_{\vec{e_n}}$

$P = \underbrace{P(a)}_{\lambda_1} \times \underbrace{1}_{\vec{x_1}} + \underbrace{P'(a)}_{\lambda_2}\underbrace{(X-a)}_{\vec{x_2}} + \underbrace{P''(a)}_{\lambda_3}\underbrace{\frac{(X-a)^2}{2!}}_{\vec{x_3}} \dots + \underbrace{P^{(n)}(a)}_{\lambda_n}\underbrace{\frac{(X-a)^n}{n!}}_{\vec{x_n}}$

$(x,y) = \lambda(1,1) + \mu(1,-1)$

Les coordonnées de $(2,-1)$ sont $\left(\frac{1}{2}, \frac{3}{2}\right)$

4. Ev de dimension finie

Cours 4 3

1. Définition

$$E = \text{vect}(\vec{x}_1, \dots, \vec{x}_n)$$

$$\mathbb{K}^n = \text{vect}(\vec{e}_1, \dots \vec{e}_n) \qquad \vec{e}_i = (\underbrace{0 \dots 0\ 1}_{i}\ 0 \dots 0)$$

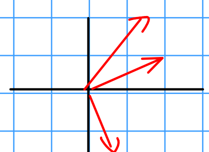
$$\mathbb{K}_n[X] = \text{vect}(1, X, \dots X^n)$$

$$\mathbb{K}[X] \cancel{=} \text{vect}(X^{d_1}, X^{d_2}, \dots X^{d_p}) \qquad d_1 < d_2 < \dots < d_p$$

$$X^{d_p+1}$$

$\forall i \in \{p+1, \dots n\} \qquad \vec{x}_i \in \text{vect}(\vec{x}_1, \dots \vec{x}_p)$.

$$\vec{x} \in E, \quad \vec{x} = \sum_{i=1}^{p} \lambda_i \vec{x}_i + \underbrace{\sum_{i=p+1}^{n} \lambda_i \vec{x}_i}_{\substack{\in \text{vect}(\vec{x}_1, \dots \vec{x}_p) \\ = \sum_{i=1}^{p} \mu_i x_i}} = \sum_{i=1}^{p} (\lambda_i + \mu_i) \vec{x}_i .$$



$$\mathbb{R}^2 = \text{vect}\left(\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right).$$