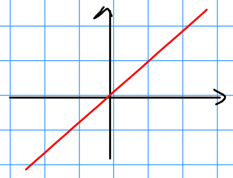
TD 3 . Sciences / Maths

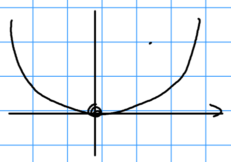
Exercice 1

- P_1 : $\exists x \in \mathbb{R},\ f(x) = 0$.

$f(x) = x$



$f(x) = x^2$

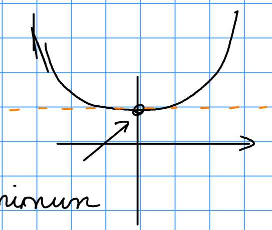


f s'annule en 0

$f(0) = 0$.

non(P_1) : $\forall x \in \mathbb{R},\ f(x) \neq 0$.

$f(x) = x^2 + 1$



minimum

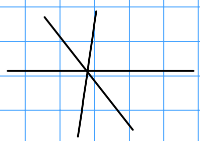
exp

- P_2 : $\forall x \in \mathbb{R},\ f(x) \geq 1$

$f(x) = x^2 + 1$

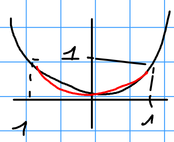
$f(x) = \exp(x) + 1$.

non(P_2) : $\exists x \in \mathbb{R},\ f(x) < 1$



$f(x) = -x$

$f(x) = x^2$



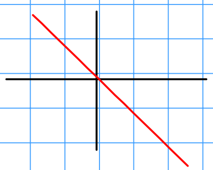
- P_3 : $\exists x \in \mathbb{R}_+,\ f(x) \geq 0$

$f(x) = x$

$f(x) = x + 1$

non(P_3) : $\forall x \in \mathbb{R}_+,\ f(x) < 0$.

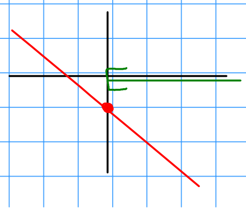
$\cancel{f(x) = -x}$



$x = 0 \in \mathbb{R}_+$

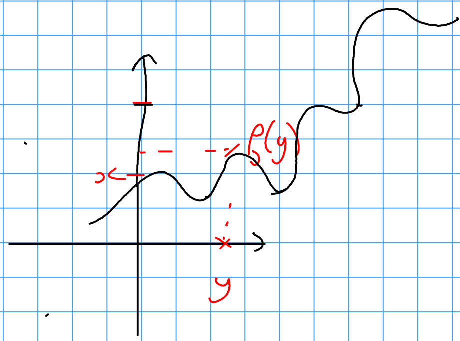
$f(0) = 0 \not< 0$

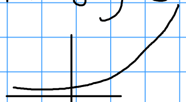
$f(x) = -x - 1$



$f(0) = -1 < 0$

• P_4 : $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}$, $f(y) \geq x$.



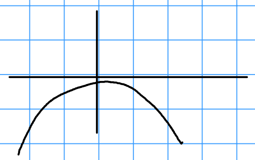
$f(x) = \exp(x)$  , $f(x) = x^2$

la fonction tend vers l'infini.

non(P_4) : $\exists x \in \mathbb{R}$, $\forall y \in \mathbb{R}$, $f(y) < x$.

$\leq$ majorée

$f(x) = -x^2$



par exemple $x = 1$. , $f(x) = \cos(x)$

Exercice 2.

Parité : paire ou impaire.

1.a) $f_1(-x) = \sin(-2x)\cos(-x)$

$= -\sin(2x)\cos(x)$

$= -f_1(x)$ Donc impaire

b) $f_2(-x) = e^{-x} - e^{-(-x)} = e^{-x} - e^{x}$

$= -(e^{x} - e^{-x})$

$= -f_2(x)$. Donc impaire.

c) $f_3(-x) = \dfrac{e^{-x}}{(e^{-x}+1)^2} = \dfrac{e^{2x}\, e^{-x}}{e^{2x}(e^{-x}+1)^2}$

$(e^{x})^2 (e^{-x}+1)^2$

$= (e^{x}(e^{-x}+1))^2$

$= \dfrac{e^{x}}{(1+e^{x})^2}$

$= f_3(x)$ Donc paire.

2 - $f_4(x+4\pi) = \sin(2(x+4\pi)) - 2\cos\left(\dfrac{x+4\pi}{2}\right)$

$= \sin(2x+8\pi) - 2\cos\left(\dfrac{x}{2} + 2\pi\right)$

$= \sin(2x) - 2\cos\left(\dfrac{x}{2}\right)$

$= f_4(x)$. Donc f_4 est 4π-périodique.

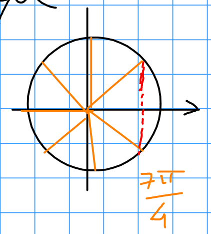
Exercice 3.

1. $\cos\left(\frac{7\pi}{4}\right) = \cos\left(2\pi - \frac{\pi}{4}\right)$ 2π-périodique

$= \cos\left(-\frac{\pi}{4}\right)$ paire

$= \cos\left(\frac{\pi}{4}\right)$

$= \frac{\sqrt{2}}{2}$.



$\sin\left(-\frac{43\pi}{6}\right) = \sin\left(-\frac{(6\times 7+1)\pi}{6}\right)$

$= \sin\left(-7\pi - \frac{\pi}{6}\right)$ impaire

$= -\sin\left(7\pi + \frac{\pi}{6}\right)$

$\sin(x + n\pi) = (-1)^n \sin(x)$

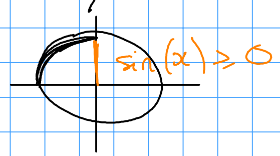
$= +\sin\left(\frac{\pi}{6}\right)$

$= \frac{1}{2}$.

2. $\cos^2(x) + \sin^2(x) = 1$

$\frac{4}{5}$: 4 cinquième

$\sin^2(x) = 1 - \left(-\frac{4}{5}\right)^2 = \frac{9}{25}$



$\sin(x) = \pm\sqrt{\frac{9}{25}} = +\frac{3}{5}$ Or $x \in \left[\frac{\pi}{2}, \pi\right]$

donc $\sin(x) \geq 0$

3. $\frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$.

$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\,\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2} + \sqrt{6}}{4}$

$$\sin\left(\frac{\pi}{12}\right) = -\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\tan\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} = 2 - \sqrt{3}$$

simplifier

[identité remarquable

$(a-b)^2 = a^2 - 2ab + b^2$

$$= \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} \times \frac{(\sqrt{6}-\sqrt{2})}{\sqrt{6}-\sqrt{2}} = \frac{6 - 2\sqrt{12} + 2}{4}$$

$$= 2 - \sqrt{3}.$$

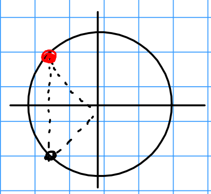
$$\sqrt{12} = \sqrt{4\times 3} = 2\sqrt{3}$$

modulo 2π

$\equiv \quad [2\pi]$

Exercice 4.

cos est 2π-périodique



1. $S = \left\{ \frac{3\pi}{4} + 2k\pi,\ k \in \mathbb{Z} \right\}$

$\cup \left\{ \frac{5\pi}{4} + 2k\pi,\ k \in \mathbb{Z} \right\}$

$\left\{ -\frac{3\pi}{4} + 2k\pi,\ k \in \mathbb{Z} \right\}$.

$\frac{1}{3}$: un tiers.

2. $S = \left\{ \frac{\pi}{3} + k\pi,\ k \in \mathbb{Z} \right\}$

tan est π-périodique.

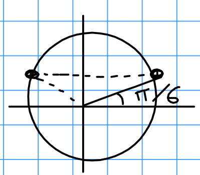
1 sur 2

ou un demi

$\frac{1}{4}$: un quart

3. $\sin(x)\cos(x) = \frac{1}{4} \iff \sin(2x) = \frac{1}{2}$

$2\sin(x)\cos(x) = \sin(2x)$



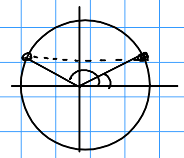
$\iff 2x = \frac{\pi}{6} + 2k\pi$ avec $k \in \mathbb{Z}$

ou $2x = \frac{5\pi}{6} + 2k\pi$ avec $k \in \mathbb{Z}$

$\iff x = \frac{\pi}{12} + k\pi$ avec $k \in \mathbb{Z}$

ou $x = \frac{5\pi}{12} + k\pi$ avec $k \in \mathbb{Z}$.

4. $\sin(5x) = \sin\left(\frac{2\pi}{3} + x\right)$



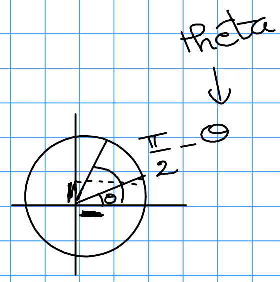
$\iff 5x = x + \frac{2\pi}{3} + 2k\pi$ avec $k \in \mathbb{Z}$

ou $5x = -x + \frac{\pi}{3} + 2k\pi$ avec $k \in \mathbb{Z}$

$\pi - \left(\frac{2\pi}{3} + x\right) + 2k\pi$

$\iff x = \frac{\pi}{6} + k\frac{\pi}{2}$ avec $k \in \mathbb{Z}$

ou $x = \frac{\pi}{18} + k\frac{\pi}{3}$ avec $k \in \mathbb{Z}$.

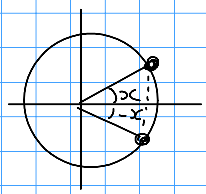


5. $\cos\left(2x - \frac{\pi}{3}\right) = \sin\left(x + \frac{3\pi}{4}\right)$

$\iff \cos\left(2x - \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{2} - \left(x + \frac{3\pi}{4}\right)\right)$

$= \cos\left(-x - \frac{\pi}{4}\right)$

$= \cos\left(x + \frac{\pi}{4}\right)$



$\iff 2x - \frac{\pi}{3} = x + \frac{\pi}{4} + 2k\pi$ avec $k \in \mathbb{Z}$

ou $2x - \frac{\pi}{3} = -x - \frac{\pi}{4} + 2k\pi$ avec $k \in \mathbb{Z}$

$\iff x = \frac{7\pi}{12} + 2k\pi$ avec $k \in \mathbb{Z}$

ou $x = \frac{\pi}{36} + k\frac{2\pi}{3}$ avec $k \in \mathbb{Z}$.

6. $\cos(2x) = \cos^2(x)$

$\iff \cos(2x) = \frac{1 + \cos(2x)}{2}$

$\iff \cos(2x) = 1$

$\iff 2x = 2k\pi$ avec $k \in \mathbb{Z}$

$\iff x = k\pi$ avec $k \in \mathbb{Z}$.

$\left(\sin(p) + \sin(q) = \ldots\right)$ with $p = x$, $q = 3x$

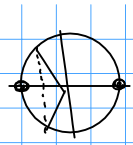
7. $\sin(x) + \sin(3x) = 2\cos(x)\sin(2x)$

$\sin(x) + \sin(2x) + \sin(3x) = 0$

$\iff \sin(2x)\left(1 + 2\cos(x)\right) = 0$

facteur de

$\iff \sin(2x) = 0$ ou $\cos(x) = -\frac{1}{2}$



$\iff x = \frac{k\pi}{2}$ avec $k \in \mathbb{Z}$ ou $x = \frac{2\pi}{3} + 2k\pi$

ou $x = -\frac{2\pi}{3} + 2k\pi$

8 - $nx = k\pi$ avec $k \in \mathbb{Z}$

$x = \frac{k\pi}{n}$ avec $k \in \mathbb{Z}$.