$$\{2k,\ k\in\mathbb{Z}\} = \{f(k),\ k\in\mathbb{Z}\}, \quad f : k \mapsto 2k.$$

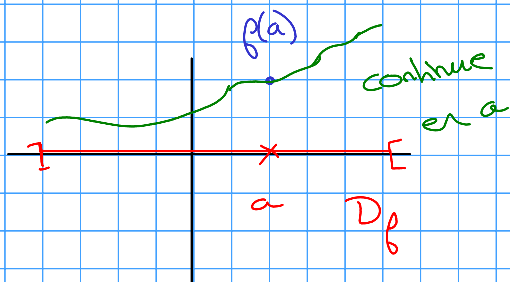
La somme de $\underset{\in\mathbb{R}}{a}$ et $\underset{\in\mathbb{R}}{b}$: $a+b$

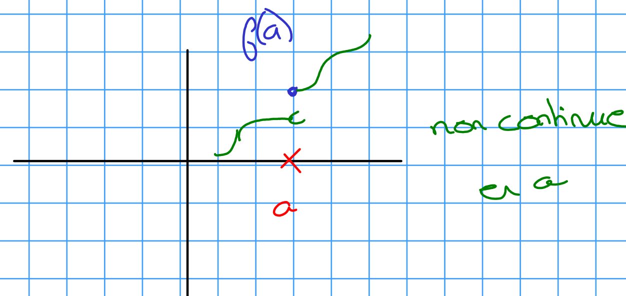
La différence de a et b : $a-b$.

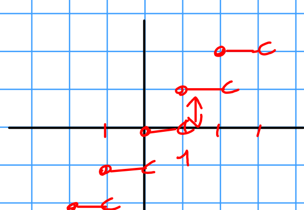
Le produit de a et b : $a\times b$.

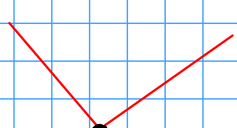
g ne s'annule pas sur D : $\forall x\in D,\ g(x)\neq 0$.

$f(D_f)\subset D_g$.







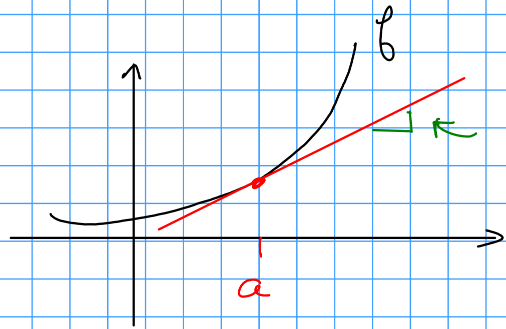


$$\mathbb{R}\to\mathbb{R}$$
$$f : x\mapsto |x|$$

$$\frac{f(x)-f(0)}{x-0} = \frac{|x|}{x} \xrightarrow[x\to 0]{} \begin{cases} 1 & \text{si } x>0 \quad \text{car } |x| = x \\ -1 & \text{si } x<0 \quad \text{car } |x| = -x \end{cases}$$

$$g : \begin{array}{l} \mathbb{R}_+ \longrightarrow \mathbb{R} \\ x \longmapsto \sqrt{x} \end{array} \qquad \frac{g(x)-g(0)}{x-0} = \frac{\sqrt{x}}{x} \xrightarrow[x \to 0]{} +\infty$$

$$= \frac{1}{\sqrt{x}}$$



Soit $f(x) = \dfrac{1}{x^2}$. Alors $f(x) = x^{-2}$,

donc $f'(x) = -2 \times x^{-2-1} = -2x^{-3}$

$$= \frac{-2}{x^3}.$$

- arccos.

$\cos :]0,\pi[\longrightarrow]-1,1[$ est bijective, dérivable sur $]0,\pi[$ et sa dérivée $-\sin$ ne s'annule pas sur $]0,\pi[$, donc

$$\arccos'(x) = \frac{1}{\cos' \circ \arccos(x)} \qquad (f^{-1})' = \frac{1}{f' \circ f^{-1}}$$

$$= \frac{-1}{\sin(\arccos(x))} \qquad \cos^2 + \sin^2 = 1$$

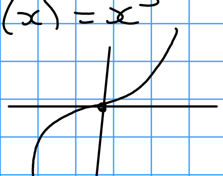
$$\sin^2 = 1 - \cos^2$$

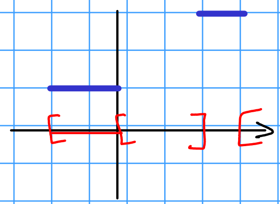
$$= \frac{-1}{\sqrt{1-\cos(\arccos(x))^2}} \qquad \sin = \sqrt{1-\cos^2}$$

$$= \frac{-1}{\sqrt{1-x^2}}.$$

$f(x) = x^3$ $\qquad f'(x) = 3x^2 > 0$ sauf si $x = 0$

$f'(0) = 0$.





$f'(x) = 0$ mais f n'est pas constante.

Posons $h = f - g$. Alors $h' = f' - g' = 0$ sur I

Donc h est constante, et $h(x_0) = f(x_0) - g(x_0)$
$= 0$.

Donc $h = 0$. Donc $f = g$.

$$\exp' = \exp$$
$$\exp'' = \exp' = \exp$$

$f_n(x) = x^n$.

$f_n'(x) = n x^{n-1}$

$f_n''(x) = n(n-1)x^{n-2}$

$f_2(x) = x^2$

$f_2'(x) = 2x$

$f_2''(x) = 2$

$f_2'''(x) = 0$

$$f_n^{(k)}(x) = \begin{cases} n(n-1)(n-2)\cdots(n-k+1)x^{n-k} & \text{si } k \leq n \\ 0 & \text{si } k > n \end{cases}$$

$$= \begin{cases} \dfrac{n!}{(n-k)!} x^{n-k} & \text{si } k \leq n \\ 0 \quad \text{si } k > n. \end{cases}$$

$n! = 1 \times 2 \times \cdots \times n$.