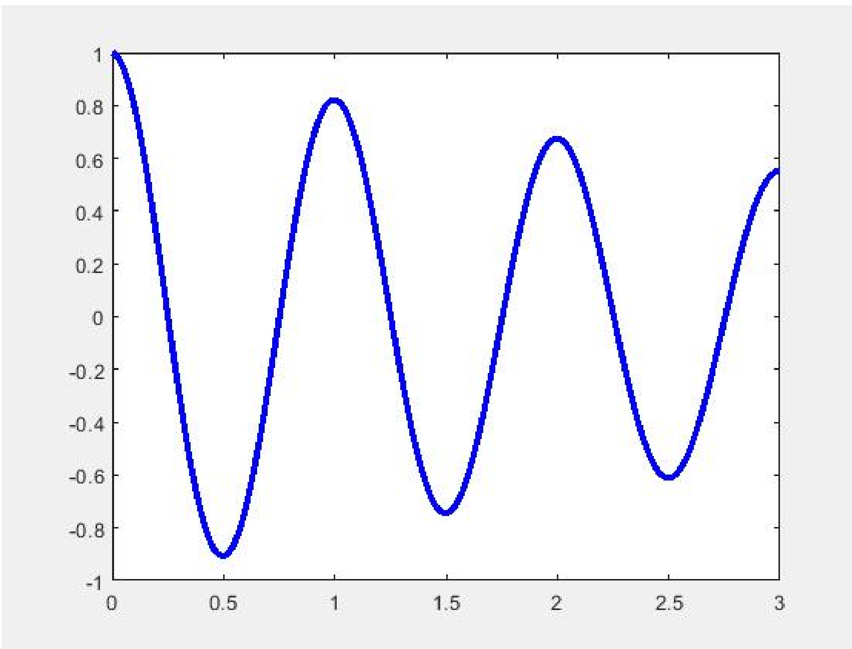
DM3

3.1--méthode 1

```
dt2=0.01;
T0=3;
w0c=4*(pi)^2;
q0=1;
dq0=0;
t2=(0:dt2:T0)';
np2=size(t2,1);
q2=zeros(np2,1);
dq2=zeros(np2,1);
q2(1)=q0;
dq2(1)=dq0;
for inc=2:np2
    q2(inc)=(q2(inc-1)+dt2*dq2(inc-1))/(1+w0c*dt2*dt2)
    ddqc=-w0c*q2(inc)
    dq2(inc)=dq2(inc-1)+dt2*ddqc
end
plot(t2,q2,'b','Linewidth',3)
```

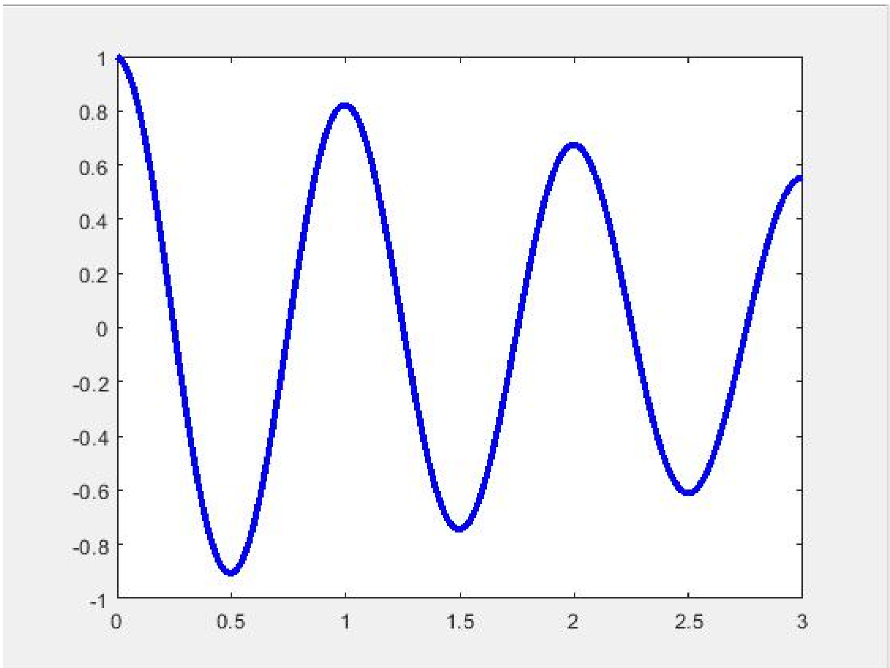
Figure:



3.1--Méthode 2

```
dt2=0.01;
I=3;
t2=(0:dt2:I0)'
np2=size(t2,1);
q=[q0;dq0];
q2b=zeros(np2,1);
dq2b=zeros(np2,1);
q2b(1)=q0;
dq2b(1)=dq0;
A=[1,dt2;-w0c*dt2,1];
A=A/(1+w0c*dt2*dt2);
for inc=2:np2
    q=A*q
    q2b(inc)=q(1)
    dq2b(inc)=q(2)
end
energ2=0.5*(dq2b.*dq2b+w0c*(q2b.^2));
plot(t2,q2b,'b','Linewidth',3)
```

Figure:

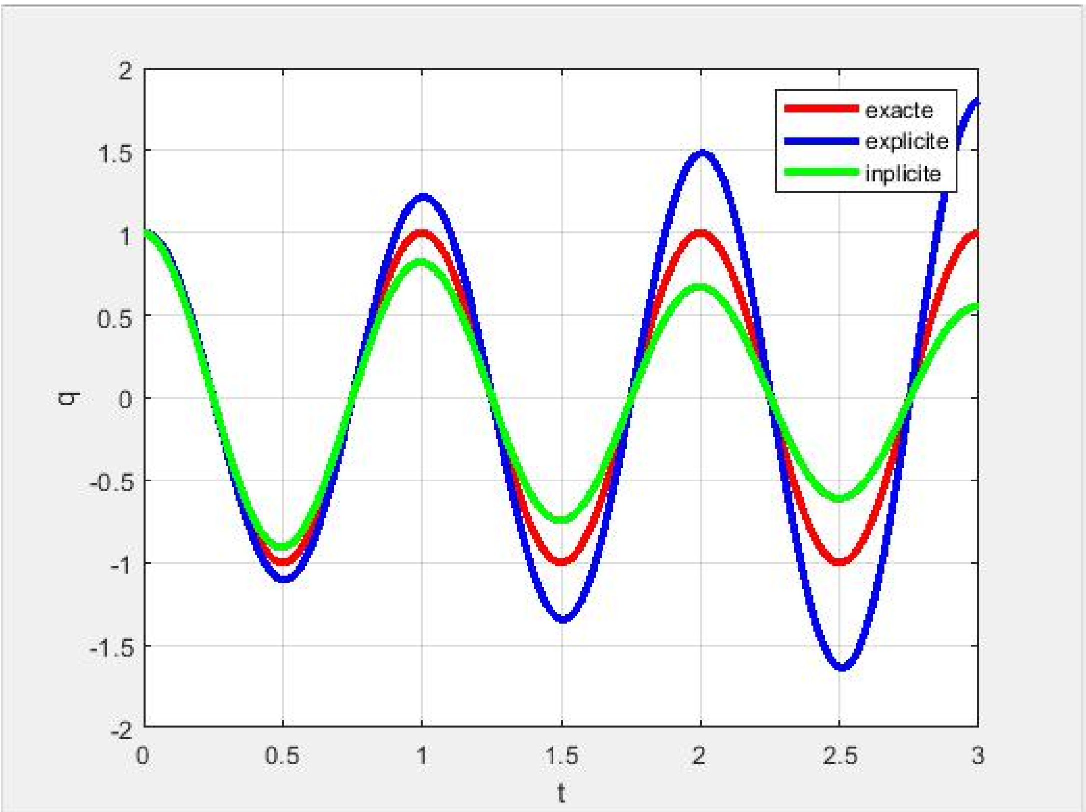


En utilisant les deux méthodes, on obtient le même résultat.

3.2

```
dt=0.01;
T0=3;
t=(0:dt:T0)';
np=size(t,1);
q=zeros(np,1);
for inc=1:np
    q(inc)=exp(-pi*t(inc)*2i)/2 + exp(pi*t(inc)*2i)/2
end
plot(t,q,'r','Linewidth',3)
hold on
plot(t1,q1b,'b','Linewidth',3)%t1,q1 obtenu par l'exercice 2-2
hold on
plot(t2,q2b,'g','Linewidth',3)%t2,q2 obtenu par l'exercice 3-1
grid on;
xlabel('t');
ylabel('q');
legend('exacte','explicite','inplicite')
```

Figure:

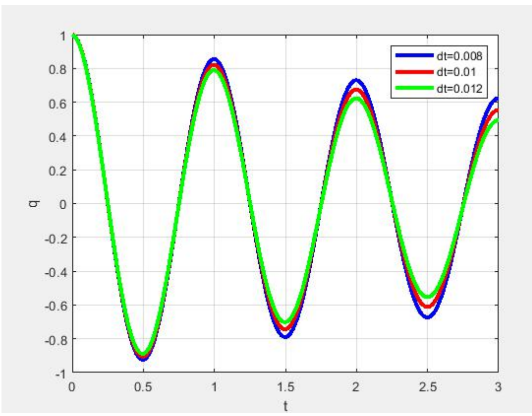


3.3

```
I=3;
for dt2=[0.008,0.01,0.012]
    t2=(0:dt2:I0)'
    np2=size(t2,1);
    q=[q0;dq0];
    q2b=zeros(np2,1);
    dq2b=zeros(np2,1);
    q2b(1)=q0;
    dq2b(1)=dq0;
    A=[1,dt2;-w0c*dt2,1];
    A=A/(1+w0c*dt2*dt2);
    for inc=2:np2
        q=A*q
        q2b(inc)=q(1)
        dq2b(inc)=q(2)
    end
    if dt2==0.008
        plot(t2,q2b,'b','Linewidth',3)
         hold on
     elseif dt2==0.01
         plot(t2,q2b,'r','Linewidth',3)
         hold on
     else plot(t2,q2b,'g','Linewidth',3)
     end

end
grid on;
xlabel('t');
ylabel('q');
legend('dt=0.008','dt=0.01','dt=0.012')
```

Figure

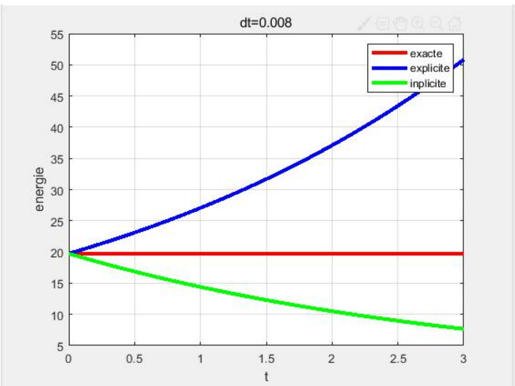


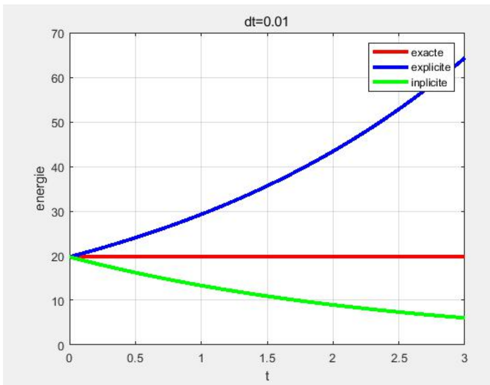
3.4

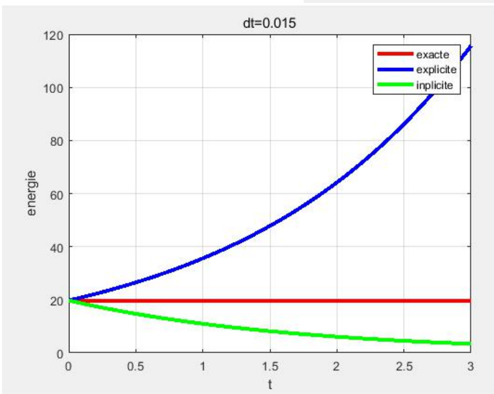
```
dt=0.015;%ou t=0.008 ou t=0.01
T0=3;
t=(0:dt:T0)';
np=size(t,1);
q=zeros(np,1);
energ=zeros(np,1);
for inc=1:np
    q(inc)=exp(-pi*t(inc)*2i)/2 + exp(pi*t(inc)*2i)/2
    energ(inc)=2*pi^2
end

plot(t,energ,'r','Linewidth',3)
hold on
plot(t1,energ1,'b','Linewidth',3)%t1 energ1 obtenu de l'exercice 2-2
hold on
plot(t2,energ2,'g','Linewidth',3)%t2 energ2 obtenu de l'exercice 3-1
grid on;
title('dt=0.015')
xlabel('t');
ylabel('energie');
legend('exacte','explicite','inplicite')
%Pour Euler implicite, plus dt est grand, plus vite l'energie baisse.
%Pour Euler explicite, plus dt est grand, plus vite l'energie s'accroit.
```

Figure:







3.5

```
[z,d]=eig(A)
%abs(d(1))=abs(d(2))=0.9956<1,donc stable
%pour tous les dt, d(1)=(1-i*w0*dt)/(1+w0^2*dt^2)
%                  d(2)=(1+i*w0*dt)/(1+w0^2*dt^2)
%donc module de d est toujours inferieur a 1,donc inconditionnellement
%stable
```

4.1

$$U_c = \begin{pmatrix} q \\ \dot{q} \end{pmatrix} \quad dU_c = \begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix}$$

$$dU_c = \begin{pmatrix} 0 & 1 \\ -w_0^2 & 0 \end{pmatrix} U_c$$

donc $dU_c(1) = U_c(2)$

$$dU(2) = -w_0^2 U_c(1)$$

4.2--définition de la fonction

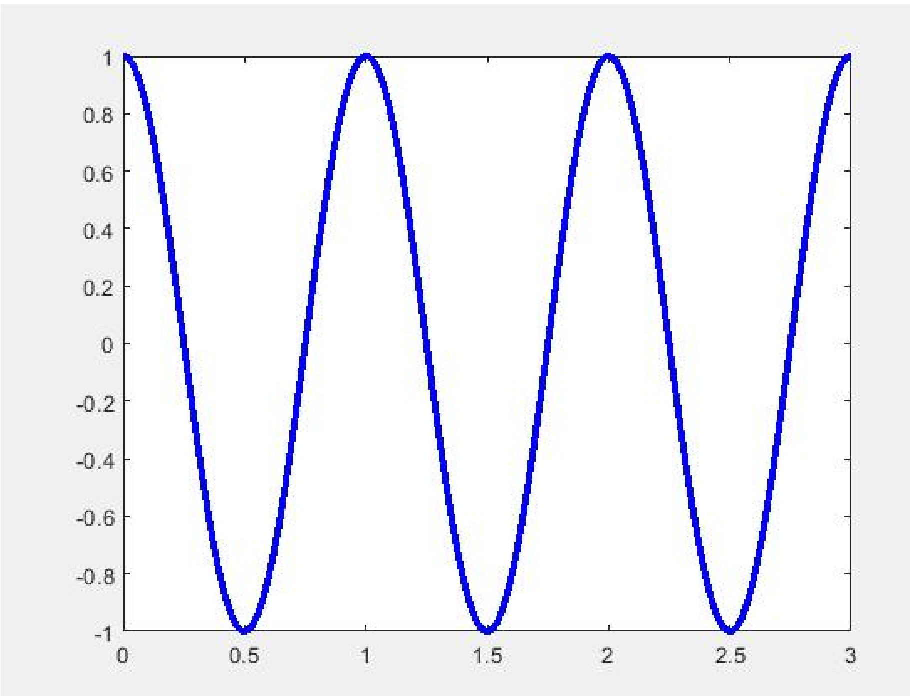
```
function[dUc]=cal_f(Uc,tc,w0c)
dUc=zeros(2,1);
dUc(1)=Uc(2);
dUc(2)=-w0c*Uc(1);
end
```

4.2

```
dt6=0.01;
t6=(0:dt6:3)';
np6=size(t6,1)
q0=1;
dq0=0;
q6=zeros(np6,1);
dq6=zeros(np6,1);
q6(1)=q0;
dq6(1)=dq0;
qj=[q0;dq0];
```

```
for inc=2:np6
    tc=t6(inc-1)
    xc=qj
    k1=cal_f(xc,tc,4*pi^2)
    xc=qj+k1*dt6/2
    k2=cal_f(xc,tc+dt6/2,4*pi^2)
    xc=qj+k2*dt6/2
    k3=cal_f(xc,tc+dt6/2,4*pi^2)
    xc=qj+k3*dt6
    k4=cal_f(xc,tc+dt6,4*pi^2)
    dq=(k1+2*k2+2*k3+k4)/6
    qj=qj+dq*dt6
    q6(inc)=qj(1)
    dq6(inc)=qj(2)
end
E6=0.5*(dq6.^2+4*pi^2*q6.^2)
plot(t6,q6,'b','Linewidth',3)
```

Figure:

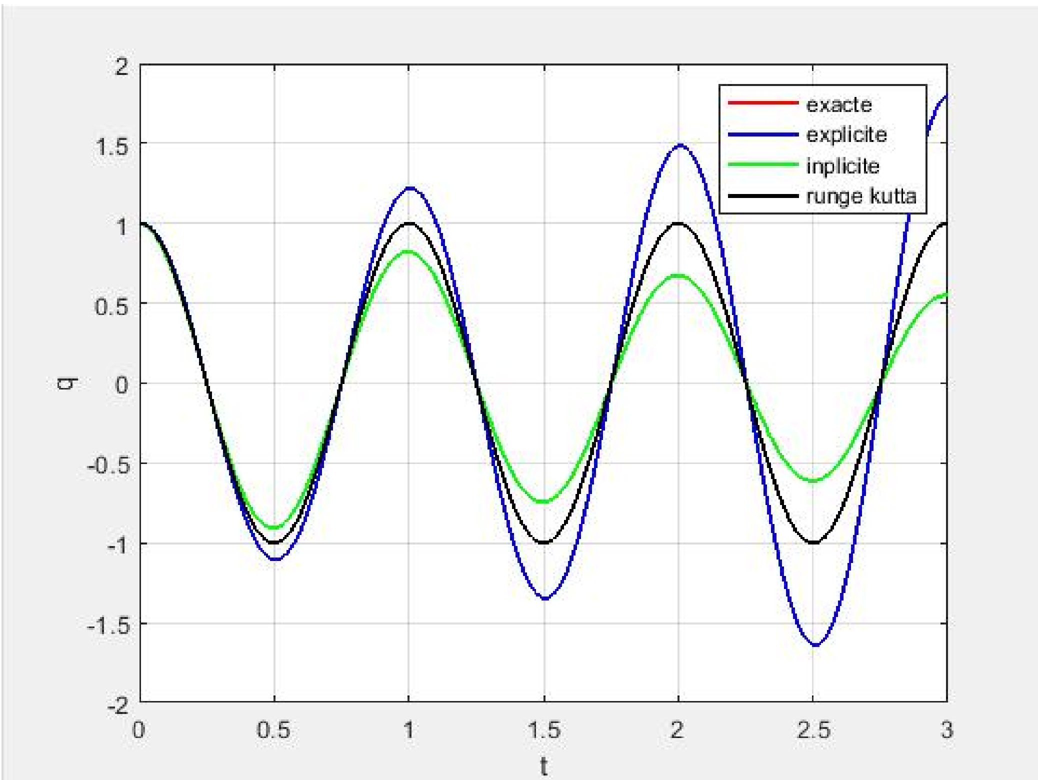


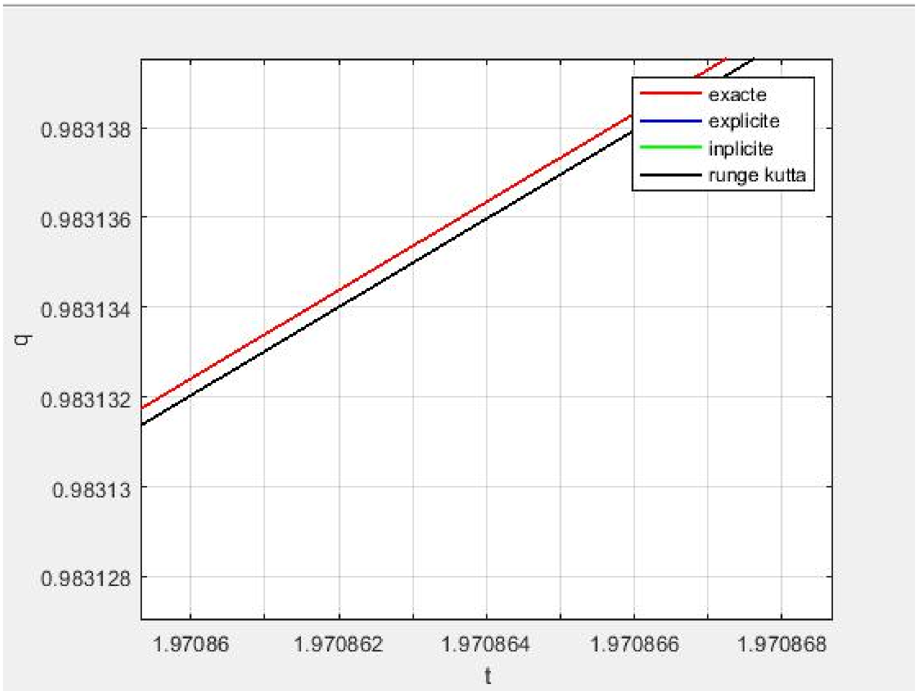
C’est très proche du résultat exacte

4.3

```
dt=0.01;
I0=3;
t=(0:dt:I0)';
np=size(t,1);
q=zeros(np,1);
for inc=1:np
    q(inc)=exp(-pi*t(inc)*2i)/2 + exp(pi*t(inc)*2i)/2
end
plot(t,q,'r','Linewidth',1.5)
hold on
plot(t1,q1b,'b','Linewidth',1.5)%t1 q1 obtenu de l'exercie 2-2
hold on
plot(t2,q2b,'g','Linewidth',1.5)%t2 q2 obtenu de l'exercie 3-1
hold on
plot(t6,q6,'k','Linewidth',1.5)%t6 q6 obtenu de l'exercie 4-2
grid on;
xlabel('t');
ylabel('q');
legend('exacte','explicite','inplicite','runge kutta')
```

Figure:



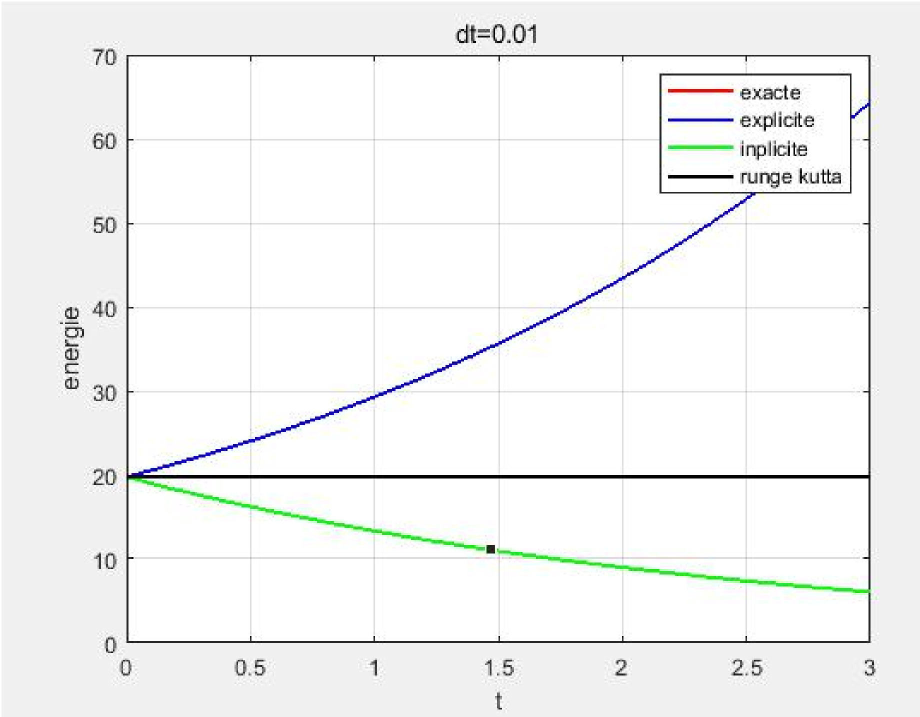


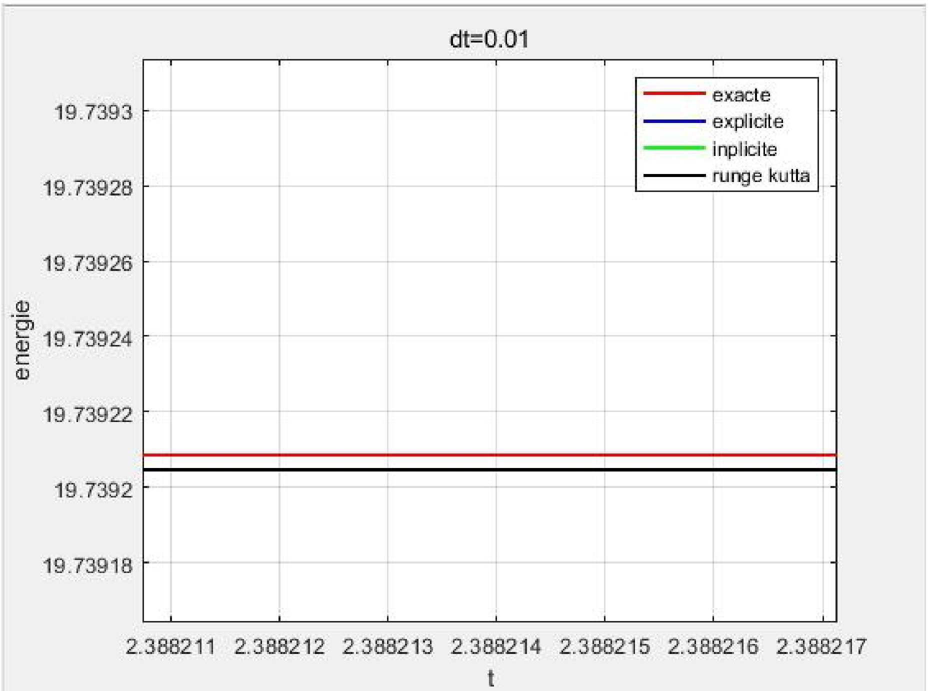
4.4

```
dt=0.01;
T0=3;
t=(0:dt:T0)';
np=size(t,1);
q=zeros(np,1);
energ=zeros(np,1);
for inc=1:np
    q(inc)=exp(-pi*t(inc)*2i)/2 + exp(pi*t(inc)*2i)/2
    energ(inc)=2*pi^2
end

plot(t,energ,'r','Linewidth',1.5)
hold on
plot(t1,energ1,'b','Linewidth',1.5)%t1 energ1 obtenu de l'exercie 2-2
hold on
plot(t2,energ2,'g','Linewidth',1.5)%t2 energ2 obtenu de l'exercie 3-1
grid on;
plot(t6,E6,'k','Linewidth',1.5)%t6 E6 obtenu de l'exercie 4-2
title('dt=0.01')
xlabel('t');
ylabel('energie');
legend('exacte','explicite','inplicite','runge kutta')
```

Figure:


dt=0.01
70
60
50
40
30
20
10
0
energie
0
0.5
1
1.5
2
2.5
3
t
exacte
explicite
inplicite
runge kutta


dt=0.01
19.7393
19.73928
19.73926
19.73924
19.73922
19.7392
19.73918
energie
2.388211
2.388212
2.388213
2.388214
2.388215
2.388216
2.388217
t
exacte
explicite
inplicite
runge kutta