

```
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```

```
omega_0 = 2*pi;
```

```
q0 =1;
```

```
dq0 = 0;
```

```
T0 =3;
```

```
% 1.1
```

```
dsolve('D2q+omega_0^2*q=0','q(0)=1','Dq(0)=0');
```

```
% 1.2
```

```
q_exacte = dsolve('D2q+omega_0^2*q=0','q(0)=1','Dq(0)=0');
```

```
syms t;
```

```
Dq_exacte = diff(q_exacte,t);
```

```
E_etoile = 1/2 * ( Dq_exacte^2 + omega_0^2 * q_exacte^2);
```

```
t=0:0.01:T0;
```

```
plot(t,eval(E_etoile));
```

```
%donc,il est conservatif
```

```
%2.1 pour la première ligne de l'équation (6), selon la première ligne de l'équation (5),
```

```
il est évident.
```

```
%pour la deuxième ligne de l'équation (6), selon l'équation 1, on sait que
```

```
%ddqj = -omega_0*qj,avec la deuxième ligne de l'équation 5, on a bien dqj+1
```

```
%=-omega_0^2*dt*qj+dqj
```

```
%2.2 2.3 2.4
```

```
j=1;
```

```
q(1) = 1;
```

```
dq(1) =0;
```

```
ddq(1)=0;
```

```
Energie(1)=1/2 * (dq(1)^2+omega_0^2 * q(1)^2);
```

```
pas=0.01;
```

```
%pas=0.001;
```

```
for t =0:pas:T0
```

```

q(j+1) = q(j)+pas*dq(j);

dq(j+1) = dq(j) +pas*ddq(j);

ddq(j+1) = -omega_0^2*q(j+1);

Energie(j+1) = 1/2 * (dq(j+1)^2+omega_0^2 * q(j+1)^2);

j=j+1;

end

t=0:pas:T0+pas;

figure(1);

subplot(3,2,1),plot(t,q);title('explicite q');

subplot(3,2,2),plot(t,Energie);title('explicite energy');

%2.5

A = [1 pas; -omega_0^2*pas 1];

[x,y] = eig(A)

%En variant le pas, on trouve que le plus de pas de temps donne le plus

%grand de la partie imaginaire de valeur propre.Avec l'itération de fois la matrix

d'amplification, cela va donne plus

%instable.

%3.1

j=1;

q(1) = 1;

dq(1) =0;

ddq(1)=0;

Energie(1)=1/2 * (dq(1)^2+omega_0^2 * q(1)^2);

pas=0.01;

%pas=0.001;

for t =0:pas:T0

q(j+1) = (q(j)+pas*dq(j))/(1+pas^2*omega_0^2);

ddq(j+1) = -omega_0^2*q(j+1);

dq(j+1) = dq(j) +pas*ddq(j);

```

```
Energie(j+1) = 1/2 * (dq(j+1)^2+omega_0^2 * q(j+1)^2);
```

```
j=j+1;
```

```
end
```

```
t=0:pas:T0+pas;
```

```
subplot(3,2,3),plot(t,q);title('implicite q');
```

```
subplot(3,2,4),plot(t,Energie);title('implicite energy');
```

```
%3.2
```

```
q(1) = 1;
```

```
dq(1) =0;
```

```
Energie=[];
```

```
Energie(1)=1/2 * (dq(1)^2+omega_0^2 * q(1)^2);
```

```
j=1;
```

```
for t=0:pas:T0
```

```
    q(j)=exp(-omega_0*t*1i)/2 + exp(omega_0*t*1i)/2;
```

```
    Energie(j)=1/2 * (dq(j)^2+omega_0^2 * q(j)^2);
```

```
    j=j+1;
```

```
end
```

```
t=0:pas:T0+pas;
```

```
subplot(3,2,5),plot(t,q);title('exacte q');
```

```
t=0:pas:T0;
```

```
subplot(3,2,6),plot(t,Energie);title('exacte energy');
```

```
%3.3
```

```
j=1;
```

```
q(1) = 1;
```

```
dq(1) =0;
```

```
ddq(1)=0;
```

```
Energie(1)=1/2 * (dq(1)^2+omega_0^2 * q(1)^2);
```

```
pas=0.01;
```

```
%pas=0.001;
```

```

[q_exactecompare1,Energie_exactecompare1]=Euler_implicite(0.01);
[q_exactecompare2,Energie_exactecompare2]=Euler_implicite(0.001);
[q_exactecompare3,Energie_exactecompare3]=Euler_implicite(0.0001);
t=0:0.01:T0+0.01;
figure(2);
subplot(3,2,1),plot(t,q_exactecompare1),title('q`exactecompare1');
subplot(3,2,2),plot(t,Energie_exactecompare1),title('Energie`exactecompare1');
t=0:0.001:T0+0.001;
subplot(3,2,3),plot(t,q_exactecompare2),title('q`exactecompare2');
subplot(3,2,4),plot(t,Energie_exactecompare2),title('Energie`exactecompare2');
t=0:0.0001:T0+0.0001;
subplot(3,2,5),plot(t,q_exactecompare3),title('q`exactecompare3');
subplot(3,2,6),plot(t,Energie_exactecompare3),title('Energie`exactecompare3');
%3.5
delta_t=0.01;
[q_implicite_compare1,A_compare1]=q_avecA_implicite(delta_t);
delta_t=0.001;
[q_implicite_compare2,A_compare2]=q_avecA_implicite(delta_t);
delta_t=0.0001;
[q_implicite_compare3,A_compare3]=q_avecA_implicite(delta_t);
figure(3)
t_start=0;
t_end=3;
delta_t=0.01;
t=t_start:delta_t:t_end;
plot(t,q_implicite_compare1);
hold on;
delta_t=0.001;
t=t_start:delta_t:t_end;

```

```

plot(t,q_implicite_compare2);

hold on;

delta_t=0.0001;

t=t_start:delta_t:t_end;

plot(t,q_implicite_compare3);

hold on;

[x1,y1]=eig(A_compare1)

[x2,y2]=eig(A_compare2)

[x3,y3]=eig(A_compare3)

%donc, on a le schema plus stable quand on a delta_t plus petit.

function [q_avecA,A_implicite] = q_avecA_implicite(delta_t)

omega_0=2*pi;

A_implicite=[1/(1+delta_t^2*omega_0^2)

delta_t/(1+delta_t^2*omega_0^2);-delta_t*omega_0^2/(1+delta_t^2*omega_0^2)

1/(1+delta_t^2*omega_0^2)];

i=0;

U=[];

t_start=0;

t_end=3;

t=t_start:delta_t:t_end;

for t_i=t

    if i==0

        U(:,1)=[1;0]

    else

        U(:,i+1)=A_implicite*U(:,i);

    end

    i=i+1;

end

q_avecA=U(1,:);

```

```

end

function [q,Energy] =Euler_implicite(pas)
j=1;
q(1) = 1;
dq(1) =0;
ddq(1)=0;
T0=3;
omega_0=2*pi;
for t =0:pas:T0
    q(j+1) = (q(j)+pas*dq(j))/(1+pas^2*omega_0^2);
    ddq(j+1) = -omega_0^2*q(j+1);
    dq(j+1) = dq(j) +pas*ddq(j);
    Energie(j+1) = 1/2 * (dq(j+1)^2+omega_0^2 * q(j+1)^2);
    j=j+1;
end
Energy=Energie;
end

```

x =

```

0.0000 - 0.1572i    0.0000 + 0.1572i
0.9876 + 0.0000i    0.9876 + 0.0000i

```

y =

```

1.0000 + 0.0628i    0.0000 + 0.0000i
0.0000 + 0.0000i    1.0000 - 0.0628i

```

U =

1

0

U =

1

0

U =

1

0

x1 =

0.0000 - 0.1572i 0.0000 + 0.1572i

0.9876 + 0.0000i 0.9876 + 0.0000i

y1 =

0.9961 + 0.0626i 0.0000 + 0.0000i

0.0000 + 0.0000i 0.9961 - 0.0626i

x2 =

$$\begin{array}{cc} 0.0000 - 0.1572i & 0.0000 + 0.1572i \\ 0.9876 + 0.0000i & 0.9876 + 0.0000i \end{array}$$

y2 =

$$\begin{array}{cc} 1.0000 + 0.0063i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 1.0000 - 0.0063i \end{array}$$

x3 =

$$\begin{array}{cc} 0.0000 - 0.1572i & 0.0000 + 0.1572i \\ 0.9876 + 0.0000i & 0.9876 + 0.0000i \end{array}$$

y3 =

$$\begin{array}{cc} 1.0000 + 0.0006i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 1.0000 - 0.0006i \end{array}$$



