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Devoir 4

Etude d'un oscillateur linéaire amorti à un degré de liberté

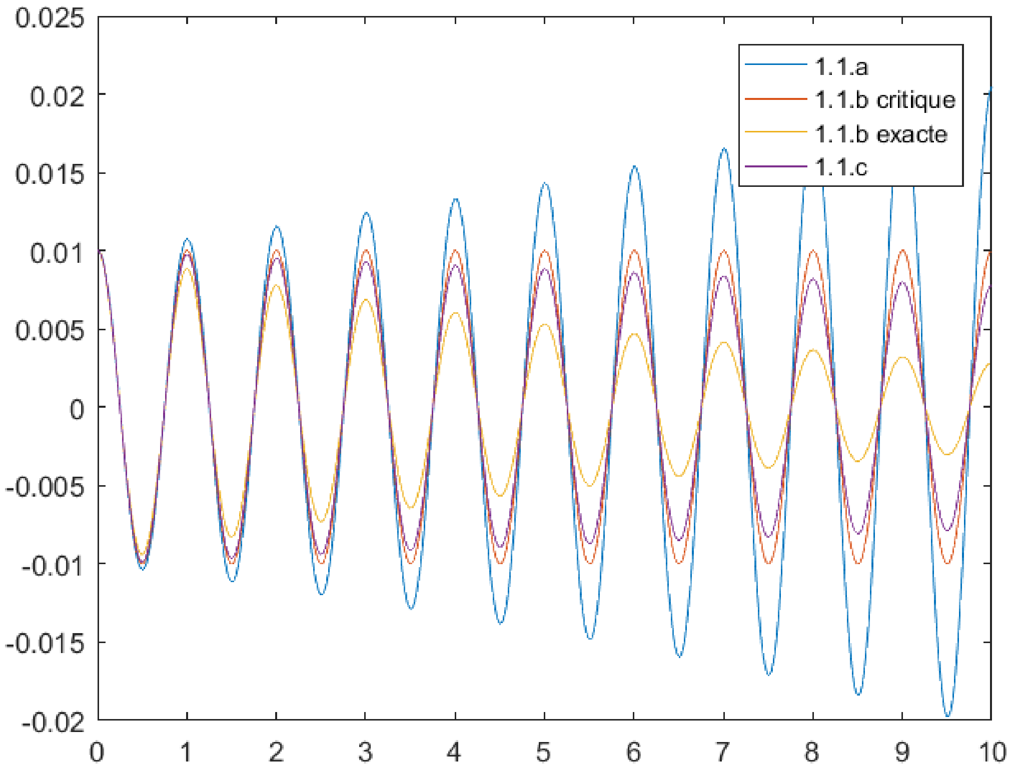
J'ai programmé pour les étudier.

Les codes sont comme suivant.

```
omega_0= 2*pi;
epsilon = 0.02;
t_critique=2*epsilon/omega_0;
T0=1;
%solution exacte
x_exacte=dsolve('D2x+2*omega_0*epsilon*Dx+omega_0^2*x
=0','x(0)=0.01','Dx(0)=0');
%on a t_critique=0.0064
%1.1.a
t_start=0;
t_end=10*T0;
delta_t=0.01;
[x_explicite_plus,A_explicite_plus]=x_avecA_explicite
(delta_t);
t=t_start:delta_t:t_end;
figure(1)
plot(t,x_explicite_plus);
hold on;
%1.1.b
delta_t=t_critique;
[x_explicite_critique,A_explicite_critique]=x_avecA_e
xplicite(delta_t);
t=t_start:delta_t:t_end;
plot(t,x_explicite_critique);
hold on;
plot(t,eval(x_exacte));
%1.1.c
delta_t=0.8*t_critique;
[x_explicite_moin,A_explicite_moin]=x_avecA_explicite
(delta_t);
t=t_start:delta_t:t_end;
plot(t,x_explicite_moin);
```

```
hold on;
```

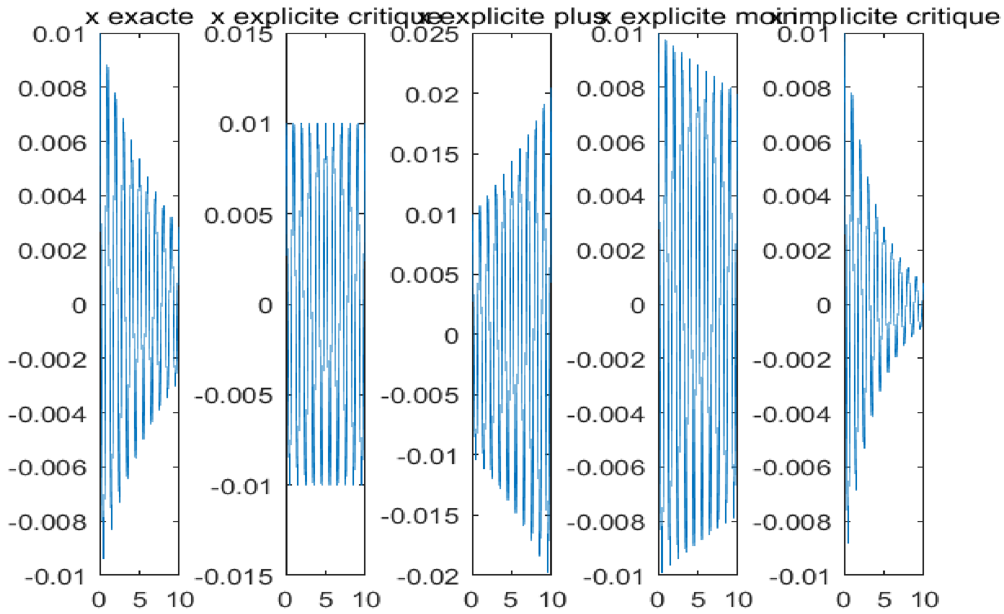
On peut obtenir la figure :



Pour voir plus claire, on a

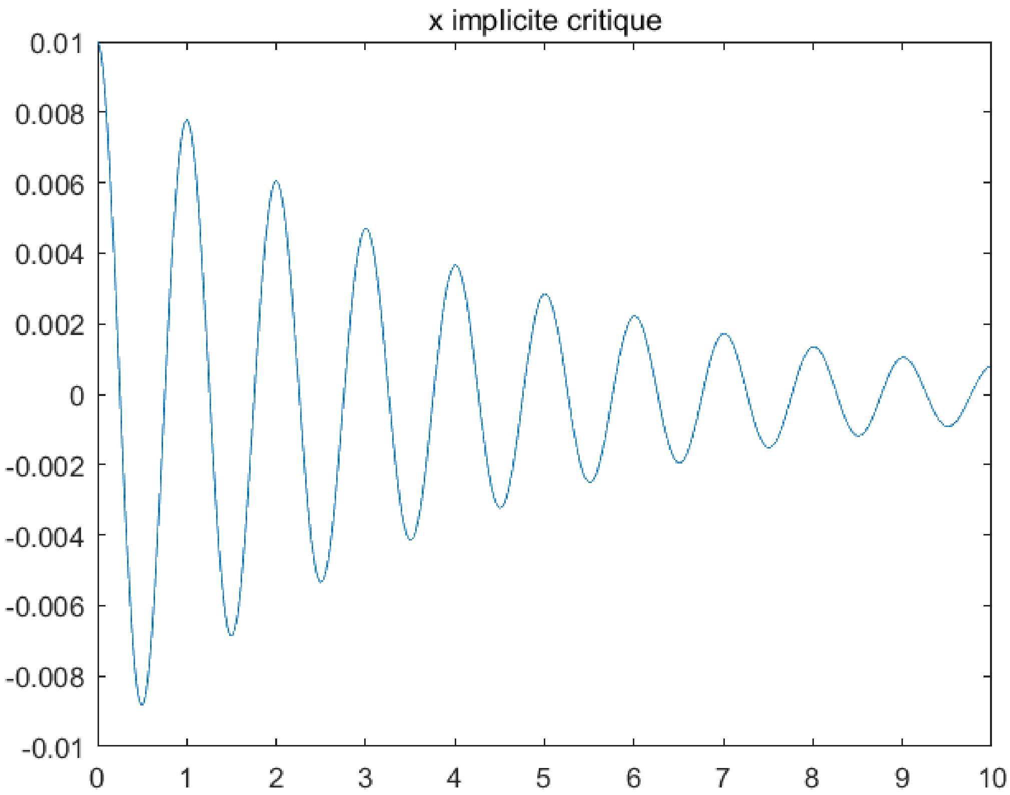
```
figure(2)
subplot(1,5,1),plot(t,eval(x_exacte)),title('x
exacte');
delta_t=t_critique;
t=t_start:delta_t:t_end;
subplot(1,5,2),plot(t,x_explicite_critique),title('x
explicite critique');
delta_t=0.01;
t=t_start:delta_t:t_end;
subplot(1,5,3),plot(t,x_explicite_plus),title('x
explicite plus');
delta_t=0.8*t_critique;
t=t_start:delta_t:t_end;
subplot(1,5,4),plot(t,x_explicite_moin),title('x
```

```
explicite moin');
t=t_start:t_critique:t_end;
[x_implicite_critique,A_implicite_critique]=x_avecA_i
mplicite(t_critique);
subplot(1,5,5),plot(t,x_implicite_critique),title('x
implicite critique');
```



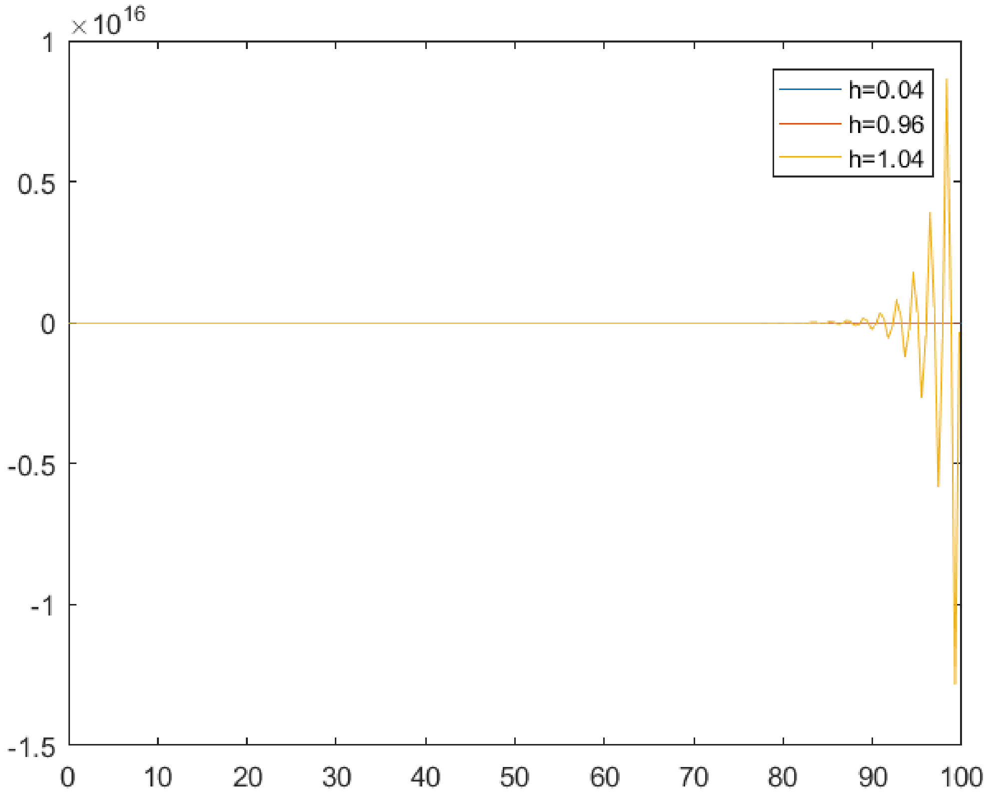
Au début de mon réponse, j'ai déjà le calculé. Ici je prend la valeur et faire un dessin.

```
%1.2
[x_implicite_critique,A_implicite_critique]=x_avecA_i
mplicite(t_critique);
figure(3)
t=t_start:t_critique:t_end;
plot(t,x_implicite_critique),title('x implicite
critique');
```




```
%1.3.a
h=[0.04,0.96,1.04];
delta_t1=h(1)*2*sqrt(2)/omega_0;
delta_t2=h(2)*2*sqrt(2)/omega_0;
delta_t3=h(3)*2*sqrt(2)/omega_0;
[x_rungekutta1,Energie_RungeKutta1]=x_Rungekutta(delt
a_t1);
[x_rungekutta2,Energie_RungeKutta2]=x_Rungekutta(delt
a_t2);
[x_rungekutta3,Energie_RungeKutta3]=x_Rungekutta(delt
a_t3);
figure(4)
t=0:delta_t1:100*T0;
plot(t,x_rungekutta1);
hold on;
t=0:delta_t2:100*T0;
plot(t,x_rungekutta2);
hold on;
t=0:delta_t3:100*T0;
plot(t,x_rungekutta3);
hold on;
legend('h=0.04','h=0.96','h=1.04');
%1.3.b
```

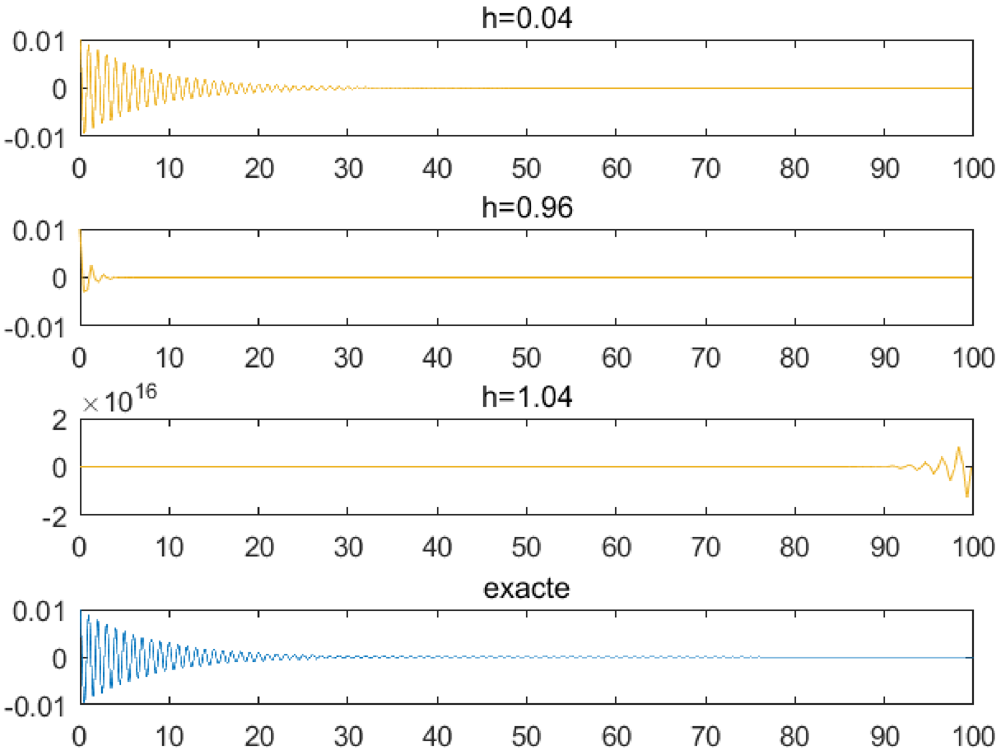
```
[x_rungekutta4,Energie_RungeKutta4]=x_Rungekutta(0.9)
;
figure(5)
t=0:0.9:100*T0;
plot(t,x_rungekutta4);
hold on;
legend('h=0.9');
```



Pour les voir plus claire,

```
figure(6)
t=0:delta_t1:100*T0;
subplot(4,1,1);
plot(t,x_rungekutta1),title('h=0.04');
hold on;
t=0:delta_t2:100*T0;
subplot(4,1,2);
plot(t,x_rungekutta2),title('h=0.96');
hold on;
t=0:delta_t3:100*T0;
subplot(4,1,3);
plot(t,x_rungekutta3),title('h=1.04');
hold on;
subplot(4,1,4);
```

```
t=t_start:t_critique:100;
plot(t,eval(x_exacte)),title('exacte');
```



On a h=0.04,0.96 stable et h=1.04 pas stable, h=0.04 est plus précis.

Je devine $\Delta t_c = \frac{2\varepsilon}{\omega_0}$

```
function [dU_c] = cal_f(U_c,t_c,omega_0_c)
epsilon=0.02;
dU_c=zeros(2,1);
dU_c(1)=U_c(2);
dU_c(2)=-omega_0_c^2*U_c(1)-
2*epsilon*omega_0_c*U_c(2);
end
function
[x_rungekutta,Energie_RungeKutta]=x_Rungekutta(delta_
t)
dt6=delta_t;
T0=1;
t6=(0:dt6:100*T0)';
```

```
np6=size(t6,1);
x0=0.01;
dx0=0;
x6=zeros(np6,1);
dx6=zeros(np6,1);
x6(1)=x0;
dx6(1)=dx0;
xj=[x0;dx0];
omega_0=2*pi;
for index=2:np6
    t_c=t6(index-1);
    x_c=xj;
    k1=cal_f(x_c,t_c,omega_0);
    x_c=xj+k1*dt6/2;
    k2=cal_f(x_c,t_c+dt6/2,omega_0);
    x_c=xj+k2*dt6/2;
    k3=cal_f(x_c,t_c+dt6/2,omega_0);
    x_c=xj+k3*dt6;
    k4=cal_f(x_c,t_c+dt6,omega_0);
    dx=(k1+2*k2+2*k3+k4)/6;
    xj=xj+dx*dt6;
    x6(index)=xj(1);
    dx6(index)=xj(2);
    Energie_RungeKutta(index)=1/2 *
(dx6(index)^2+omega_0^2 * x6(index)^2);
end
x_rungekutta=x6;
end
function [x_implicite,A_implicite]
=x_avecA_implicite(delta_t)
omega_0=2*pi;
epsilon=0.02;
T0=1;
A_implicite_inv=[1 -delta_t;omega_0^2*delta_t
1+2*epsilon*omega_0*delta_t];
U=[];
i=0;
t_start=0;
t_end=10*T0;
t=t_start:delta_t:t_end;
for t_i=t
    if i==0
        U(:,1)=[0.01;0];
    else
```

```
        U(:,i+1)=A_implicite_inv\U(:,i);
    end
    i=i+1;
end
x_implicite=U(1,:);
A_implicite=inv(A_implicite_inv);
end

function [x_explicite,A_explicite] =
x_avecA_explicite(delta_t)
omega_0= 2*pi;
epsilon = 0.02;
T0=1;
A_explicite=[1 delta_t;-omega_0^2*delta_t 1-
2*epsilon*omega_0*delta_t];
i=0;
U=[];
t_start=0;
t_end=10*T0;
t=t_start:delta_t:t_end;
for t_i=t
    if i==0
        U(:,1)=[0.01;0];
    else
        U(:,i+1)=A_explicite*U(:,i);
    end
    i=i+1;
end
x_explicite=U(1,:);
end
```

Etude d'un double pendule avec l'hypothèse des petits mouvements

1.On peut écrire la matrice d'amplification par calculer B et C d'abord.

```
clear all;
m=2;
```

```
a=0.5;
g=9.81;
F0=20;
T0=8;
syms t;
beta=0;
gamma =0.5;
syms delta_t;
omega=2*pi;
theta1_0=0;
theta2_0=0;
q0=[theta1_0;theta2_0];
dtheta1_0=-1.31519275;
dtheta2_0=-1.85996342;
grand_M=m*a^2*[2 1;1 1];
grand_K=m*g*a*[2 0;0 1];
grand_F=F0*sin(omega*t)*[a;a/sqrt(2)];
%grand_B = zeros(4,4);
%grand_C = zeros(4,4);
syms grand_B;
syms grand_C;
grand_B(1:2,1:2) =
eye(2)+beta*delta_t^2*inv(grand_M)*grand_K;
grand_B(1:2,3:4) = 0;
grand_B(3:4,1:2) =
gamma*delta_t*eye(2)*inv(grand_M)*grand_K;
grand_B(3:4,3:4) = eye(2);
grand_C(1:2,1:2) = eye(2)-delta_t^2*(0.5-
beta)*inv(grand_M)*grand_K;
grand_C(1:2,3:4) = delta_t*eye(2);
grand_C(3:4,1:2) = -delta_t*(1-
gamma)*inv(grand_M)*grand_K;
grand_C(3:4,3:4) = eye(2);
grand_A=grand_B/grand_C;
```

on obtient

A =

[-(962361*delta_t^4 -

20000)/(962361*delta_t^4 + 392400*delta_t^2 + 20000), (196200*delta_t^2)/(962361*delta_t^4 + 392400*delta_t^2 + 20000), (200*delta_t*(981*delta_t^2 + 100))/(962361*delta_t^4 + 392400*delta_t^2 + 20000), (98100*delta_t^3)/(962361*delta_t^4 + 392400*delta_t^2 + 20000)]

[(392400*delta_t^2)/(962361*delta_t^4 + 392400*delta_t^2 + 20000), -(962361*delta_t^4 - 20000)/(962361*delta_t^4 + 392400*delta_t^2 + 20000), (196200*delta_t^3)/(962361*delta_t^4 + 392400*delta_t^2 + 20000), (200*delta_t*(981*delta_t^2 + 100))/(962361*delta_t^4 + 392400*delta_t^2 + 20000)]

[-(3924*delta_t*(981*delta_t^2 + 200))/(962361*delta_t^4 + 392400*delta_t^2 + 20000), (392400*delta_t)/(962361*delta_t^4 + 392400*delta_t^2 + 20000), -(962361*delta_t^4 - 20000)/(962361*delta_t^4 + 392400*delta_t^2 + 20000), (196200*delta_t^2)/(962361*delta_t^4 + 392400*delta_t^2 + 20000)]

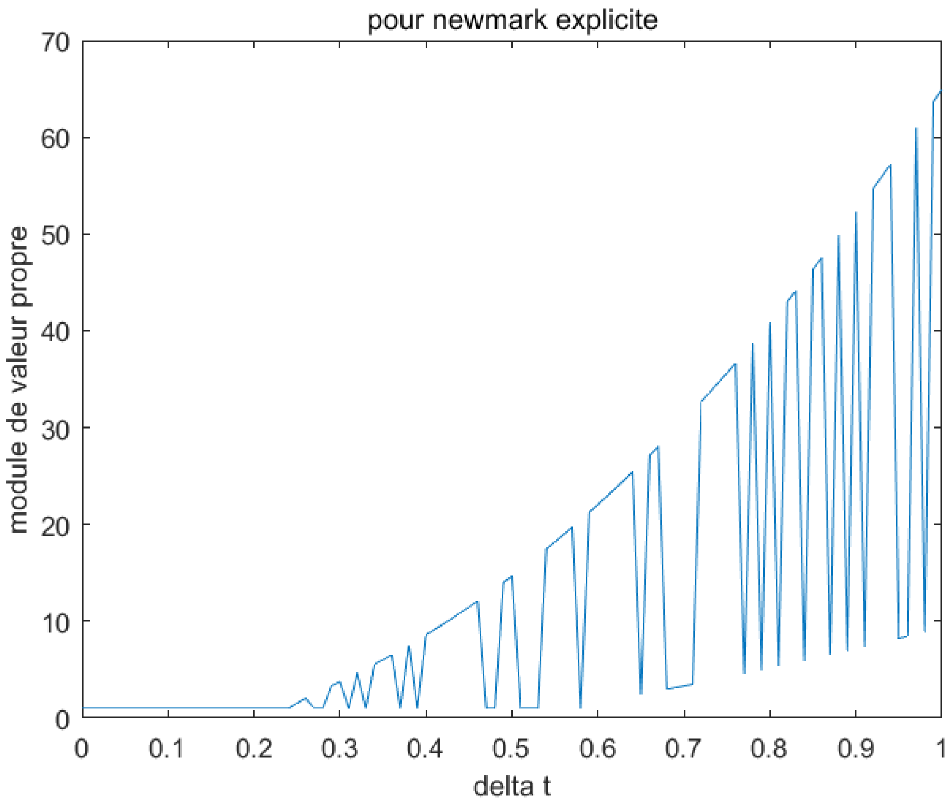
[(784800*delta_t)/(962361*delta_t^4 + 392400*delta_t^2 + 20000), -(3924*(981*delta_t^3 + 200*delta_t))/(962361*delta_t^4 + 392400*delta_t^2 +

20000),

(392400*delta_t^2)/(962361*delta_t^4 + 392400*delta_t^2 + 20000), -(962361*delta_t^4 - 20000)/(962361*delta_t^4 + 392400*delta_t^2 + 20000)]

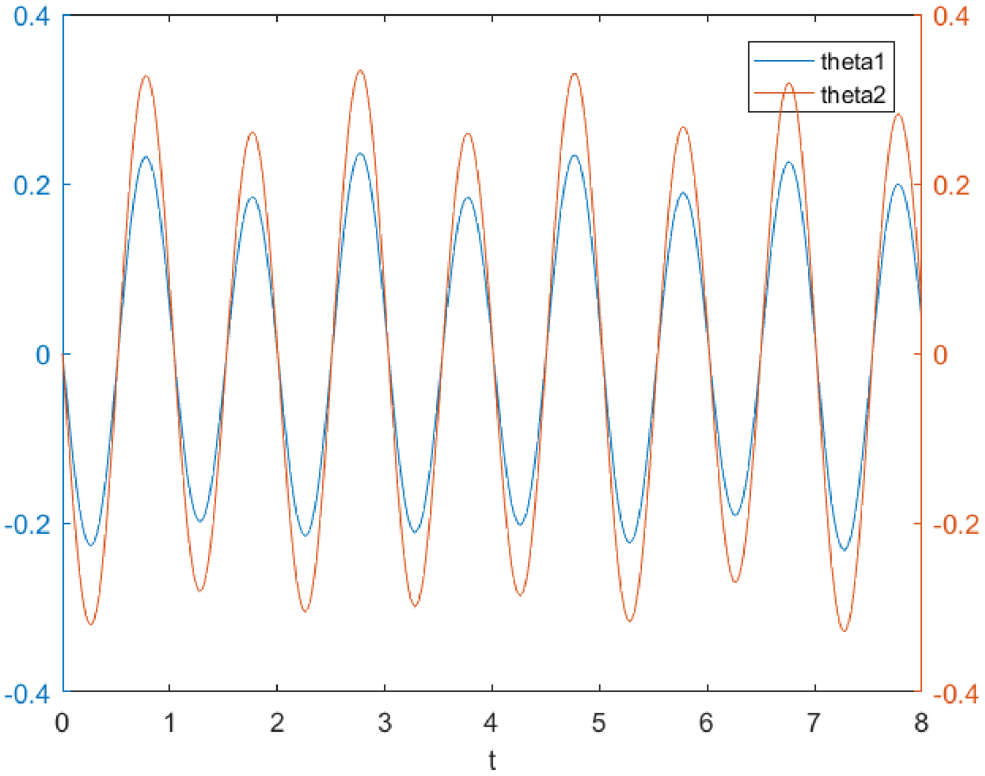
pour trouver le pas de temps critique,

```
i=1;
for delta_ti=0:0.01:1
[vec,Valeur_propre]=eig(subs(grand_A,delta_t,delta_ti
));
module(i)=abs(Valeur_propre(1,1));
i=i+1;
end
delta_ti=0:0.01:1;
figure(1);
plot(delta_ti,module);
xlabel('delta t');
ylabel('module de valeur propre');
title('pour newmark explicite');
```




```
%1.3
ddq0=inv(grand_M)*(grand_F-grand_K*q0);
ddq0=[0;0];
%1.4
dq0=[dtheta1_0;dtheta2_0];
j=1
delta_t=0.02;
q=[];
q(:,1)=q0;
dq=[];
dq(:,1)=dq0;
for t_i=0:delta_t:T0

q(:,j+1)=q(:,j)+delta_t*dq(:,j)+0.5*delta_t^2*ddq0;
    ddq=inv(grand_M)*(subs(grand_F,t,t_i)-
grand_K*q(:,j+1));
    dq(:,j+1)=dq(:,j)+0.5*delta_t*(ddq0+ddq);
    ddq0=ddq;
    j=j+1;
end
q(:,j)=[];
figure(2);
t=0:delta_t:T0;
```

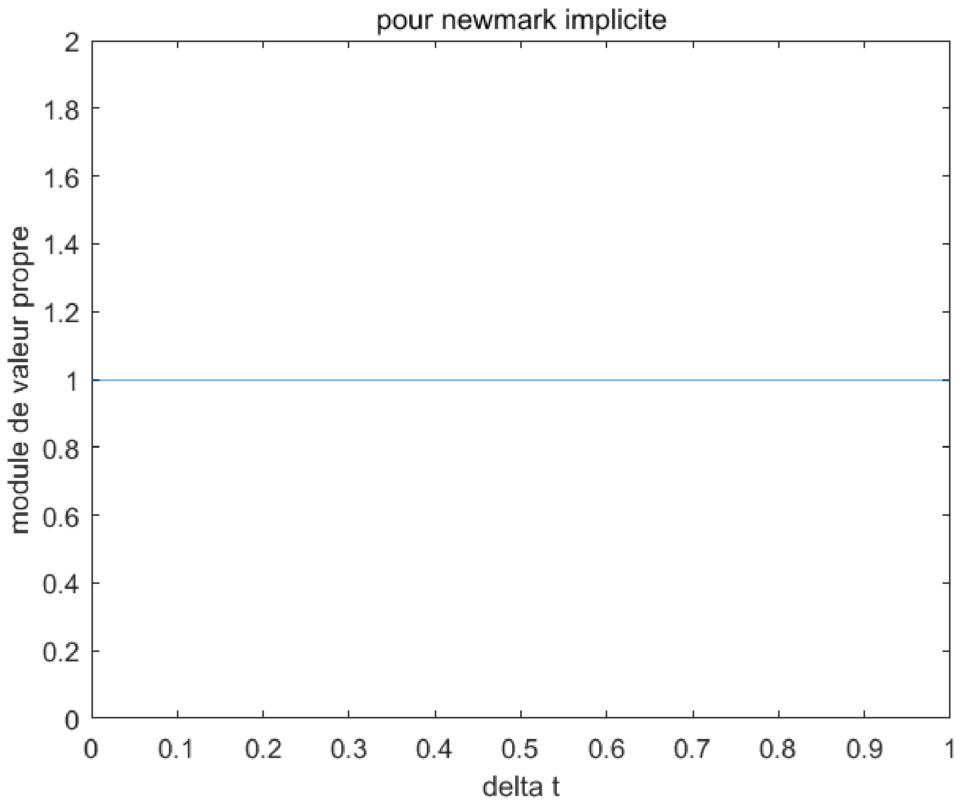
```
plotyy(t,q(1,:),t,q(2,:));
legend('theta1','theta2')
xlabel('t');
```




```
%1.6
q(1,1);
q(2,1);
q(1,2);
q(2,2);
q(1,3);
q(2,3);
q(1,26);
q(2,26);

ans =

     0


ans =

     0
```

```
ans =

   -0.0263


ans =

   -0.0372


ans =

   -0.0525


ans =

   -0.0742


ans =

   -0.0242
%2.1

syms t;
beta=0.25;
gamma =0.5;
syms delta_t;
omega=2*pi;
theta1_0=0;
theta2_0=0;
q0=[theta1_0;theta2_0];
dtheta1_0=-1.31519275;
dtheta2_0=-1.85996342;
grand_M=m*a^2*[2 1;1 1];
grand_K=m*g*a*[2 0;0 1];
grand_F=F0*sin(omega*t)*[a;a/sqrt(2)];
%grand_B = zeros(4,4);
%grand_C = zeros(4,4);
syms grand_B;
syms grand_C;
grand_B(1:2,1:2) =
eye(2)+beta*delta_t^2*inv(grand_M)*grand_K;
```

```
grand_B(1:2,3:4) = 0;
grand_B(3:4,1:2) =
gamma*delta_t*eye(2)*inv(grand_M)*grand_K;
grand_B(3:4,3:4) = eye(2);
grand_C(1:2,1:2) = eye(2)-delta_t^2*(0.5-
beta)*inv(grand_M)*grand_K;
grand_C(1:2,3:4) = delta_t*eye(2);
grand_C(3:4,1:2) = -delta_t*(1-
gamma)*inv(grand_M)*grand_K;
grand_C(3:4,3:4) = eye(2);
grand_A_implicite=grand_B/grand_C;
i=1;
%2.2
for delta_ti=0:0.01:1
[vec,Valeur_propre]=eig(subs(grand_A_implicite,delta_
t,delta_ti));
module(i)=abs(Valeur_propre(1,1));
i=i+1;
end
delta_ti=0:0.01:1;
figure(3);
plot(delta_ti,module);
xlabel('delta t');
ylabel('module de valeur propre');
title('pour newmark implicite');
```



On peut trouver que n'importe delta t, module de valeur propre est 1.

2.4

Avant j'ai déjà écrit.

2.5 2.6

```
clear all
syms delta_t
global F0 omega a;
m = 2;
a = 0.5;
g = 9.81;
M = m*a^2*[2 1; 1 1];
K = m*g*a*[2 0; 0 1];

omega = 2*pi;
F0 = 20;
beta = 0.25
gamma = 0.5
```

```
B(1:2,1:2) = eye(2)+beta*delta_t^2*inv(M)*K;
B(1:2,3:4) = 0;
B(3:4,1:2) = gamma*delta_t*inv(M)*K;
B(3:4,3:4) = eye(2);

C(1:2,1:2) = eye(2)-delta_t^2*(0.5-beta)*inv(M)*K;
C(1:2,3:4) = delta_t*eye(2);
C(3:4,1:2) = -delta_t*(1-gamma)*inv(M)*K;
C(3:4,3:4) = eye(2);
A = B\C
delta_t = 0.02;
t = 0:delta_t:8;
B_num = eval(B);
C_num = eval(C);
A_num = eval(A);
global M K q0 dq0 ddq0;
q0 = [0;0]
dq0 = [-1.31519275;-1.85996342]
ddq0 = inv(M)*F(0) - inv(M)*K*q0
i = 0;
U = [];
ddq=[];
for t_i=t
    if i==0
        U(:,1) = [q0;dq0];
        ddq(:,1) = ddq0;
    else
        t_i_1 = t_i - delta_t;
        U(:,i+1) =
A_num*U(:,i)+inv(B_num)*[beta*delta_t^2*inv(M)*F(t_i)
+delta_t^2*(0.5-beta)*F(t_i);
gamma*delta_t*inv(M)*F(t_i_1)+delta_t*(1-
gamma)*F(t_i_1)];
        ddq(:,i+1) = inv(M)*F(t_i)-
inv(M)*K*U(1:2,i+1);
    end
    i=i+1;
end
q=U(1:2,:);
dq=U(3:4,:);
q(:,1)
dq(:,1)
q(:,2)
dq(:,2)
```

```
q(:,3)
dq(:,3)
q(:,(0.5/0.02+1))
dq(:,(0.5/0.02+1))
function [F1] = F(t)
    global F0 omega a;
    F1 = F0 * sin(omega*t) * [a;a/sqrt(2)];
end
ans =

     0
     0
ans =

   -1.3152
   -1.8600
ans =

   -0.0261
   -0.0370
ans =

   -1.3122
   -1.8557
ans =

   -0.0518
   -0.0736
ans =

   -1.2835
   -1.8236
ans =

    0.0679
    0.0136
ans =

    1.4235
    2.2889
```

Oscillateur non linéaire à un degré de liberté

1.1

On a

Equation (2)

$$\ddot{q} + {\omega_0}^2 q(1 + aq^2) = 0$$

Equation (4)

$$q_{j+1} = q_j + \Delta t\dot{q}_J + \Delta t^2(0.5 - \beta)\ddot{q}_J + \beta\Delta t^2\ddot{q}_{j+1}$$

Equation (5)

$$\dot{q}_{j+1} = \dot{q}_j + \Delta t(1 - \gamma)\ddot{q}_j + \gamma\Delta t\ddot{q}_{j+1}$$

Avec

$$\gamma = 0.5$$

$$\beta = 0$$

On peut écrire la relation comme

$$q_{j+1} = q_j + \Delta t\dot{q}_J + \frac{1}{2}\Delta t^2 \ddot{q_{j+1}}$$

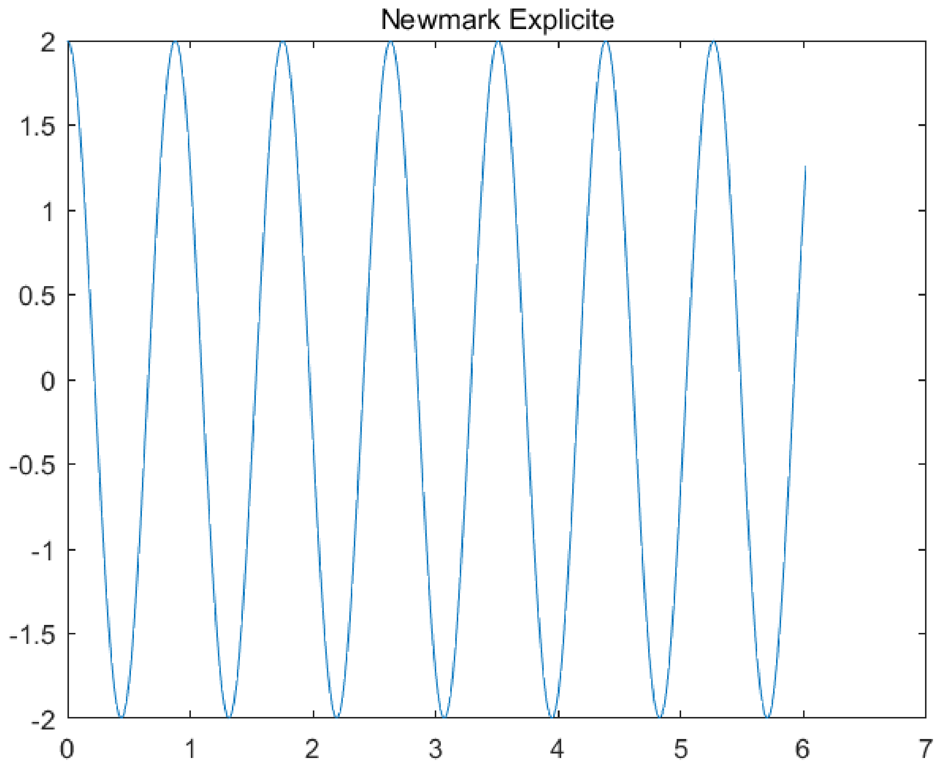
$$\dot{q}_{j+1} = \dot{q}_j + \frac{1}{2}\Delta t\ddot{q}_j + \frac{1}{2}\Delta t\ddot{q}_{j+1}$$

$$\ddot{q}_{j+1} = -{\omega_0}^2 q_{j+1}\left(1 + a{q_{j+1}}^2\right)$$

1.2

```
q0=2;
dq0=0;
q(1)=q0;
dq(1)=dq0;
omega_0=2*pi;
a=0.1;
delta_t=0.02;
T0=6;
```

```
ddq0c=-omega_0^2*q0*(1+a*q0^2);
j=1;
for t=0:delta_t:T0

energie_etoile_explicite(j)=0.5*dq(j)^2+0.5*omega_0^2
*q(j)^2;
    q(j+1)=q(j)+delta_t*dq(j)+0.5*delta_t^2*ddq0c;
    ddqc=-omega_0^2*q(j+1)*(1+a*q(j+1)^2);
    dq(j+1)=dq(j)+0.5*delta_t*(ddq0c+ddqc);
    ddq0c=ddqc;
    j=j+1;
end
t=0:delta_t:T0+delta_t;
figure(1)
plot(t,q),title('Newmark Explicite');
```



1.3

t=0s q=2

t=0.02s q=1.9779

t=0.04s q=1.9123

t=T0 q=1.0329

2.1

On doit chercher à minimiser le résidu.

2.2

Selon le cour, on a $\Delta\ddot{q}_{n+1} = -\frac{f(\ddot{q}_{n+1}^{*}, \dot{q}_{n+1}^{*}, q_{n+1}^{*})}{\frac{\partial f}{\partial \ddot{q}_{n+1}^{*}} + \frac{\partial f}{\partial q_{n+1}^{*}}\beta\Delta t^2}$

Donc $\Delta\ddot{q}_{n+1} = -\frac{\ddot{q}_{n+1}^{*} + \omega_0^2 q_{n+1}^{*}(1 + a{q_{n+1}^{*}}^2)}{0.25dt^2\omega_0^2\left(1 + 3a{q_{n+1}^{*}}^2\right) + 1}$

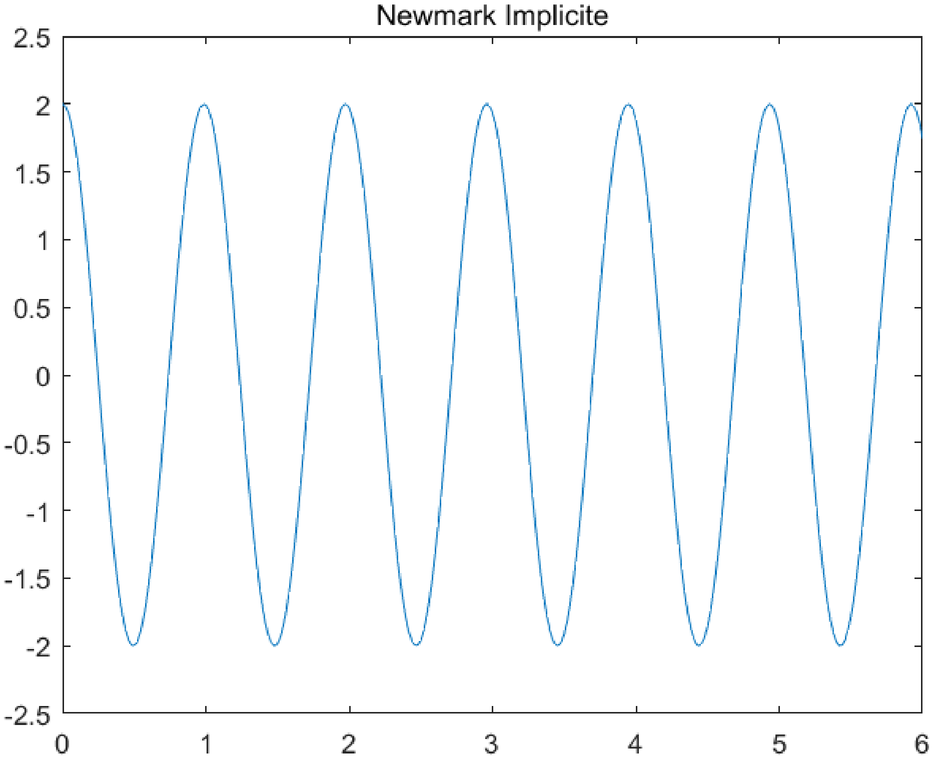
$$\Delta q_{n+1} = 0.25dt^2\Delta\ddot{q}_{n+1}$$

$$\Delta\dot{q}_{n+1} = 0.5dt\Delta\ddot{q}_{n+1}$$

2.3

```
omega_0=2*pi;
a=0.01;
T0=6;
q(1) = 2;
dq(1) = 0;
ddq(1) = -omega_0^2*q(1)*(1+a*q(1)^2);
delta_t = 0.02;
for j=1:300
    ddq(j+1)=0;
    dq(j+1)=dq(j)+delta_t*0.5*ddq(j);
    q(j+1)=q(j)+delta_t*dq(j)+delta_t^2*0.25*ddq(j);
    d2q_test=-omega_0^2*q(j+1)*(1+a*q(j+1)^2);
    while(abs(ddq(j+1)-d2q_test)>0.0001)
        ddq(j+1)=d2q_test;
        dq(j+1)=dq(j)+delta_t*0.5*(ddq(j)+ddq(j+1));

q(j+1)=q(j)+delta_t*dq(j)+delta_t^2*0.25*(ddq(j)+ddq(
j+1));
        d2q_test=-omega_0^2*q(j+1)*(1+a*q(j+1)^2);
    end
end

figure(2)
t=0:delta_t:T0;
```

```
plot(t,q),title('Newmark Implicite');
```



2.4

t=0s q=2

t=0.02s q= 1.9836

t=0.04s q=1.9349

t=T0 q=1.7446

3.1

Pour définie l'énergie mécanique pour cet oscillateur non linéaire, on peut diviser l'énergie en partie cinétique et partie potentielle. Donc, la partie cinétique est $\frac{1}{2}m\dot{q}^2$ et la partie potentielle est $\frac{1}{2}kq^2 + \frac{1}{4}kaq^4$. Pour les calculassions simples, on prend E_étoile par diviser les deux parties par m.

3.2

Comme avant,

```
energie_etoile_explicite(j)=0.5*dq(j)^2+0.5*omega_0^2
*q(j)^2+0.25*omega_0^2*a*q(j)^4;
```

Par ailleurs,

```
for i=1:301

energie_etoile_implicite(i)=0.5*dq(i)^2+0.5*omega_0^2
*q(i)^2+0.25*omega_0^2*a*q(i)^4;
end
```

Enfin on peut obtenir les énergies.

3.3

```
figure(3)
plot(t,energie_etoile_explicite);
hold on;
plot(t,energie_etoile_implicite);
legend('explicite','implicite');
```

Les énergies ne sont pas les mêmes.