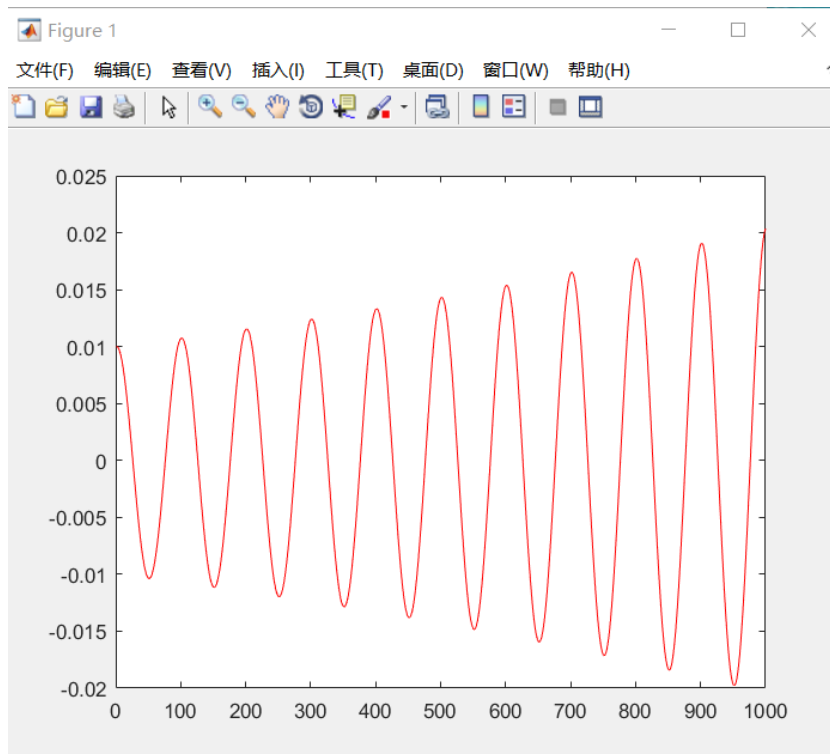


Etude d'un oscillateur linéaire amorti à un degré de liberté

$$\begin{bmatrix} x_{n+1} \\ \dot{x}_{n+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ -w_0^2 \Delta t & 1 - 2\Delta t \epsilon w_0 \end{bmatrix} \begin{bmatrix} x_n \\ \dot{x}_n \end{bmatrix}$$

1.1.a

```
clear all;
w0=2*pi;
x0=0.01;
dx0=0;
T0=1;
n=1000;
dt=10*T0/n;%dt>2*epsilon/2pi=6.4*10^-3,ici,dt=0.01s
epsilon=0.02;
A=[1 dt;-w0^2*dt 1-2*dt*epsilon*w0];
[X1,A1]=eig(A);
U(:,1)=[x0;dx0];
for i=1:n-1
    U(:,i+1)=A*U(:,i);
    E(i)=1/2*((U(2,i))^2+w0^2*(U(1,i))^2);
end
plot(1:n,U(1,:), 'r-')
```



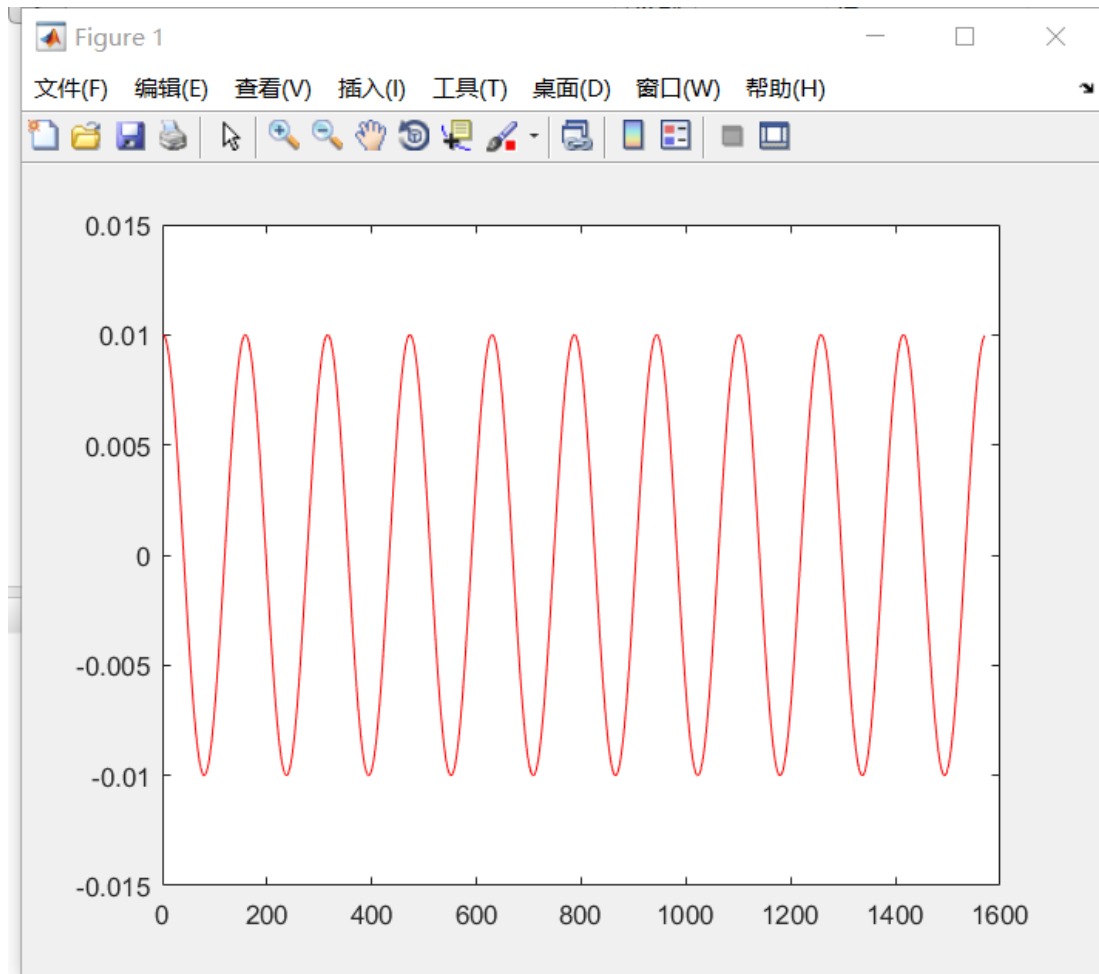
il diverge.

1.1.b

```
w0=2*pi;
x0=0.01;
dx0=0;
```

```
epsilon=0.02;  
dt=2*epsilon/w0  
n=floor(500*pi)
```

.....



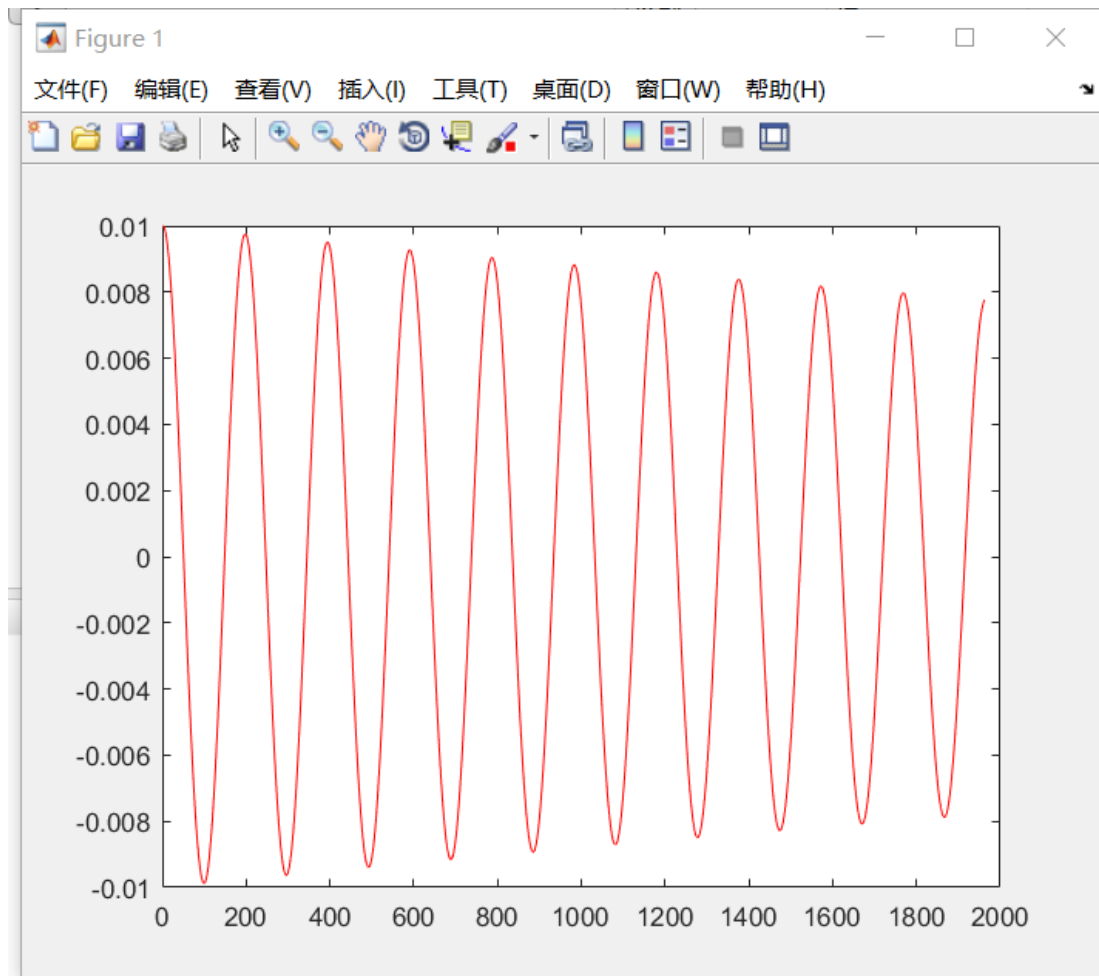
ni diverge ni converge

1.1.c

.....

```
dt=0.8*2*epsilon/w0  
n=floor(625*pi)
```

.....



it converge.

1.1.d

$$|1 - \varepsilon w_0 \Delta t \pm i w_0 \Delta t (1 - \varepsilon^2)^{0.5}| < 1$$

$$\frac{\Delta t}{\frac{2\varepsilon}{w_0}} < 1$$

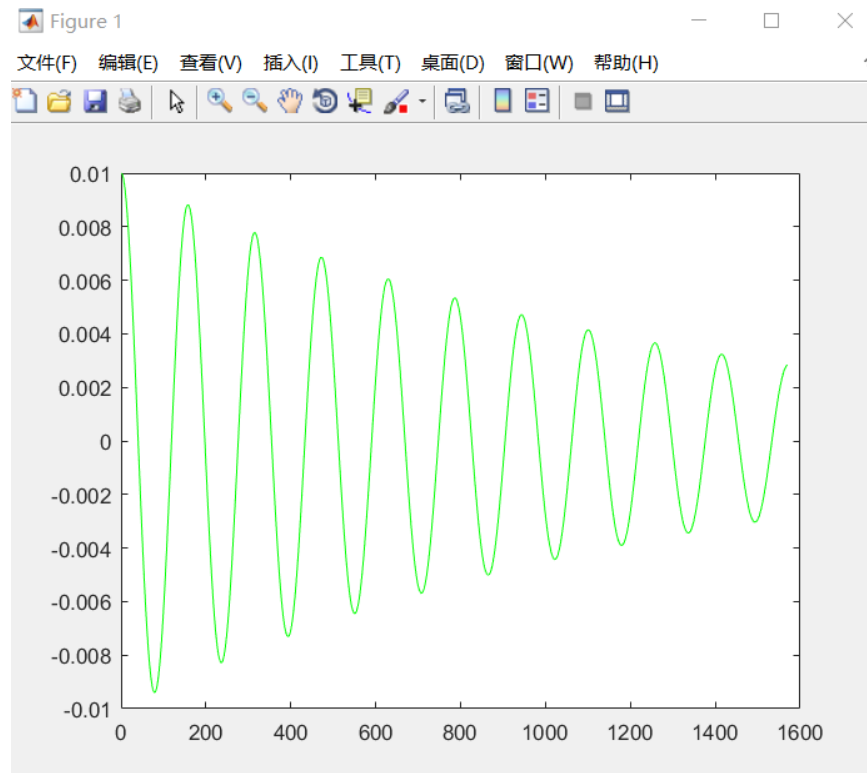
1.2

```
clear all
w0=2*pi;
x0=0.01;
dx0=0;
epsilon=0.02;
dt=2*epsilon/w0;
n=10/dt;
A=1/(1+(w0*dt)^2)*[1 dt;-w0^2*dt 1];
[X1,A1]=eig(A);
U(:,1)=[x0;dx0];
for i=1:n-1
    U(:,i+1)=A*U(:,i);
    E2(i)=1/2*((U(2,i))^2+w0^2*(U(1,i))^2);
```

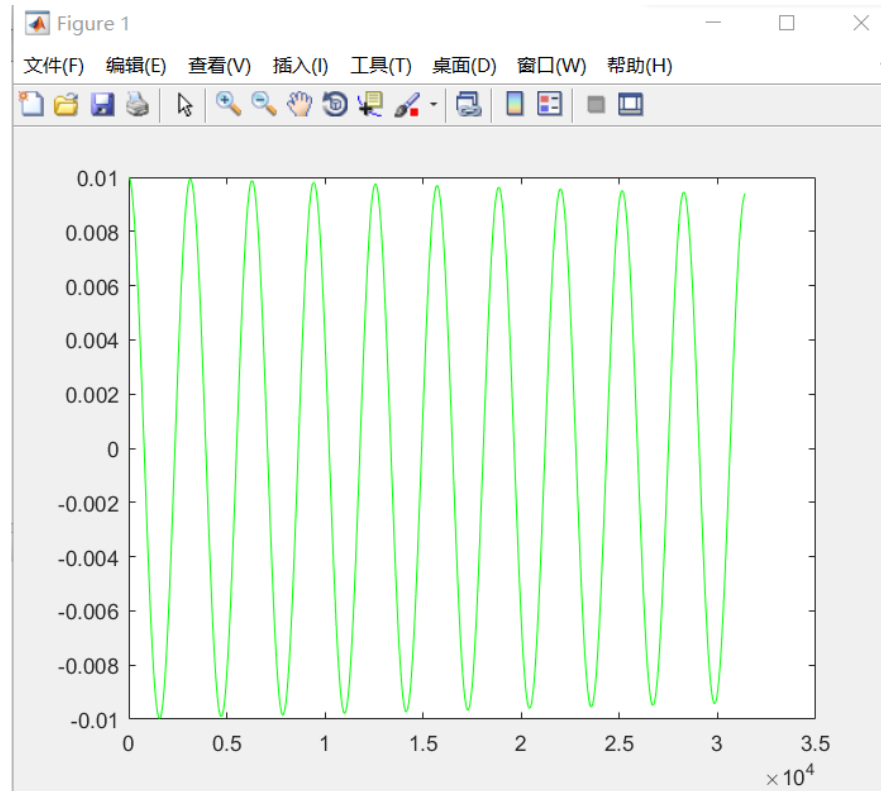
end

```
plot(1:n,U(1,:), 'g-')
```

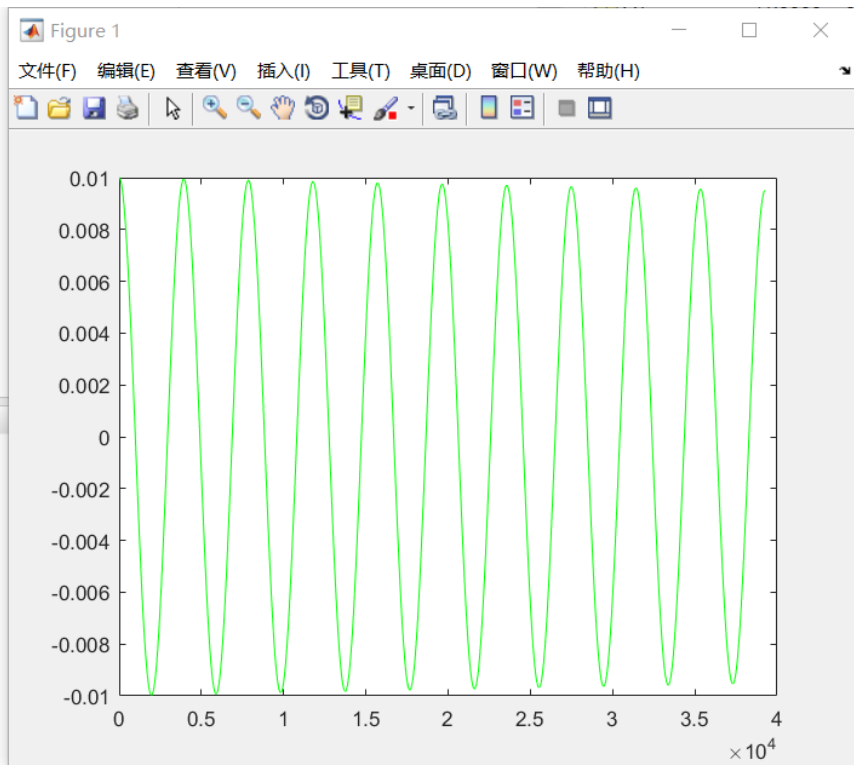
$dt=2*\epsilon/w_0$



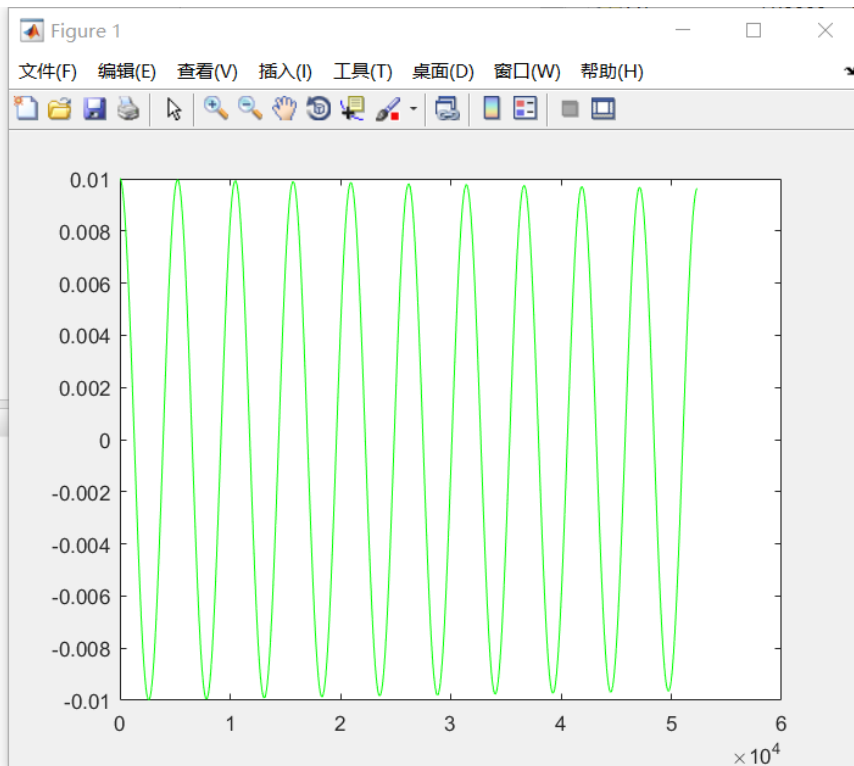
$dt=0.05*2*\epsilon/w_0$



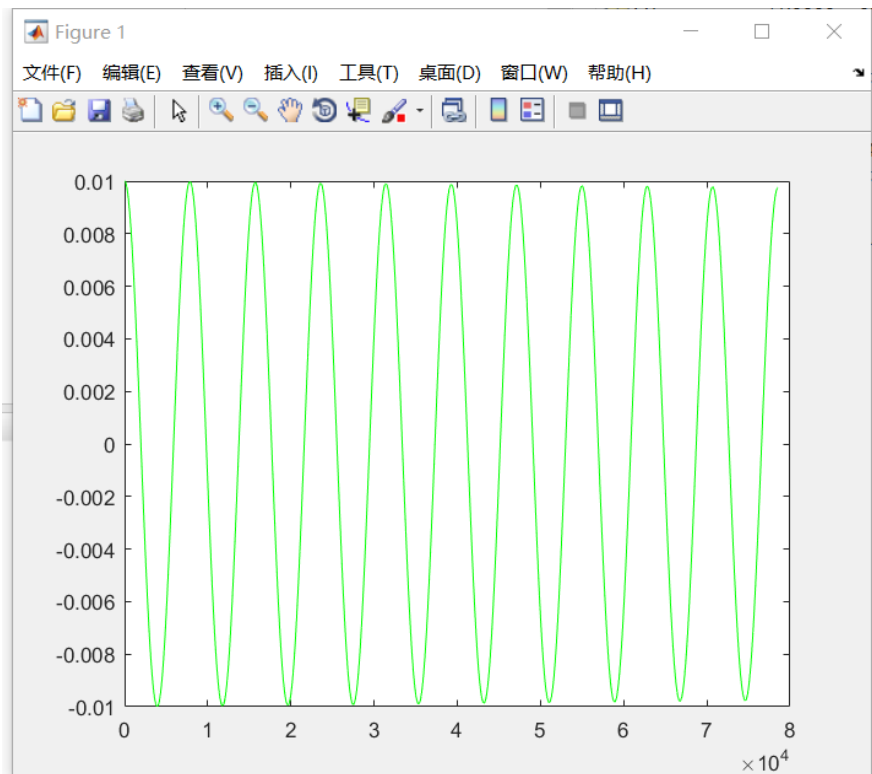
$dt=0.04*2*\epsilon/w_0$



$dt=0.03*2*\epsilon/w_0$



$dt=0.02*2*\epsilon/w_0$



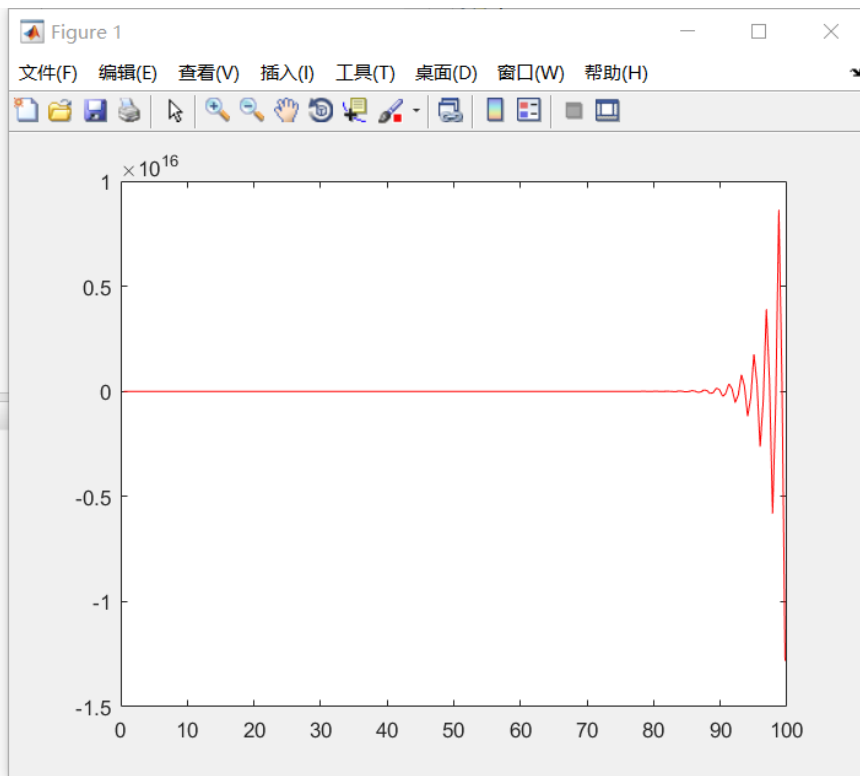
ici, il ne converge pas.

donc, $dt=0.02*2*\epsilon/w_0=1.27*10^{-3}s$

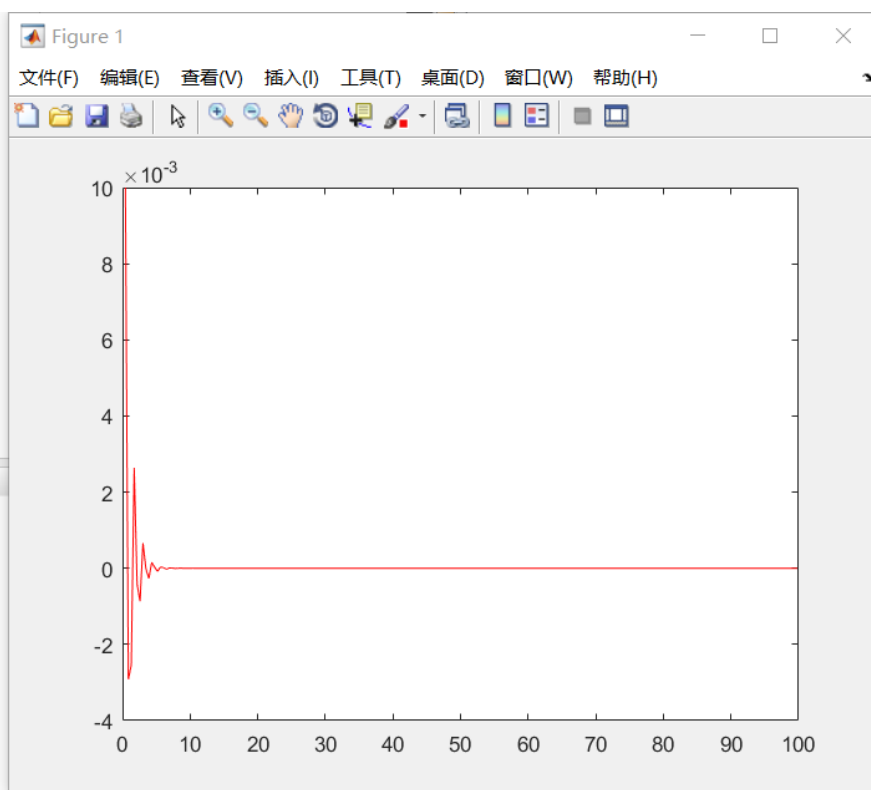
1.3.a

```
clear all
w0=2*pi;
x0=0.01;
dx0=0;
epsilon=0.02;
dt=1.04*2*2^0.5/w0;
n=100/dt;
C=[0 1;-w0^2 -2*epsilon*w0];
U(:,1)=[x0;dx0];
for i=1:n-1
    k1=C*U(:,i);
    k2=C*(U(:,i)+1/2*k1*dt);
    k3=C*(U(:,i)+1/2*k2*dt);
    k4=C*(U(:,i)+k3*dt);
    U(:,i+1)=U(:,i)+(k1+2*k2+2*k3+k4)/6*dt;
end
plot((1:n)*dt,U(1,:), 'r-')
```

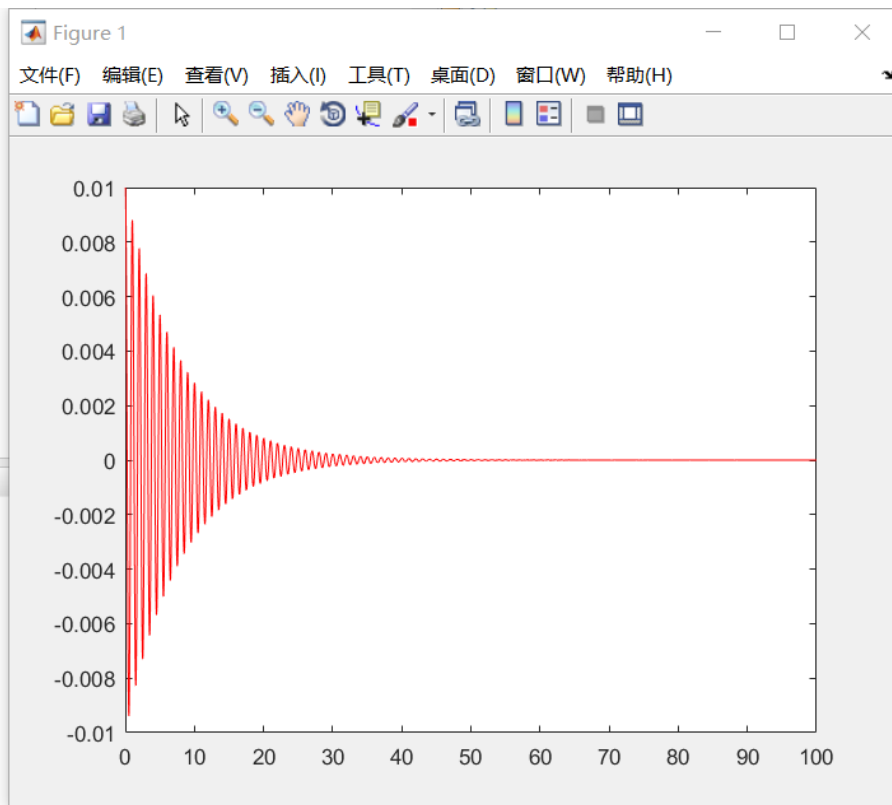
h=1.04



h=0.96



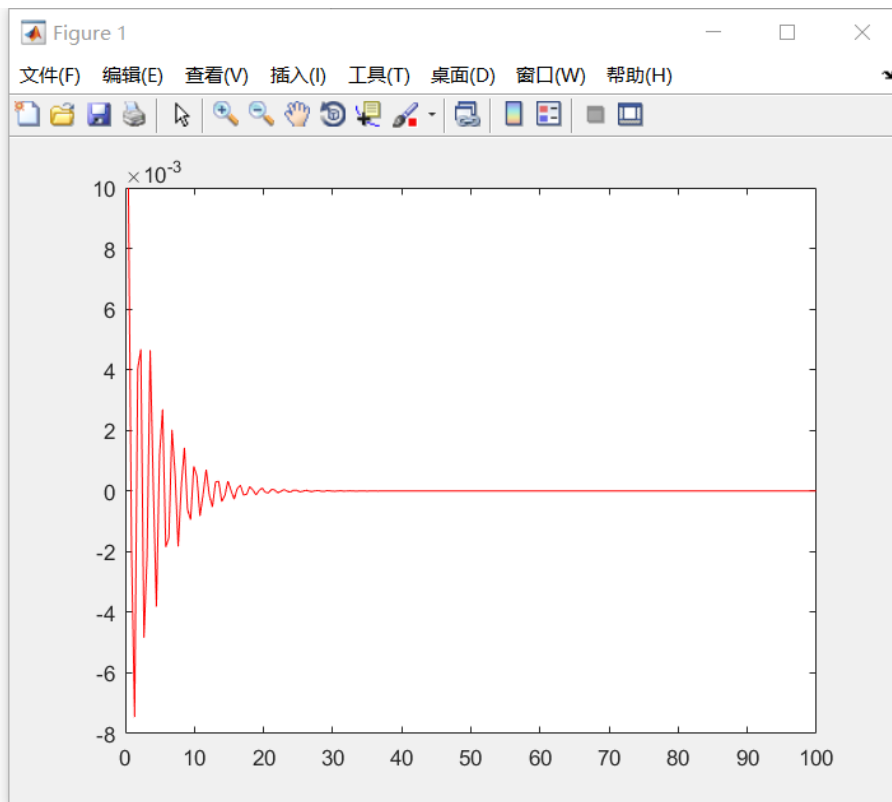
h=0.04



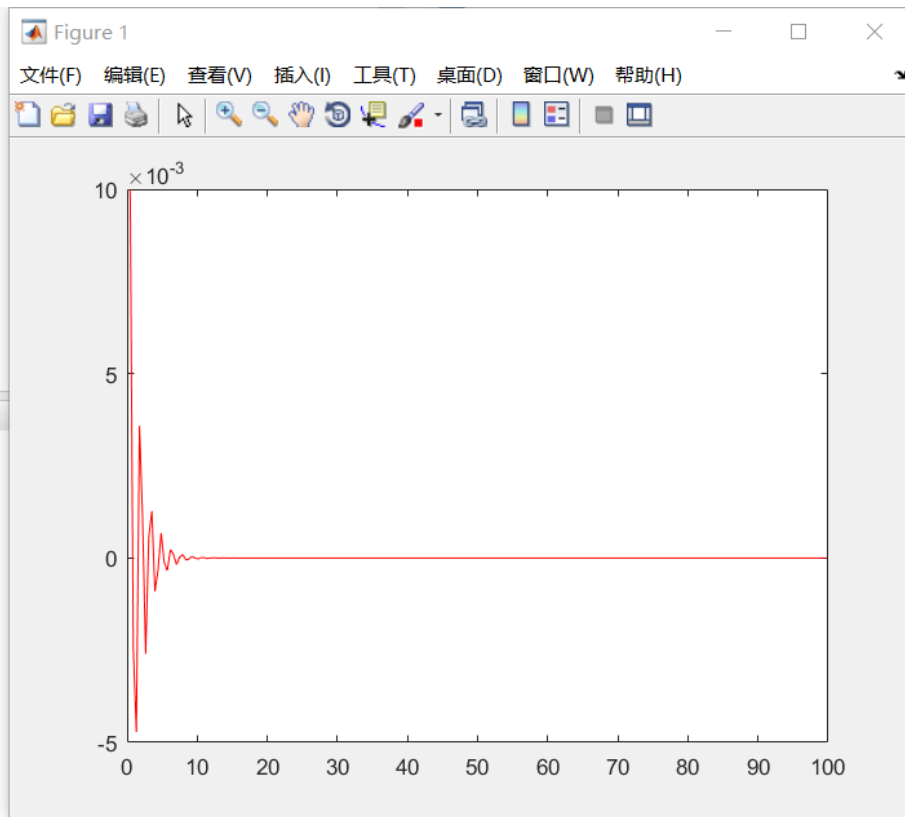
pour $h=1.04$, il diverge en fin, et pour $h=0.04$ et pour $h=0.96$ il converge.
quand $h=0.96$, il converge plus rapide, il est donc plus stable et précis.

1.3.b

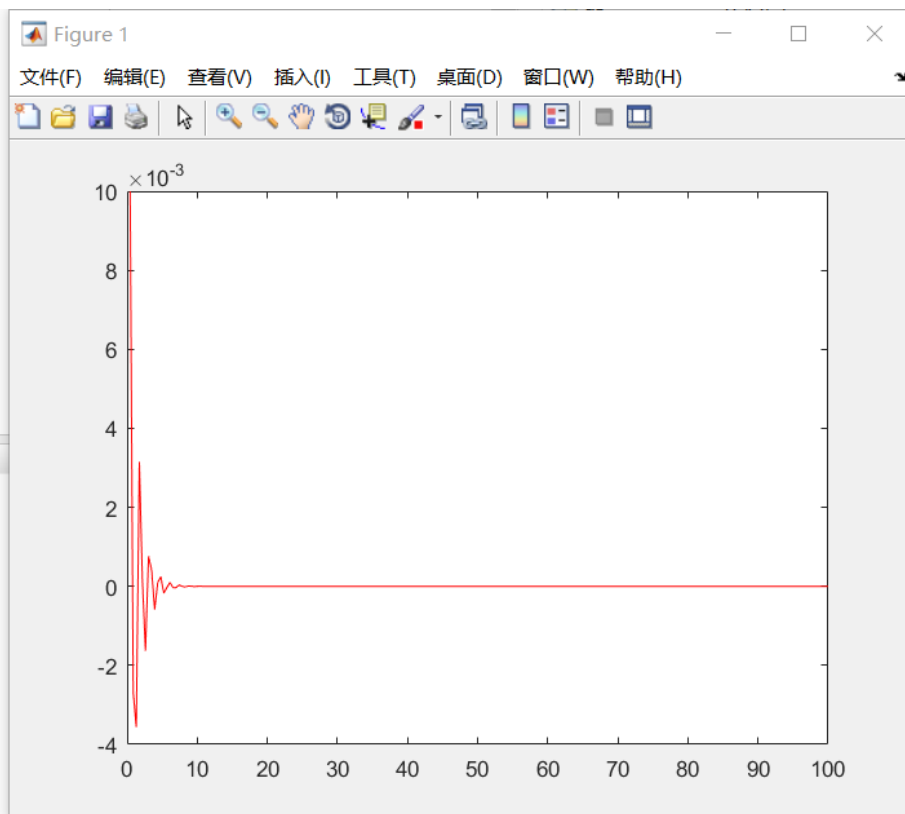
h=1



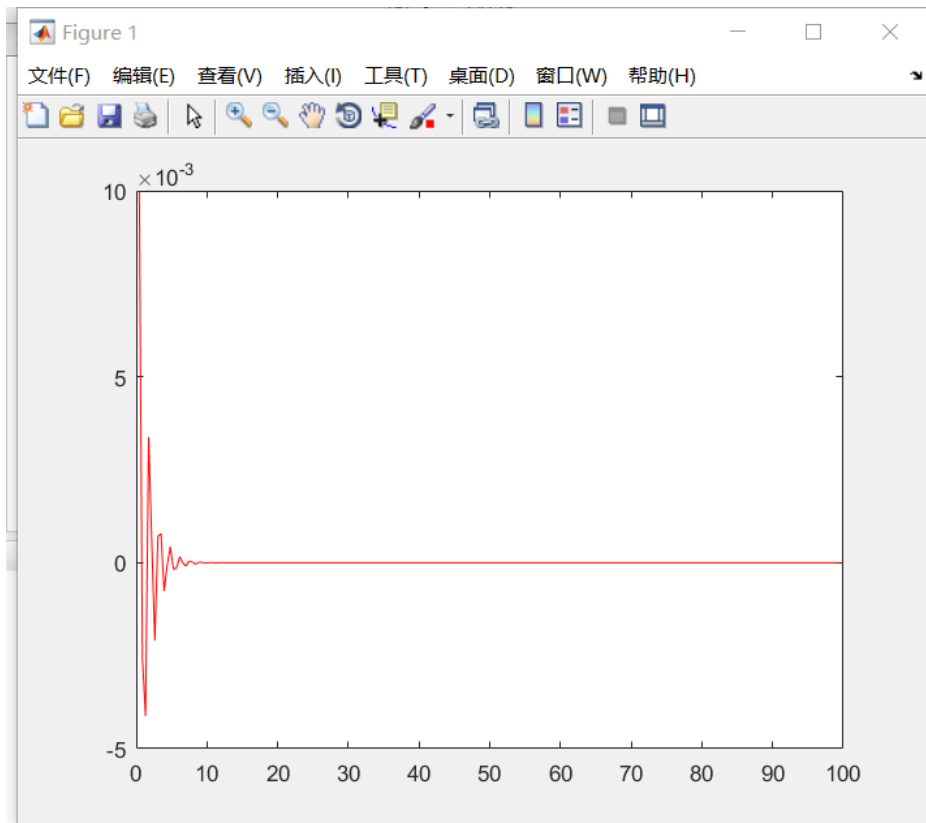
h=0.98



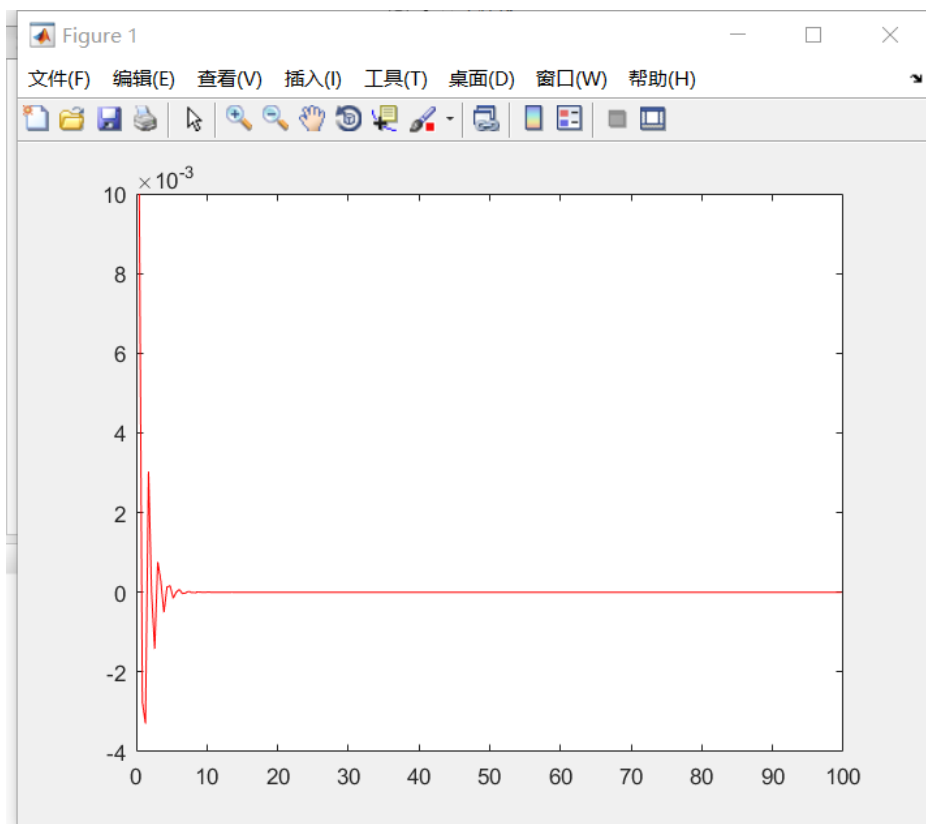
h=0.97



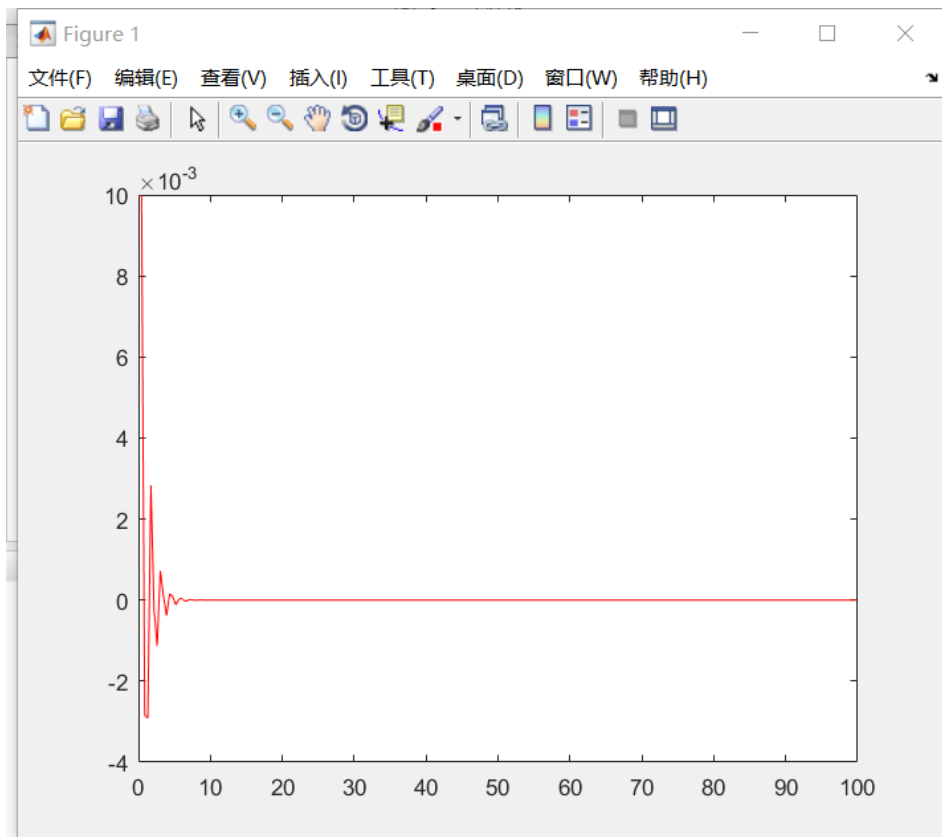
h=0.975



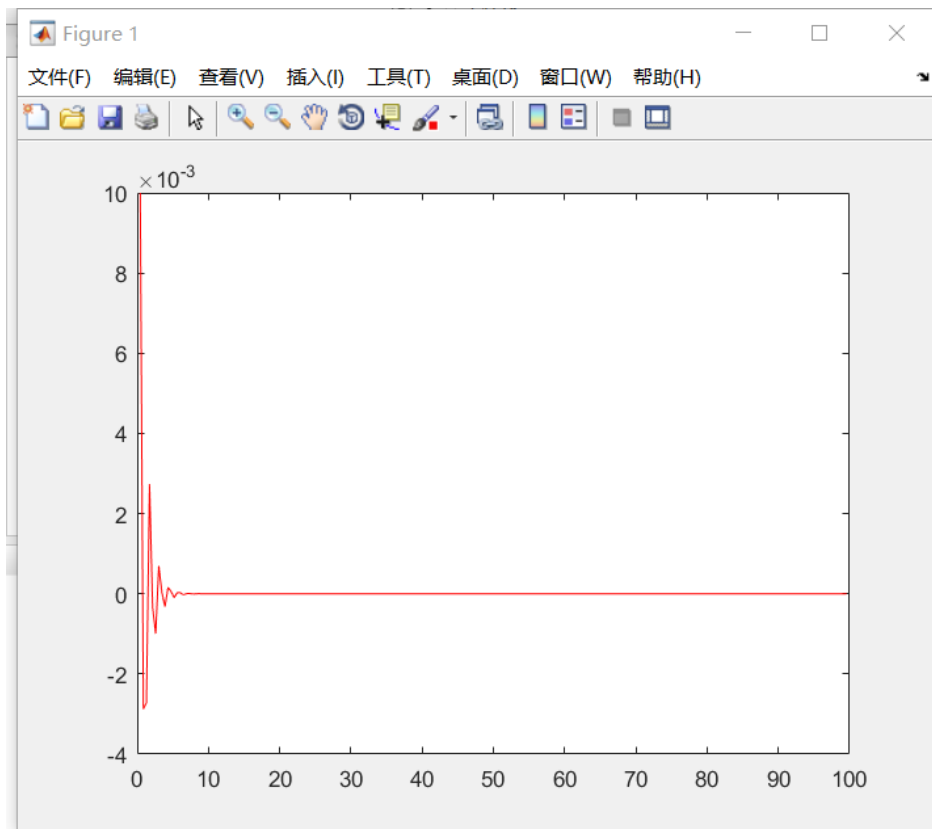
h=0.9675



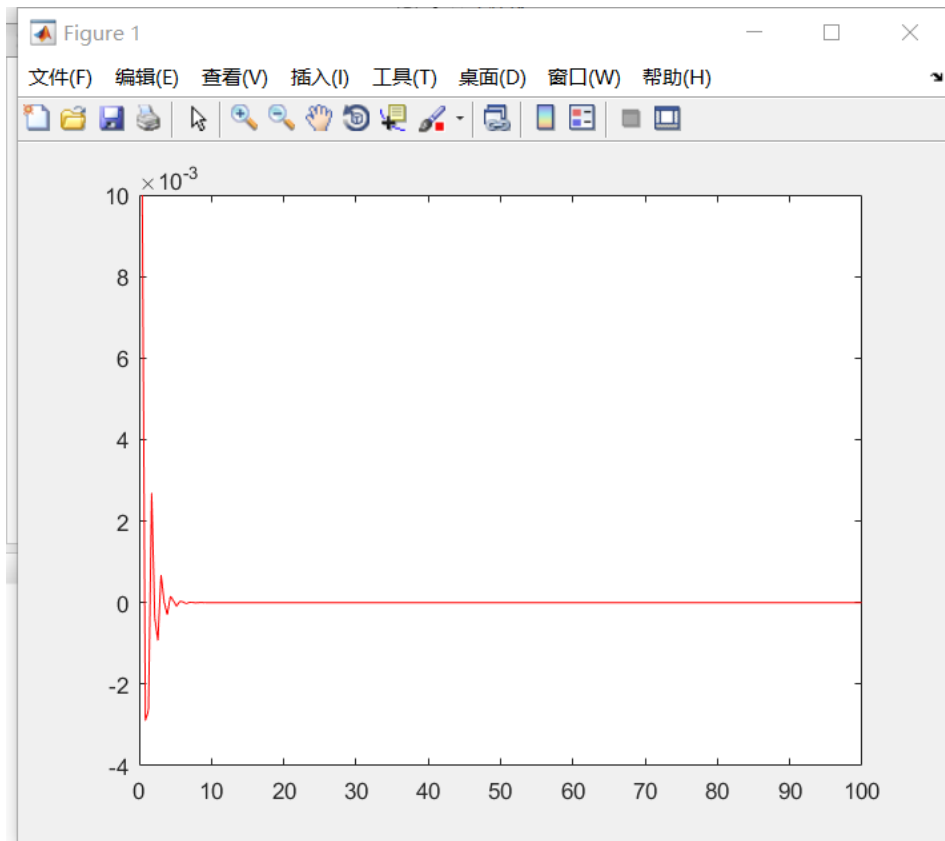
h=0.96375



h=0.961875



$h=0.9609375$



$hc=0.9609375$

donc, $dtc=0.9609375 \cdot 2 \cdot 2^{0.5} / (2 \cdot \pi) = 0.4326s$

Etude d'un double pendule avec l'hypothèse des petits mouvements

1.1

$$q = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

$$m a^2 M_1 \ddot{q} + m g a q = F_0 \sin \omega t M_3$$

$$\Rightarrow \ddot{q} = \frac{1}{M_1} \left(\frac{F_0 \sin \omega t}{m a^2} M_3 - \frac{g}{a} M_2 q \right)$$

$$\Rightarrow \ddot{q} = \sin \omega t M_5 + M_4 q$$

$$q_{n+1} = q_n + \Delta t \dot{q}_n + \Delta t^2 \left(\frac{1}{2} - \beta \right) \ddot{q}_n + \Delta t^3 \beta \ddot{q}_{n+1}$$

$$\dot{q}_{n+1} = \dot{q}_n + \Delta t (1 - \gamma) \dot{q}_n + \Delta t \gamma \ddot{q}_{n+1}$$

$$\Rightarrow \begin{bmatrix} I - \Delta t^2 \beta M_4 \\ I + \Delta t^2 \left(\frac{1}{2} - \beta \right) M_4 \end{bmatrix} \cdot q_{n+1} = \begin{bmatrix} I + \Delta t^2 \left(\frac{1}{2} - \beta \right) M_4 \\ I + \Delta t^2 \beta M_4 \end{bmatrix} \cdot q_n + \begin{bmatrix} I \Delta t \\ \Delta t^2 \left(\frac{1}{2} - \beta \right) \sin(\omega t_n) M_5 + \Delta t^3 \beta M_5 \sin(\omega t_{n+1}) \end{bmatrix} M_9$$

$$\& \quad - \Delta t M_{10} \dot{q}_{n+1} + I \dot{q}_{n+1} = (1 - \gamma) \Delta t M_{10} \dot{q}_n + I \dot{q}_n + (1 - \gamma) \sin(\omega t_n) \Delta t M_{15} + \Delta t^2 M_{15} \sin(\omega t_{n+1})$$

$$U = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}; \quad \begin{bmatrix} M_6 & 0 \\ M_{10} & M_{11} \end{bmatrix} U_{n+1} = \begin{bmatrix} M_7 & M_8 \\ M_{12} & M_{13} \end{bmatrix} U_n + \begin{bmatrix} M_9 \\ M_{14} \end{bmatrix} M_{17}$$

$$\Rightarrow U_{n+1} = A U_n + B$$

$$A = M_{15}^{-1} M_{16}; \quad B = M_{15}^{-1} M_{17}$$

```

syms dt n;
eigmax=[];
m = 2;
a = 0.5;
g = 9.81;
F0 = 20;
w = 2 * pi;
beta = 0;
gamma = 0.5;
theta1_0 = 0;
theta2_0 = 0;
dtheta1_0 = - 1.31519275;
dtheta2_0 = - 1.85996342;
I = [1, 0; 0, 1];
M1 = [2, 1; 1, 1];
M2 = [2, 0; 0, 1];
M3 = [a; a / sqrt(2)];
M4 = - inv(M1) * g / a * M2;
M5 = inv(M1) * F0 / m / a / a * M3;
M6 = I - dt * dt * beta * M4;
M7 = I + dt * dt * (0.5 - beta) * M4;
M8 = I * dt;
M9 = dt * dt * (0.5 - beta) * M5 * sin(w * n * dt) +
dt * dt * beta * M5 * sin(w * (n + 1) * dt);
M10 = - dt * gamma * M4;
M11 = I;
M12 = dt * (1 - gamma) * M4;
M13 = I;
M14 = dt * (1 - gamma) * M5 * sin(w * n * dt) + dt *
gamma * M5 * sin(w * (n + 1) * dt);
M15 = [M6, 0; M10, M11];
M16 = [M7, M8; M12, M13];
M17 = [M9; M14];
A = inv(M15) * M16
B = inv(M15) * M17

```

A =

```

[
(981*dt^2)/100, dt, 0] 1 - (981*dt^2)/50,
[
1 - (981*dt^2)/50, 0, dt] (981*dt^2)/50,
[ (981*dt*((981*dt^2)/50 - 1))/50 - (981*dt)/50 + (962361*dt^3)/5000, (981*dt)/100 -
(981*dt*((981*dt^2)/50 - 1))/100 - (962361*dt^3)/5000, 1 - (981*dt^2)/50,

```

```
(981*dt^2)/100]
[ (981*dt)/50 - (981*dt*((981*dt^2)/50 - 1))/50 - (962361*dt^3)/2500,
(981*dt*((981*dt^2)/50 - 1))/50 - (981*dt)/50 + (962361*dt^3)/5000, (981*dt^2)/50, 1
- (981*dt^2)/50]
```

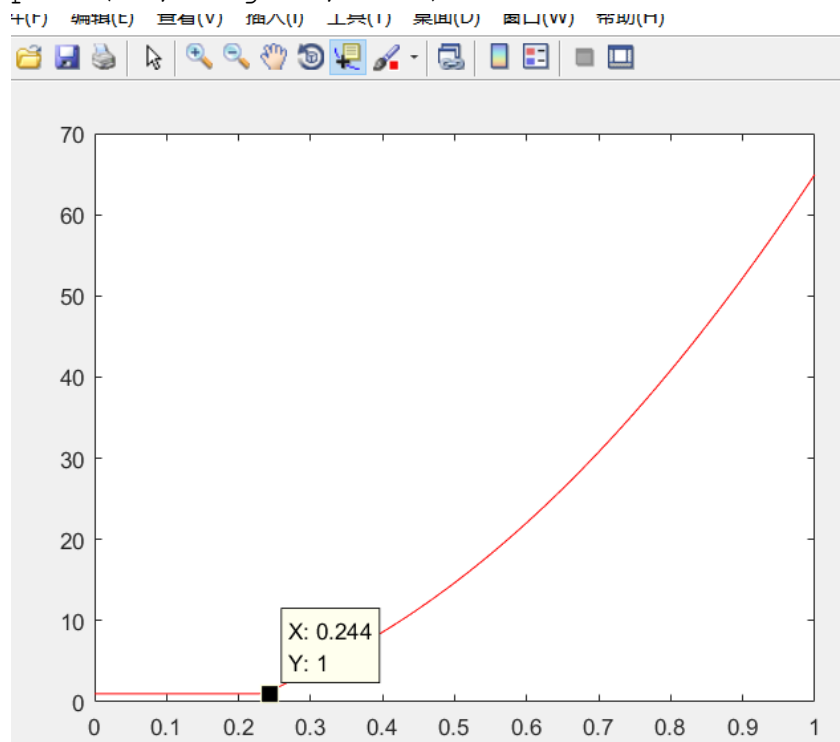
B =

```
(3297684477769025*dt^2*sin(2*pi*dt*n))/1125899906842624

(2331815056444095*dt^2*sin(2*pi*dt*n))/562949953421312
(3297684477769025*dt*sin(2*pi*dt*(n + 1)))/1125899906842624 -
(94751790231975633*dt^3*sin(2*pi*dt*n))/5629499534213120 +
(3297684477769025*dt*sin(2*pi*dt*n))/1125899906842624
(2331815056444095*dt*sin(2*pi*dt*(n + 1)))/562949953421312 -
(267998533610380173*dt^3*sin(2*pi*dt*n))/11258999068426240 +
(2331815056444095*dt*sin(2*pi*dt*n))/562949953421312
```

1.2

```
.....
for dt=0:0.001:1;
    eigmax=[eigmax,max(abs(eig(eval(A))))];
end
dt=0:0.001:1;
plot(dt, eigmax, 'r-')
```



le pas est inférieure à 0.244, tous les modules de valeur propre est presque égale à 1, et quand le pas est supérieure à 0.244, les modules de valeur propre supérieure à 1.

1.3

$$\ddot{q} = M_5 \sin \omega t + M_4 q$$

t=0;

$$\ddot{q}_0 = M_4 q_0$$

q0 = [theta1_0; theta2_0];

dq0 = [dtheta1_0; dtheta2_0];

d2q0 = M4 * q0;

1.4

$$\begin{bmatrix} 1 & 0 & -\beta \Delta t^2 \\ 0 & 1 & -\gamma \Delta t \\ \omega^2 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_j \\ \dot{q}_{j+1} \\ \ddot{q}_{j+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & \Delta t(0.5-\beta) \\ 0 & 1 & \Delta t(1-\beta) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_j \\ \dot{q}_j \\ \ddot{q}_j \end{bmatrix}$$

$$\begin{cases} \ddot{q}_j = \sin \omega t_j \cdot M_5 + M_4 q_j \\ \dot{q}_{j+1} = \sin \omega t_{j+1} \cdot M_5 + M_4 q_{j+1} \end{cases}$$

$$U_{n+1} = A \cdot U_n + B.$$

1.5

T0 = 8;

dt = 0.02;

m=T0 / dt - 1;

q = [q0];

dq = [dq0];

d2q = [d2q0];

U = [q0; dq0];

for n = 0 : m

U = eval(A) * U + eval(B);

q = [q, U(1:2)];

dq = [dq, U(3:4)];

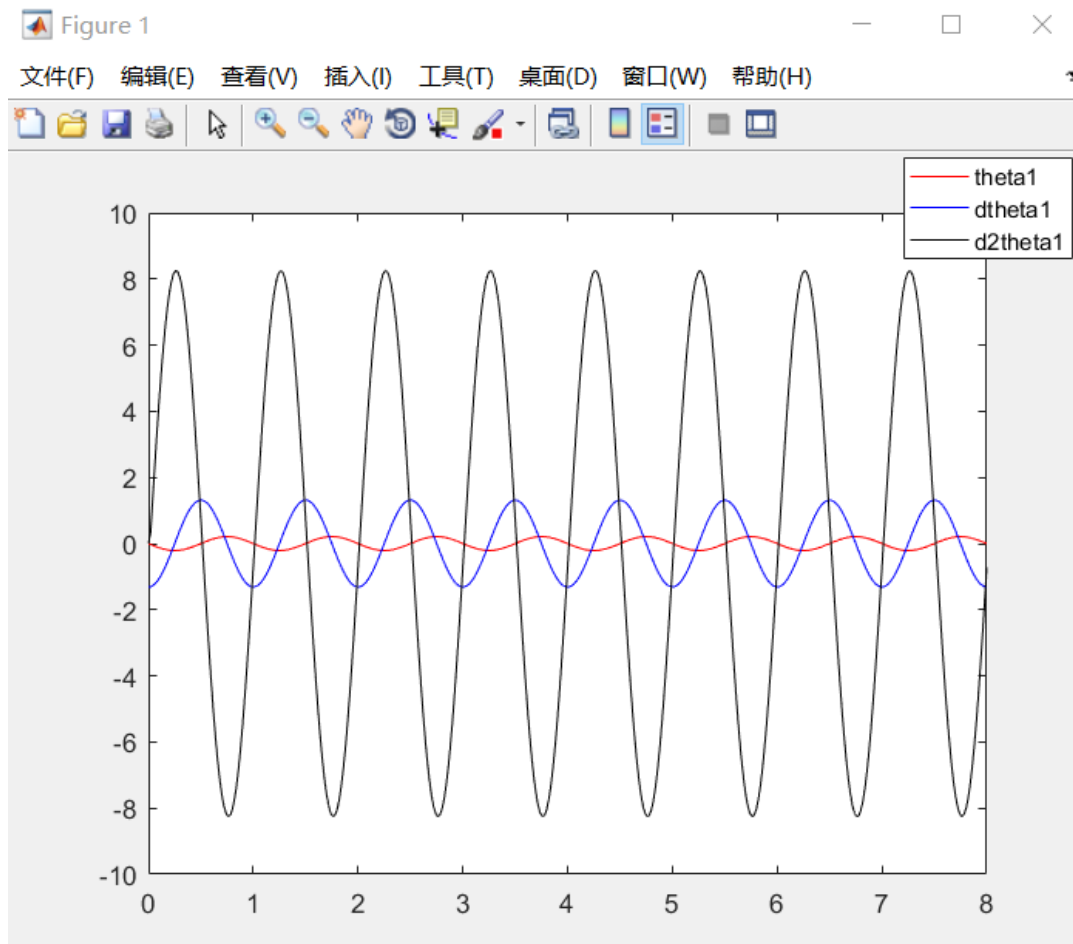
d2q = [d2q, M4 * U(1:2) + M5 * sin(w * n * dt)];

end

t = (0 : (m+1)) * dt;

plot(t, q(1, :), 'r-');

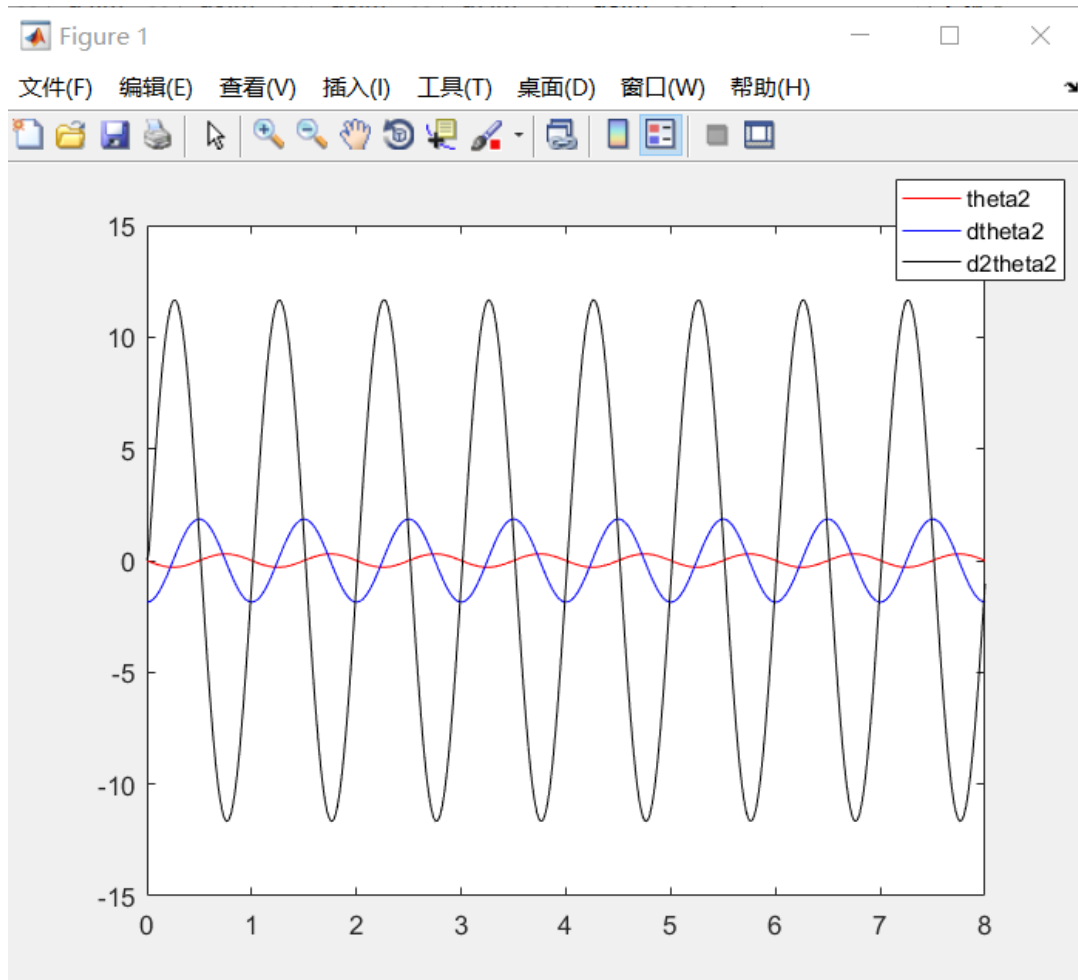
```
hold on;  
plot(t, dq(1, :), 'b-');  
hold on;  
plot(t, d2q(1, :), 'k-');
```




```

plot(t, q(2, :), 'r-');
hold on;
plot(t, dq(2, :), 'b-');
hold on;
plot(t, d2q(2, :), 'k-');

```



1.6

pour t=0, dt et 2dt

```
q(:, 1 : 3)
```

ans =

```
0 -0.0263 -0.0522
```

```
0 -0.0372 -0.0738
```

pour t=0.5s

```
q(:, 0.5 / dt + 1)
```

ans =

```
1.0e-03 *
```

```
-0.2988
```

```

-0.4226
de même
dq(:, 1 : 3)
ans =

```

```

-1.3152 -1.3048 -1.2739
-1.8600 -1.8453 -1.8016
dq(:, 0.5 / dt + 1)
ans =

```

```

1.3143
1.8587
d2q(:, 1 : 3)
ans =

```

```

0 0.3023 1.3340
0 0.4275 1.8866
d2q(:, 0.5 / dt + 1)
ans =

```

```

0.7376
1.0432

```

2.1.

```

.....

```

```

beta = 0.25;

```

```

gamma = 0.5;

```

```

.....

```

```

A =

```

```

[
(962361*dt^4)/(962361*dt^4 + 392400*dt^2 + 20000) -
(200*(981*dt^2 + 100)*((981*dt^2)/100 - 1))/(962361*dt^4 + 392400*dt^2 + 20000),
(981*dt^2*(981*dt^2 + 100))/(962361*dt^4 + 392400*dt^2 + 20000) -
(98100*dt^2*((981*dt^2)/100 - 1))/(962361*dt^4 + 392400*dt^2 + 20000),
(200*dt*(981*dt^2 + 100))/(962361*dt^4 + 392400*dt^2 + 20000),
(98100*dt^3)/(962361*dt^4 + 392400*dt^2 + 20000)]

```

```

[
(1962*dt^2*(981*dt^2 + 100))/(962361*dt^4 + 392400*dt^2 +
20000) - (196200*dt^2*((981*dt^2)/100 - 1))/(962361*dt^4 + 392400*dt^2 + 20000),
(962361*dt^4)/(962361*dt^4 + 392400*dt^2 + 20000) - (200*(981*dt^2 +
100)*((981*dt^2)/100 - 1))/(962361*dt^4 + 392400*dt^2 + 20000),
(196200*dt^3)/(962361*dt^4 + 392400*dt^2 + 20000), (200*dt*(981*dt^2 +
100))/(962361*dt^4 + 392400*dt^2 + 20000)]

```

```

[
(1924722*dt^3)/(962361*dt^4 + 392400*dt^2 + 20000) - (981*dt)/50 +
(1962*(981*dt^3 + 200*dt)*((981*dt^2)/100 - 1))/(962361*dt^4 + 392400*dt^2 + 20000),
(981*dt)/100 - (196200*dt*((981*dt^2)/100 - 1))/(962361*dt^4 + 392400*dt^2 + 20000) -

```

$$\begin{aligned}
& (962361*dt^2*(981*dt^3 + 200*dt))/(100*(962361*dt^4 + 392400*dt^2 + 20000)), 1 - \\
& (1962*dt*(981*dt^3 + 200*dt))/(962361*dt^4 + 392400*dt^2 + 20000), \\
& (196200*dt^2)/(962361*dt^4 + 392400*dt^2 + 20000)] \\
& [(981*dt)/50 - (392400*dt*((981*dt^2)/100 - 1))/(962361*dt^4 + 392400*dt^2 + 20000) - \\
& (962361*dt^2*(981*dt^3 + 200*dt))/(50*(962361*dt^4 + 392400*dt^2 + 20000)), \\
& (1924722*dt^3)/(962361*dt^4 + 392400*dt^2 + 20000) - (981*dt)/50 + (1962*(981*dt^3 + \\
& 200*dt)*((981*dt^2)/100 - 1))/(962361*dt^4 + 392400*dt^2 + 20000), \\
& (392400*dt^2)/(962361*dt^4 + 392400*dt^2 + 20000), 1 - (1962*dt*(981*dt^3 + \\
& 200*dt))/(962361*dt^4 + 392400*dt^2 + 20000)]
\end{aligned}$$

B =

$$\begin{aligned}
& (98100*dt^2*((2331815056444095*dt^2*\sin(2*pi*dt*n))/1125899906842624 \\
& (2331815056444095*dt^2*\sin(2*pi*dt*(n + 1)))/1125899906842624))/(962361*dt^4 + \\
& 392400*dt^2 + 20000) + (200*(981*dt^2 \\
& 100)*((3297684477769025*dt^2*\sin(2*pi*dt*n))/2251799813685248 \\
& (3297684477769025*dt^2*\sin(2*pi*dt*(n + 1)))/2251799813685248))/(962361*dt^4 + \\
& 392400*dt^2 + 20000)
\end{aligned}$$

$$\begin{aligned}
& (196200*dt^2*((3297684477769025*dt^2*\sin(2*pi*dt*n))/2251799813685248 \\
& (3297684477769025*dt^2*\sin(2*pi*dt*(n + 1)))/2251799813685248))/(962361*dt^4 + \\
& 392400*dt^2 + 20000) + (200*(981*dt^2 \\
& 100)*((2331815056444095*dt^2*\sin(2*pi*dt*n))/1125899906842624 \\
& (2331815056444095*dt^2*\sin(2*pi*dt*(n + 1)))/1125899906842624))/(962361*dt^4 + \\
& 392400*dt^2 + 20000)
\end{aligned}$$

$$\begin{aligned}
& (3297684477769025*dt*\sin(2*pi*dt*(n + 1)))/1125899906842624 + \\
& (3297684477769025*dt*\sin(2*pi*dt*n))/1125899906842624 - (1962*(981*dt^3 \\
& 200*dt)*((3297684477769025*dt^2*\sin(2*pi*dt*n))/2251799813685248 \\
& (3297684477769025*dt^2*\sin(2*pi*dt*(n + 1)))/2251799813685248))/(962361*dt^4 + \\
& 392400*dt^2 + 20000)
\end{aligned}$$

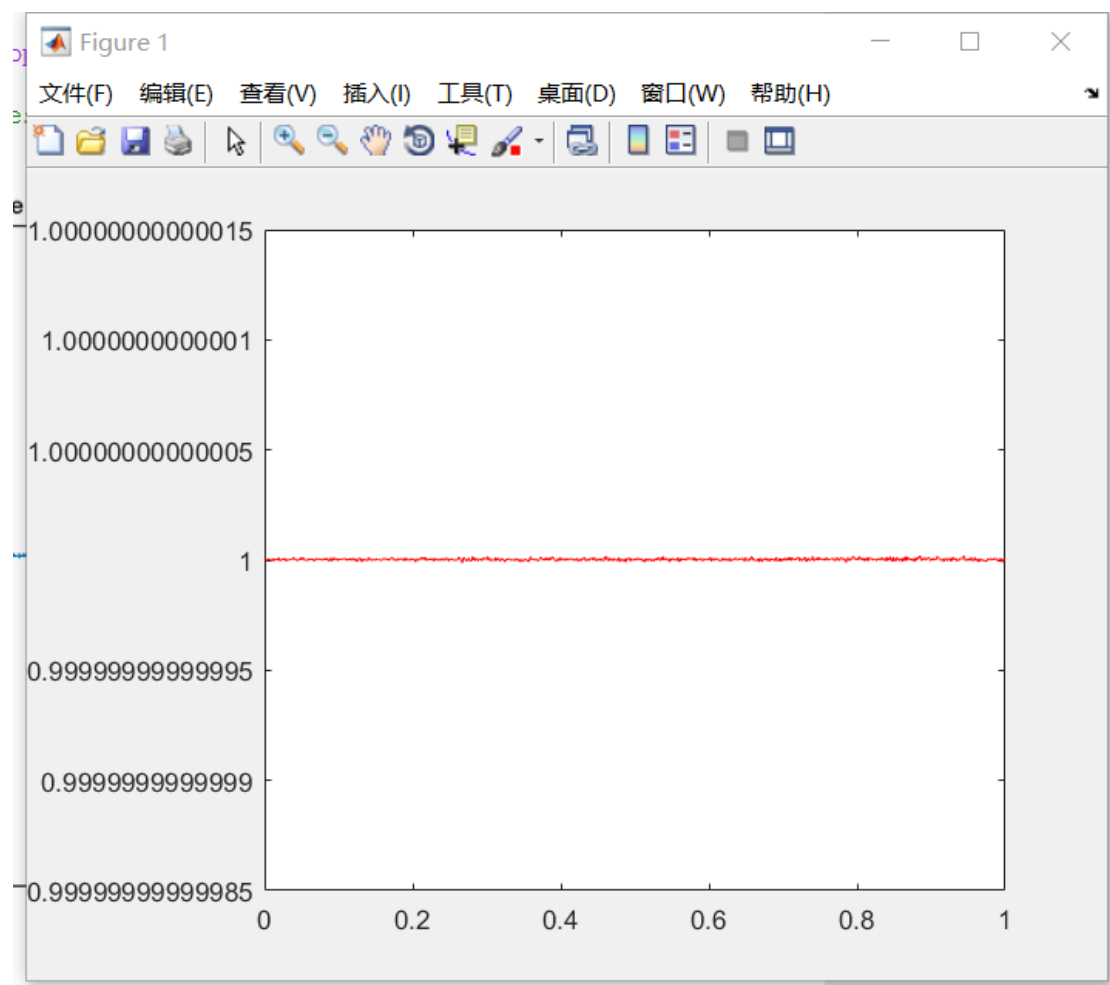
$$\begin{aligned}
& (196200*dt*((2331815056444095*dt^2*\sin(2*pi*dt*n))/1125899906842624 \\
& (2331815056444095*dt^2*\sin(2*pi*dt*(n + 1)))/1125899906842624))/(962361*dt^4 + \\
& 392400*dt^2 + 20000)
\end{aligned}$$

$$\begin{aligned}
& (2331815056444095*dt*\sin(2*pi*dt*(n + 1)))/562949953421312 + \\
& (2331815056444095*dt*\sin(2*pi*dt*n))/562949953421312 - (1962*(981*dt^3 \\
& 200*dt)*((2331815056444095*dt^2*\sin(2*pi*dt*n))/1125899906842624 \\
& (2331815056444095*dt^2*\sin(2*pi*dt*(n + 1)))/1125899906842624))/(962361*dt^4 + \\
& 392400*dt^2 + 20000)
\end{aligned}$$

$$\begin{aligned}
& (392400*dt*((3297684477769025*dt^2*\sin(2*pi*dt*n))/2251799813685248 \\
& (3297684477769025*dt^2*\sin(2*pi*dt*(n + 1)))/2251799813685248))/(962361*dt^4 + \\
& 392400*dt^2 + 20000)
\end{aligned}$$

.

2.2



Il est quasiment toujours égale à 1.

2.3

on seulement change le valeur de beta, et il vaut changer quelque valeur lié avec beta dans les matrice, mais la relation ne change pas.

$$\ddot{q} = M_5 \sin \omega t + M_4 q$$

t=0;

$$\ddot{q}_0 = M_4 q_0$$

```
q0 = [theta1_0; theta2_0];
```

```
dq0 = [dtheta1_0; dtheta2_0];
```

```
d2q0 = M4 * q0;
```

2.4

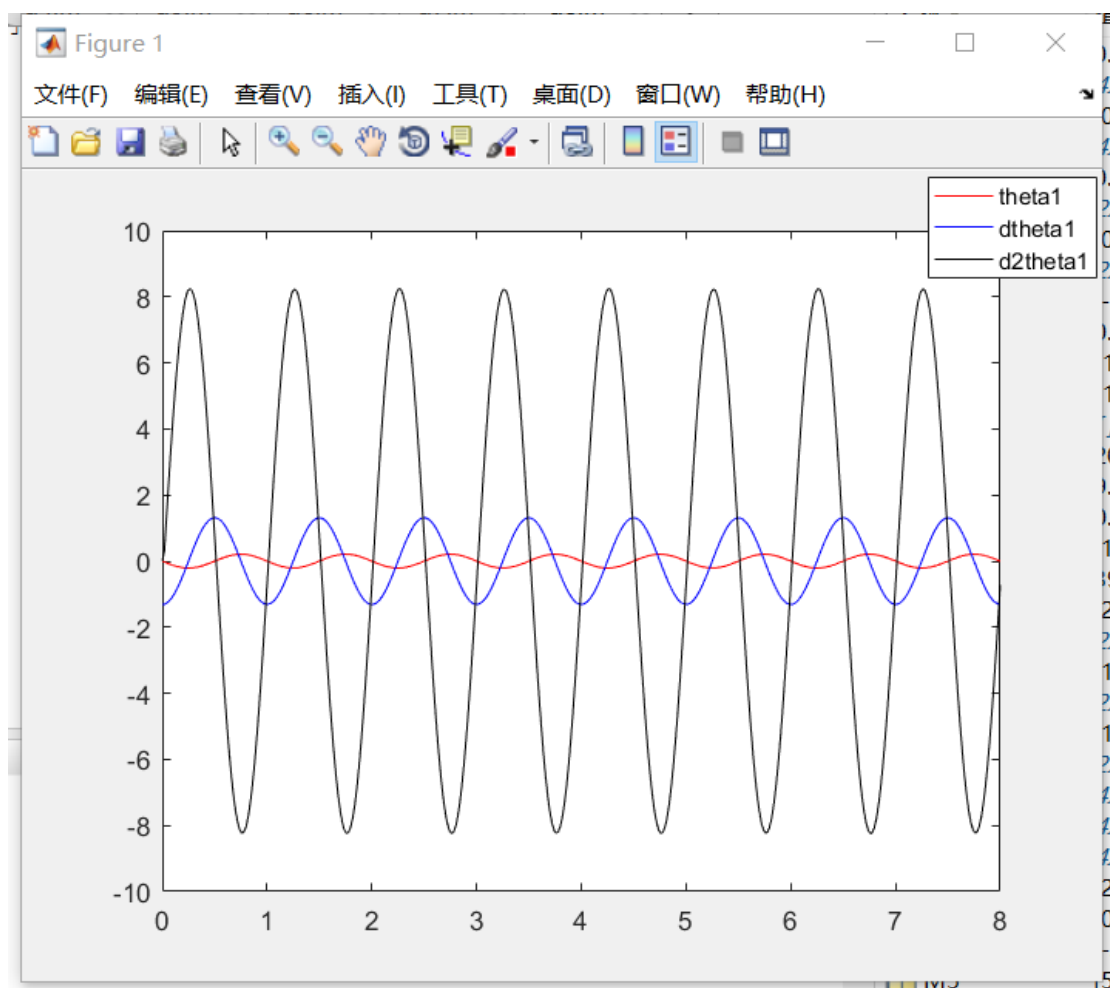
de même, les relation de symbole ne change pas.

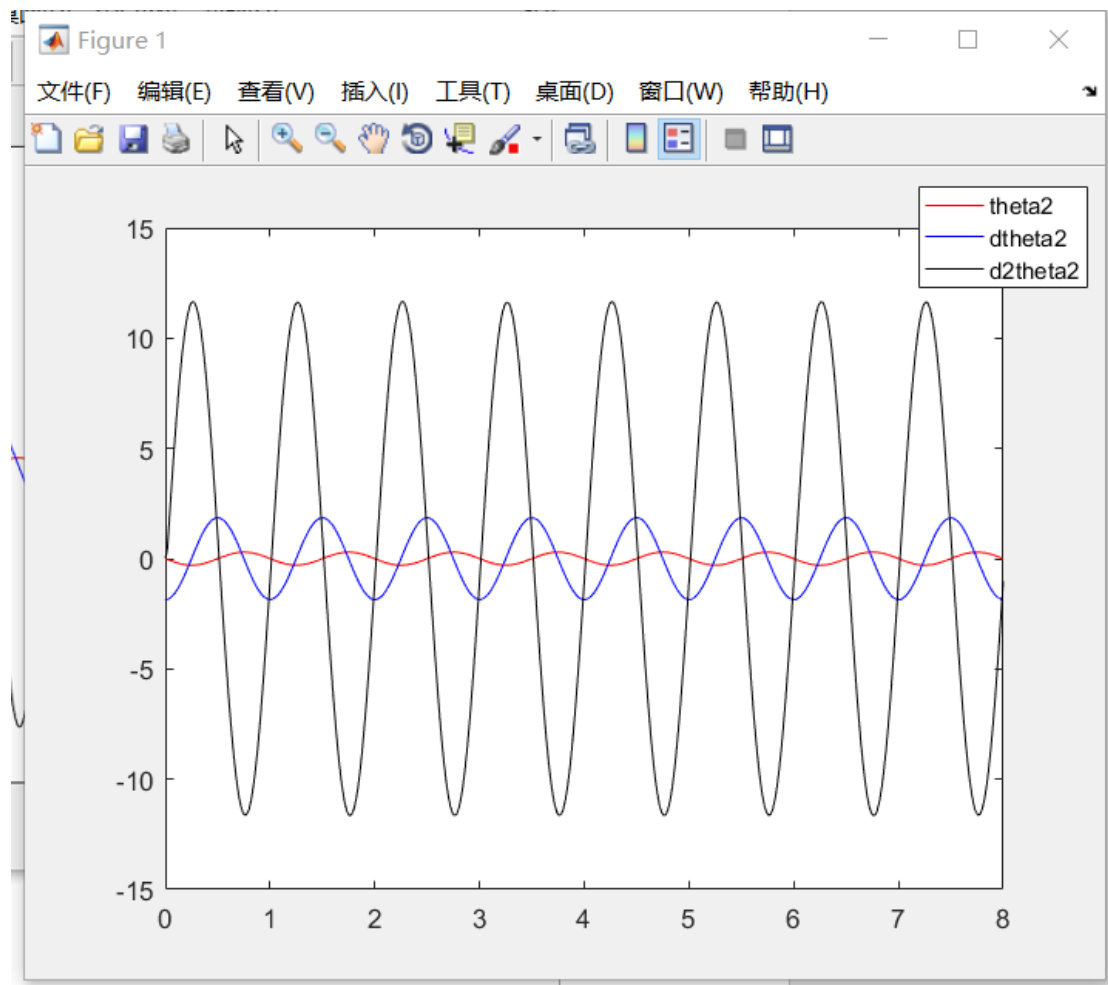
$$\begin{bmatrix} 1 & 0 & -\beta \Delta t^2 \\ 0 & 1 & -\gamma \Delta t \\ \omega_0^2 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_j \\ \dot{q}_{j+1} \\ \ddot{q}_{j+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & \Delta t(0.5-\beta) \\ 0 & 1 & \Delta t(1-\beta) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_j \\ \dot{q}_j \\ \ddot{q}_j \end{bmatrix}$$

$$\begin{cases} \ddot{q}_j = \sin \omega t_j \cdot M_5 + M_4 \dot{q}_j \\ \ddot{q}_{j+1} = \sin \omega t_{j+1} \cdot M_5 + M_4 \dot{q}_{j+1} \end{cases}$$

$$U_{n+1} = A \cdot U_n + B.$$

2.5





2.6

pour $t=0$, dt et $2dt$

```
q(:, 1 : 3)
```

ans =

```
0 -0.0262 -0.0520
```

```
0 -0.0371 -0.0735
```

```
dq(:, 1 : 3)
```

ans =

```
-1.3152 -1.3048 -1.2739
```

```
-1.8600 -1.8453 -1.8016
```

```
d2q(:, 1 : 3)
```

ans =

```
0 0.3011 1.3317
```

```
0 0.4259 1.8833
```

```
q(:, 0.5 / dt + 1)
```

ans =

```

-0.0009
-0.0013
dq(:, 0.5 / dt + 1)
ans =

1.3124
1.8561
d2q(:, 0.5 / dt + 1)
ans =

0.7448
1.0533

```

oscillateur non linéaire à un degré de liberté

1.1

avec $\beta = 0$; $\gamma = 0.5$

$$q_{j+1} = q_j + \Delta t \dot{q}_j + \frac{\Delta t^2}{2} \ddot{q}_{j+1}$$

$$\ddot{q}_{j+1} = -w_0^2 q_{j+1} (1 + a q_{j+1}^2)$$

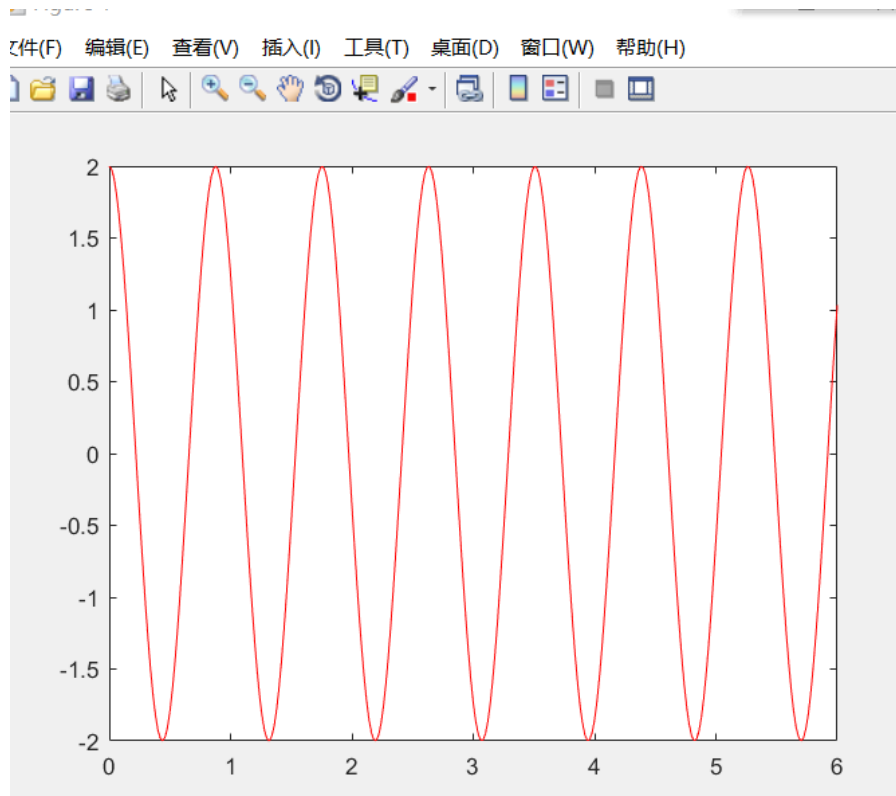
$$\dot{q}_{j+1} = \dot{q}_j + \frac{\Delta t}{2} (\ddot{q}_j + \ddot{q}_{j+1})$$

1.2

```

q0=2;
d1q0=0;
d2q0=-11.2*pi^2;
A=[q0,d1q0,d2q0];
dt=0.02;
w0=2*pi;
a=0.1;
I=1;
t=0:0.02:6;
for j=2:301
    A(j,1)=A(j-1,1)+dt*A(j-1,2)+dt^2/2*A(j-1,3);
    A(j,3)=-w0^2*A(j,1)*(I+a*A(j,1)^2);
    A(j,2)=A(j-1,2)+dt/2*(A(j-1,3)+A(j,3));
end;
plot(t,A(:,1),'r-')

```



1.3

pour $t=0, dt$ et $2dt$
 $q = [A(:, 1)];$
 $q(1:3)$
ans =

2.0000
1.9779
1.9123

pour $t=T0$
 $q(301, 1)$
ans =

1.0329

2.1

On doit minimiser la valeur de $\omega_0^2 q_{j+1}^* (1 + a q_{j+1}^{*2})$

2.2

$$\Delta q_{j+1}^* = \beta \Delta t^2 \Delta \ddot{q}_{j+1}$$

$$\Delta \dot{q}_{j+1}^* = \gamma \Delta t \Delta \ddot{q}_{j+1}$$

en utilisant la correction

$$\ddot{q}_{j+1}^* + \Delta \ddot{q}_{j+1} + \omega_0^2 (q_{j+1}^* + \Delta q_{j+1}) (1 + a (q_{j+1}^* + \Delta q_{j+1})^2) = 0$$

2.3

$$f = \omega_0^2 q_{j+1}^* (1 + a q_{j+1}^{*2})$$

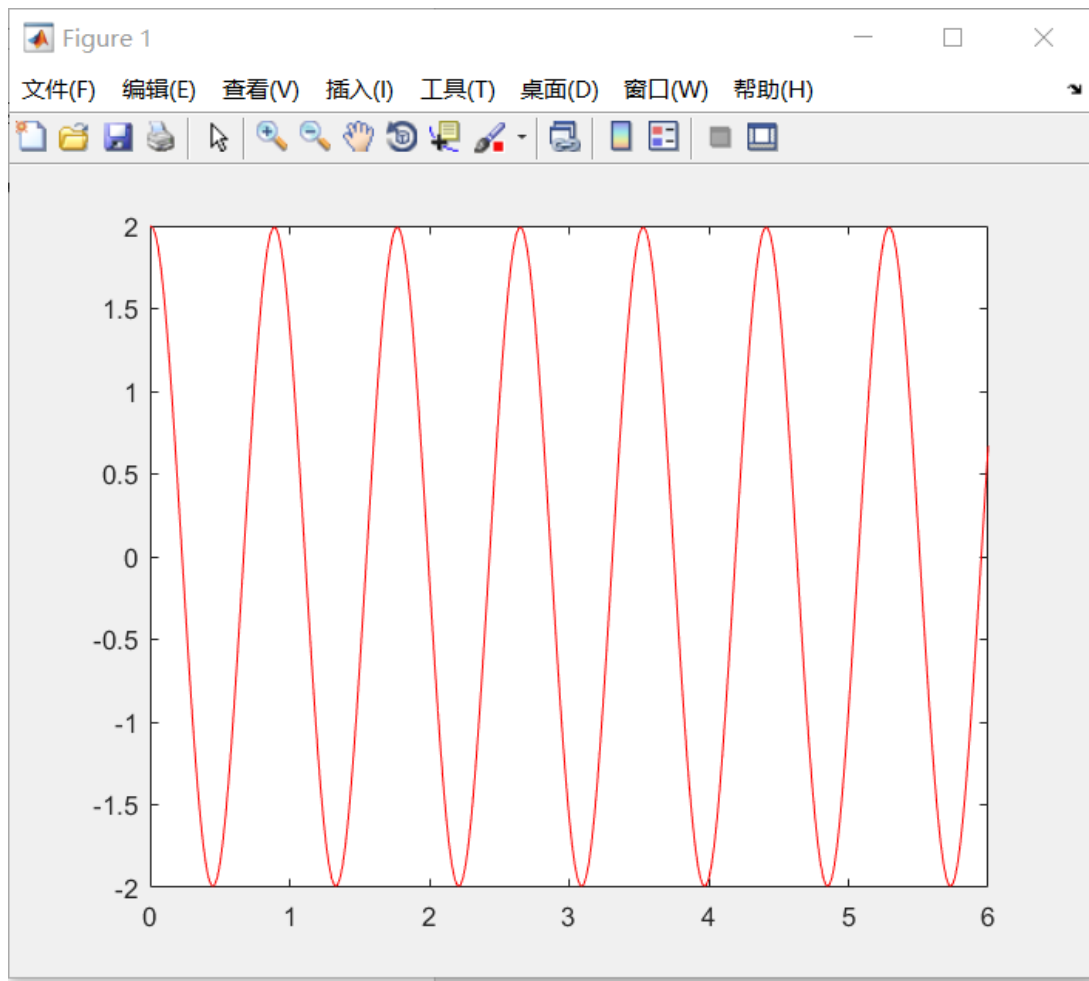
$$\Delta \ddot{q}_{j+1} = - \frac{f}{\frac{\partial f}{\partial \ddot{q}_{j+1}} + \frac{\partial f}{\partial q_{j+1}} \beta \Delta t^2} = \frac{-\ddot{q}_{j+1}^* - w_0 q_{j+1}^* (1 + a q_{j+1}^{*2})}{1 + w_0^2 \beta \Delta t^2 (1 + 3a q_{j+1}^{*2})}$$

```

q1=2;
d1q1=0;
d2q1=0;
B=[q1,d1q1,d2q1];
dt=0.02;
w0=2*pi;
a=0.1;
I=1;
t=0:0.02:6;
delta_q1=0;
d1delta_q1=0;
d2delta_q1=0;
C=[delta_q1,d1delta_q1,d2delta_q1];
for j=2:301
    B(j,3)=0;
    B(j,2)=B(j-1,2)+0.5*dt*B(j-1,3);
    B(j,1)=B(j-1,1)+dt*B(j-1,2)+dt^2*0.25*B(j-1,3);

while(abs(B(j,3)+(w0^2)*B(j,1)*(1+a*B(j,1)^2))>0.01)
    C(j,3)=(-B(j,3)-
(w0^2)*B(j,1)*(1+a*B(j,1)^2)/(1+(w0^2)*(1+3*a*B(j,1)^
2)*0.25*dt^2));
    C(j,2)=0.5*dt*C(j,3);
    C(j,1)=0.25*dt^2*C(j,3);
    B(j,3)=B(j,3)+C(j,3);
    B(j,2)=B(j,2)+C(j,2);
    B(j,1)=B(j,1)+C(j,1);
end
end
plot(t,B(:,1),'r-')
q1=[B(:,1)]
q1(1:3)
q1(301,1)

```



2.4

pour $t=0, dt$ et $2dt$

ans =

2.0000

1.9890

1.9456

pour $t=6s$

ans =

0.6694

3.1

$$E = \frac{1}{2} m \dot{q}^2 + \frac{1}{2} m k q^2 + \frac{1}{4} a m k q^4$$

3.2

soit $m=1\text{kg}$,

$$e = \frac{1}{2} \dot{q}^2 + \frac{1}{2} k q^2 + \frac{1}{4} a k q^4$$

NM-EX :

```

q0=2;
d1q0=0;
d2q0=-11.2*pi^2;
A=[q0,d1q0,d2q0];
dt=0.02;
w0=2*pi;
k=w0^2;
a=0.1;
I=1;
t=0:0.02:6;
e=[k*2+a*k*4];
for j=2:301
    A(j,1)=A(j-1,1)+dt*A(j-1,2)+dt^2/2*A(j-1,3);
    A(j,3)=-w0^2*A(j,1)*(I+a*A(j,1)^2);
    A(j,2)=A(j-1,2)+dt/2*(A(j-1,3)+A(j,3));
    e(j)=0.5*A(j,2)^2;
end;
plot(t,e,'b-')
NM-IM:
q1=2;
d1q1=0;
d2q1=0;
B=[q1,d1q1,d2q1];
dt=0.02;
w0=2*pi;
k=w0^2;
a=0.1;
I=1;
t=0:0.02:6;
delta_q1=0;
d1delta_q1=0;
d2delta_q1=0;
C=[delta_q1,d1delta_q1,d2delta_q1];
e1=[k*2+a*k*4];
for j=2:301
    B(j,3)=0;
    B(j,2)=B(j-1,2)+0.5*dt*B(j-1,3);
    B(j,1)=B(j-1,1)+dt*B(j-1,2)+dt^2*0.25*B(j-1,3);

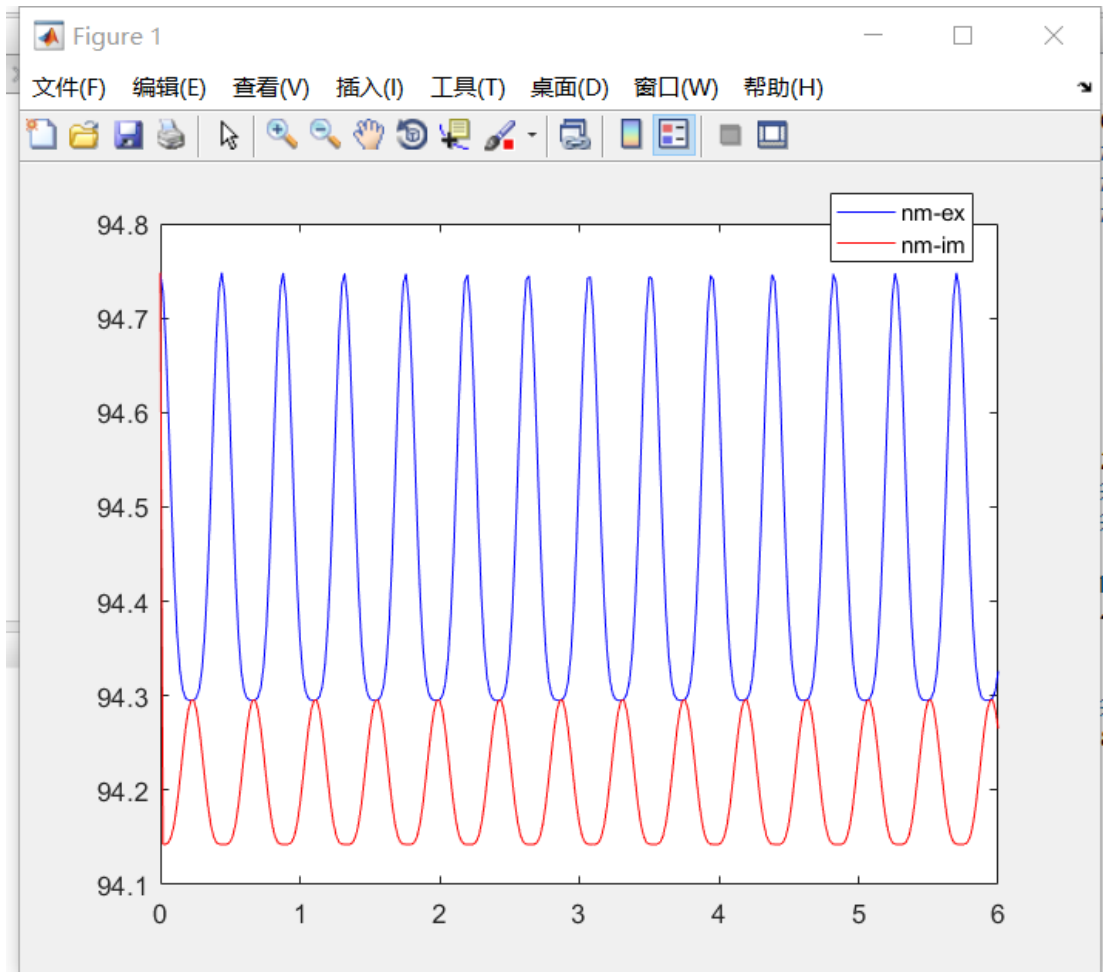
while(abs(B(j,3)+(w0^2)*B(j,1)*(1+a*B(j,1)^2))>0.01)
    C(j,3)=(-B(j,3)-
(w0^2)*B(j,1)*(1+a*B(j,1)^2)/(1+(w0^2)*(1+3*a*B(j,1)^
2)*0.25*dt^2));
    C(j,2)=0.5*dt*C(j,3);

```

```

C(j,1)=0.25*dt^2*C(j,3);
B(j,3)=B(j,3)+C(j,3);
B(j,2)=B(j,2)+C(j,2);
B(j,1)=B(j,1)+C(j,1);
end
e1(j)=0.5*B(j,2)^2+k/2*B(j,1)^2+a*k/4*B(j,1)^4;
end
plot(t,e1(1,:), 'r-')

```



3.3

Energie de NM-EX est toujours grande que celle de NM-IM.