Étude d’un oscillateur linéaire amorti à un degré de liberté

1.1

```
clear all;
T0=1;
w0=2*pi/T0;
epsilon=0.02;
x0=0.01;
dx0=0;
omega=w0*sqrt(1-epsilon^2);
```

a)

```
dt1=(2*epsilon/w0)*1.2;
U1(:,1)=[x0;dx0];
A1=[1 dt1;-w0^2*dt1 1-2*epsilon*w0*dt1];
t1=0:dt1:10*T0;
for i=2:length(t1)
    U1(:,i)=A1*U1(:,i-1);
end
plot(t1,U1(1,:))
hold on;
plot(t1,exp(-epsilon*w0*t1).*(x0*cos(omega*t1)+((epsilon*w0*x0+dx
0)/omega)*sin(omega*t1)))
legend('Euler explicite','Solution exacte')
```

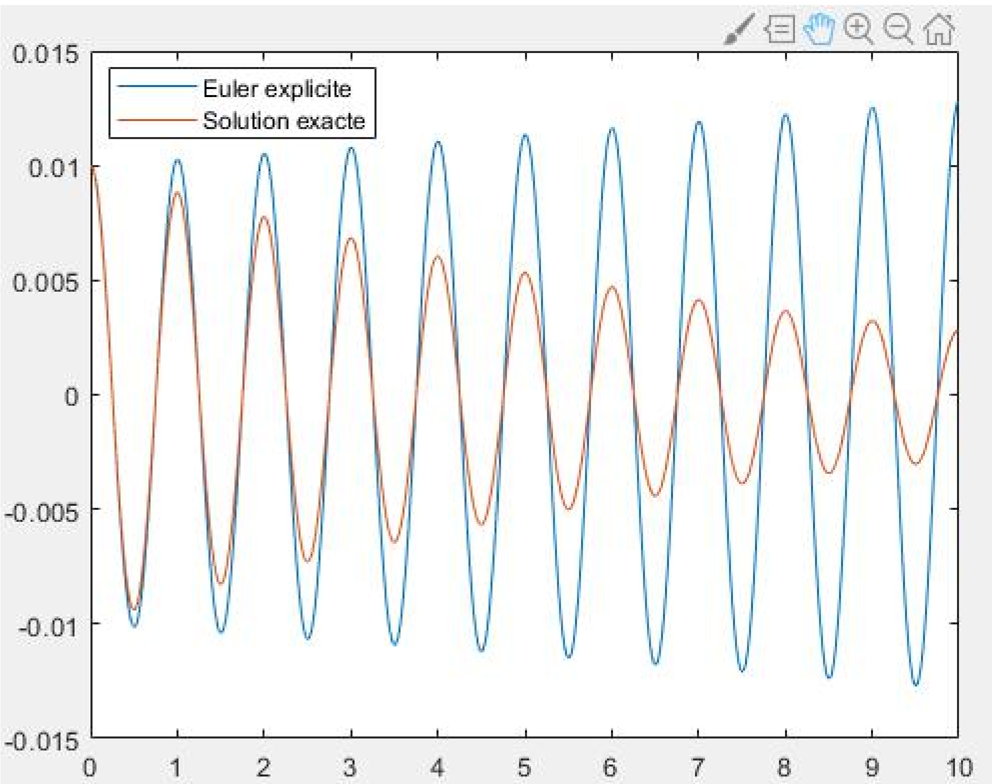


图 1 $\Delta t > \frac{2\varepsilon}{\omega_0}$

b)

```
dt1=(2*epsilon/w0)*1;
U1(:,1)=[x0;dx0];
A1=[1 dt1;-w0^2*dt1 1-2*epsilon*w0*dt1];
t1=0:dt1:10*T0;
for i=2:length(t1)
    U1(:,i)=A1*U1(:,i-1);
end
plot(t1,U1(1,:))
hold on;
plot(t1,exp(-epsilon*w0*t1).*(x0*cos(omega*t1)+((epsilon*w0*x0+dx
0)/omega)*sin(omega*t1)))
legend('Euler explicite','Solution exacte')
```

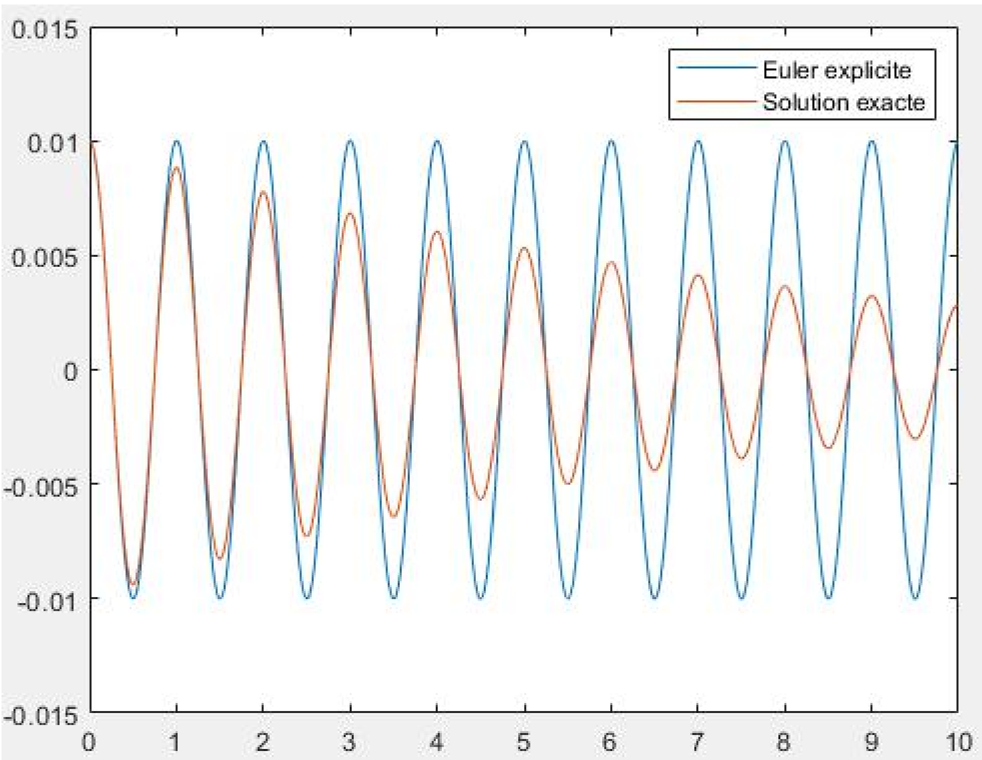


图 2 $\Delta t = \frac{2\varepsilon}{\omega_0}$

c)

```
dt1=(2*epsilon/w0)*0.8;
U1(:,1)=[x0;dx0];
```

```
A1=[1 dt1;-w0^2*dt1 1-2*epsilon*w0*dt1];
t1=0:dt1:10*T0;
for i=2:length(t1)
    U1(:,i)=A1*U1(:,i-1);
end
plot(t1,U1(1,:))
hold on;
plot(t1,exp(-epsilon*w0*t1).*(x0*cos(omega*t1)+((epsilon*w0*x0+dx0)/omega)*sin(omega*t1)))
legend('Euler explicite','Solution exacte')
```

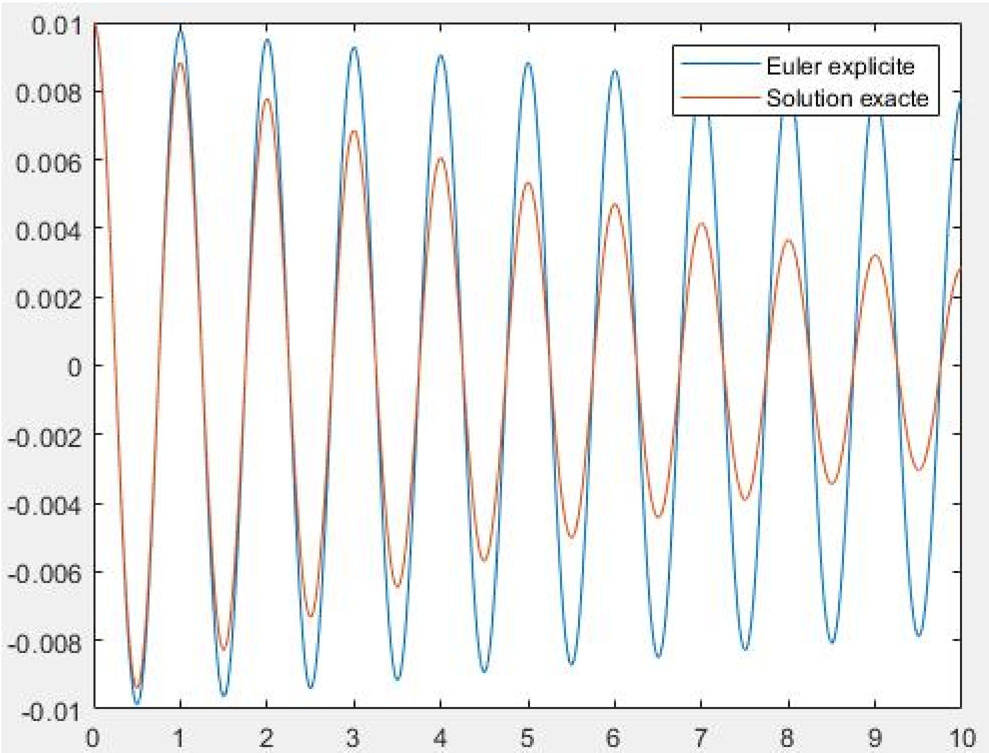


图 3 $\Delta t = 0.8 \times \frac{2\varepsilon}{\omega_0}$

Quand $\Delta t > \frac{2\varepsilon}{\omega_0}$, le résultat diverge; quand $\Delta t = \frac{2\varepsilon}{\omega_0}$, l'amplitude ne change pas; quand $\Delta t = 0.8 \times \frac{2\varepsilon}{\omega_0}$, il converge.

d)

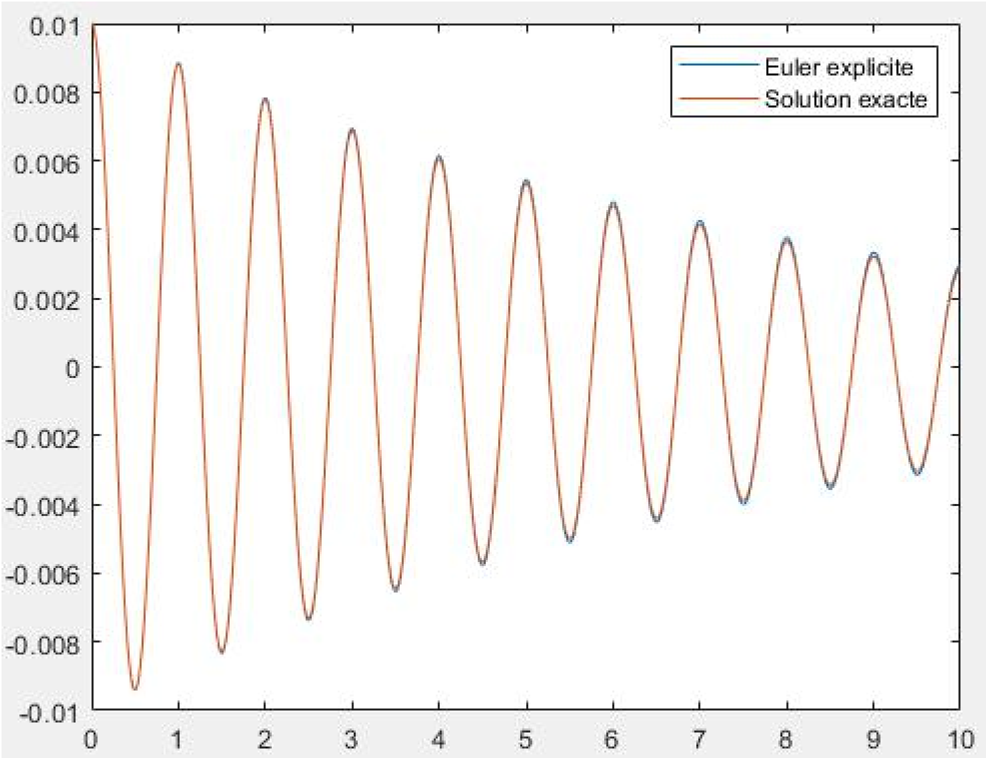


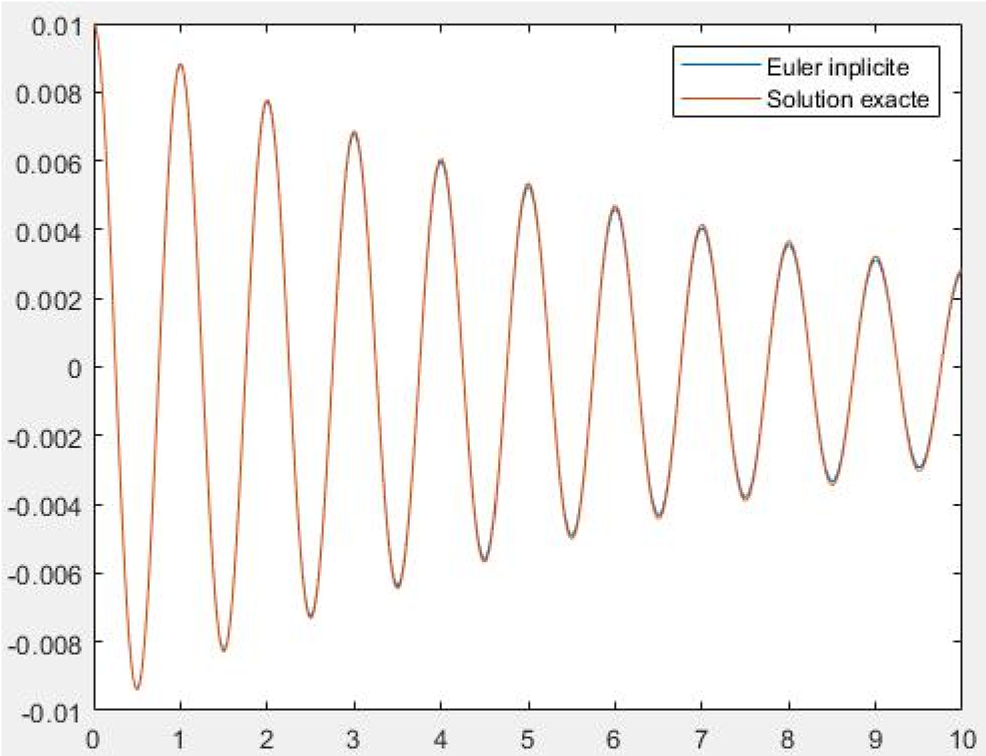
图 4 $\Delta t = 0.03 \times \frac{2\varepsilon}{\omega_0}$

Pour le résultat , il doit être précis en amplitude et en période.

Quand $\Delta t = 0.03 \times \frac{2\varepsilon}{\omega_0}$, le résultat calculée présente une précision suffisante.

1.2

```
% 1.2
dt2=(2*epsilon/w0)*0.03;
U2(:,1)=[x0;dx0];
A2=[1 -dt1;w0^2*dt2 1+2*epsilon*w0*dt2];
t2=0:dt2:10*T0;
for i=2:length(t2)
    U2(:,i)=inv(A2)*U2(:,i-1);
end
plot(t2,U2(1,:))
hold on;
plot(t2,exp(-epsilon*w0*t2).*(x0*cos(omega*t2)+((epsilon*w0*x0+dx
0)/omega)*sin(omega*t2)))
legend('Euler implicite','Solution exacte')
[X,Y]=eig(inv(A2))
module=abs(Y)
```



Y =

1.0000 + 0.0012i   0.0000 + 0.0000i

0.0000 + 0.0000i   1.0000 - 0.0012i

module =

1.0000   0

0   1.0000

On peut voir que le module de valeur propre est 1. Donc le pas de temps critique est $\Delta t = 0.03 \times \dfrac{2\varepsilon}{\omega_0}$.

1.3

a)

```
% 1.3
h=0.04;
dt3=h*2*sqrt(2)/w0;
t3=0:dt3:100*T0;
A3=[0,1;-w0^2,-2*epsilon*w0];
U3(:,1)=[x0;dx0];
for j=1:length(t3)-1
    k1=A3*U3(:,j);
    k2=A3*(U3(:,j)+0.5*dt3*k1);
    k3=A3*(U3(:,j)+0.5*dt3*k2);
    k4=A3*(U3(:,j)+dt3*k3);
```

```
    U3(:,j+1)=U3(:,j)+1/6*dt3*(k1+2*k2+2*k3+k4);
end
plot(t3,U3(1,:))
hold on;
plot(t3,exp(-epsilon*w0*t3).*(x0*cos(omega*t3)+((epsilon*w0*x0+dx
0)/omega)*sin(omega*t3)))
legend('Runge Kutta','Solution exacte')
```

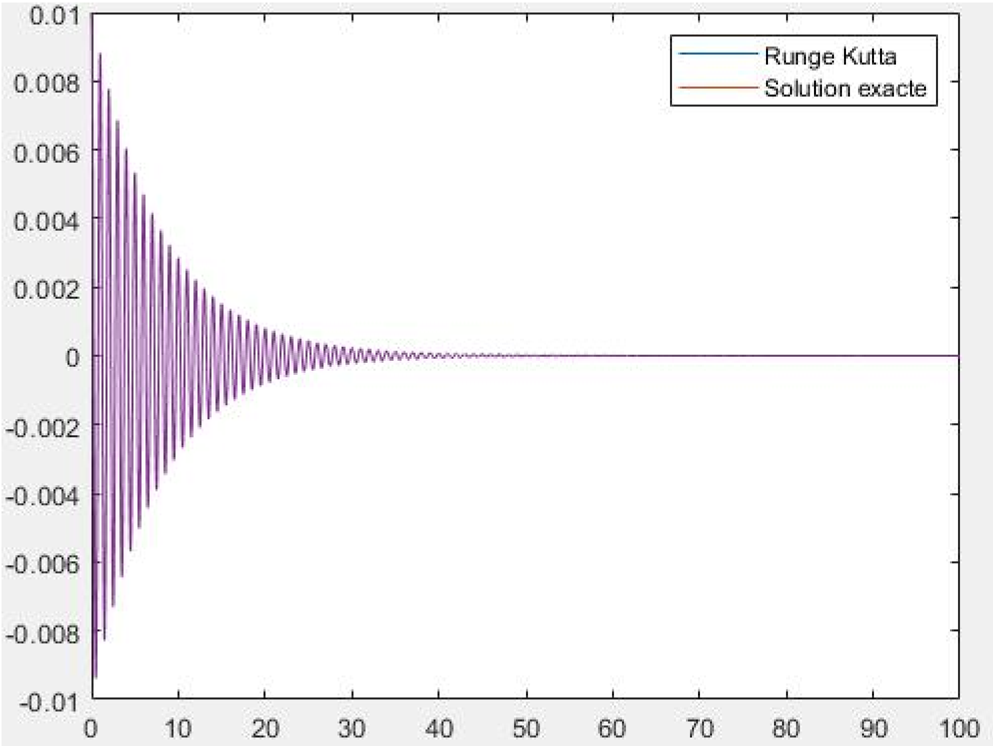


图 5 h=0.04

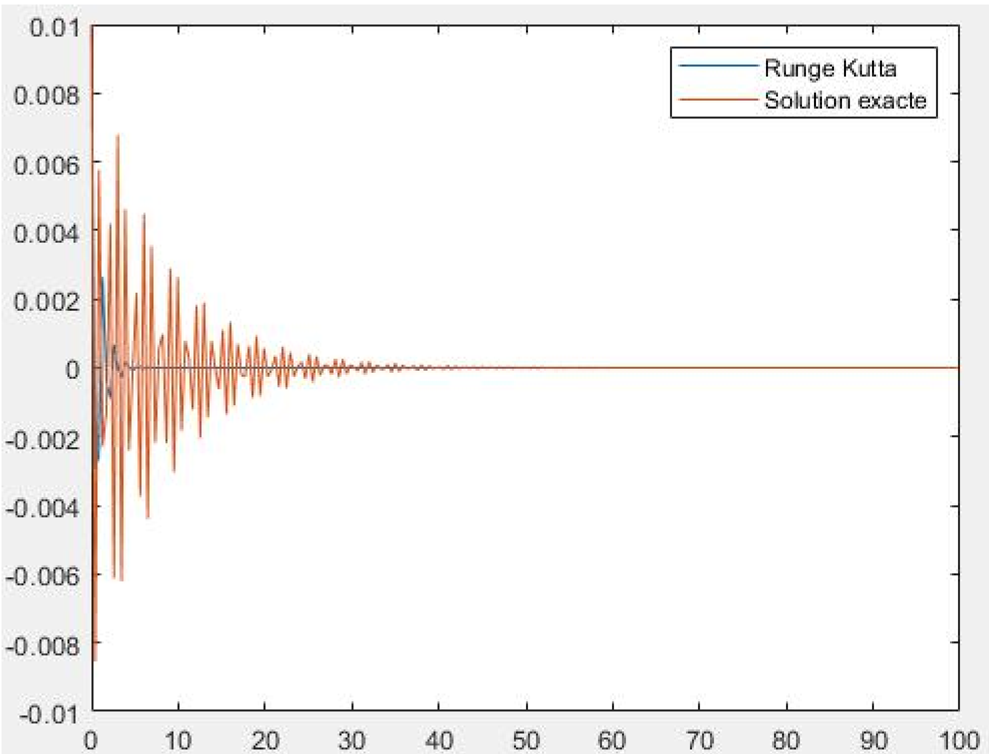


图 6 h=0.96

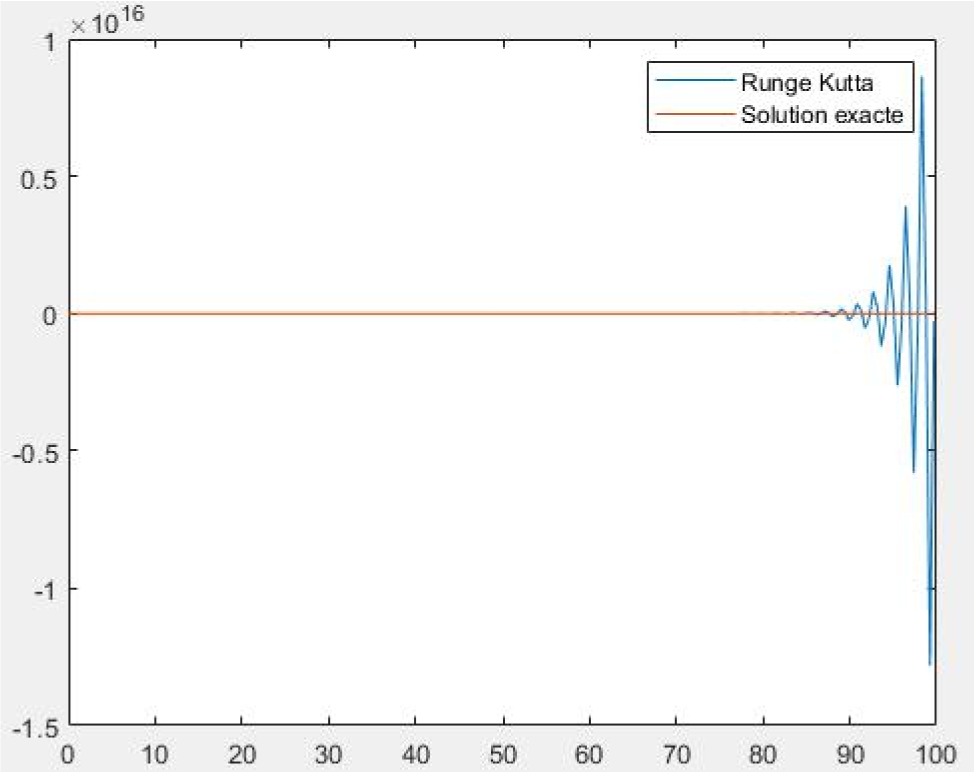


图 7 h=1.04

Quand h<1, il est stable, et quang h est plus petit, le résultat est plus précis.

b)

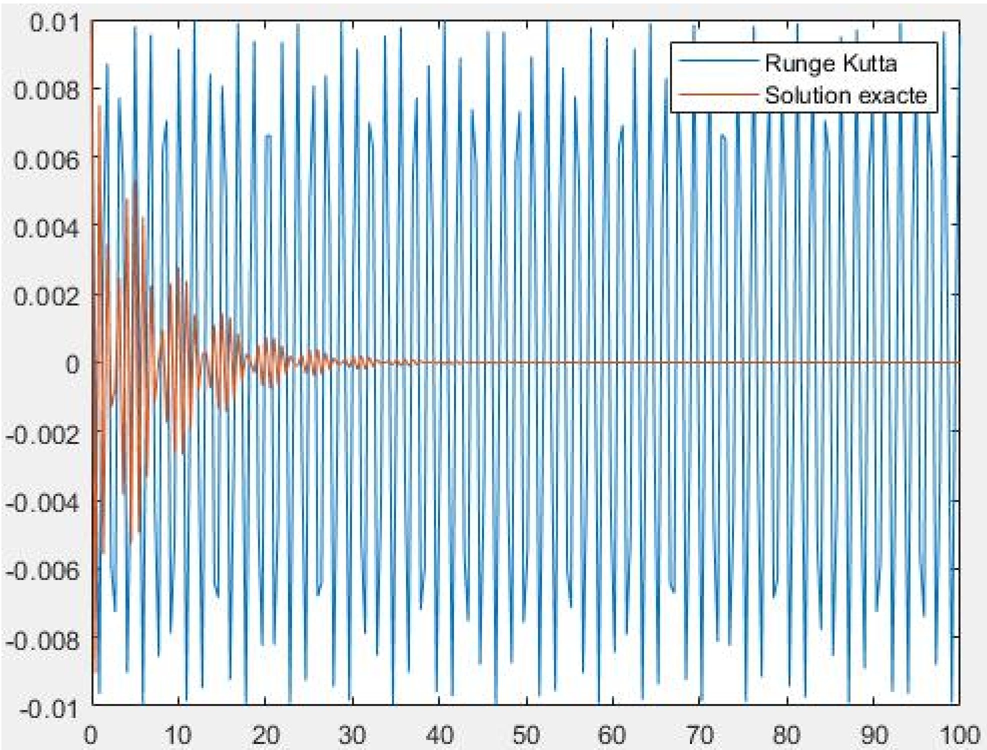


图 8 h=1.0135

Donc, on peut chiosir $h_c = 1.0135$ et $h_{\min} = 1.0134$ et $h_{\max} = 1.0136$.

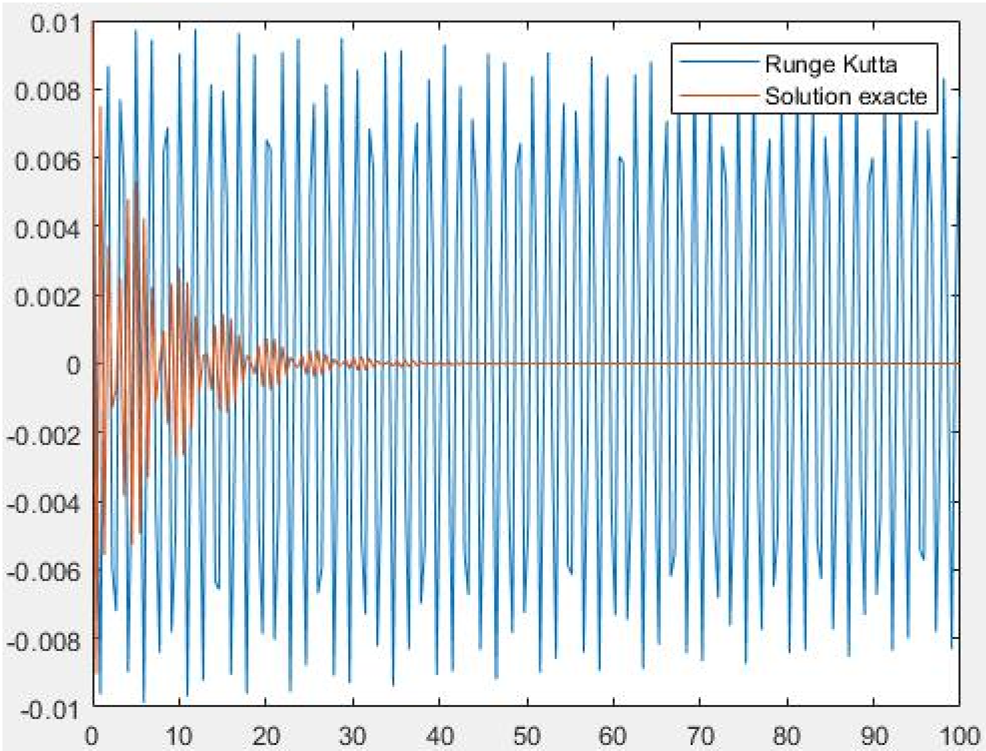


图 9 h=1.0134

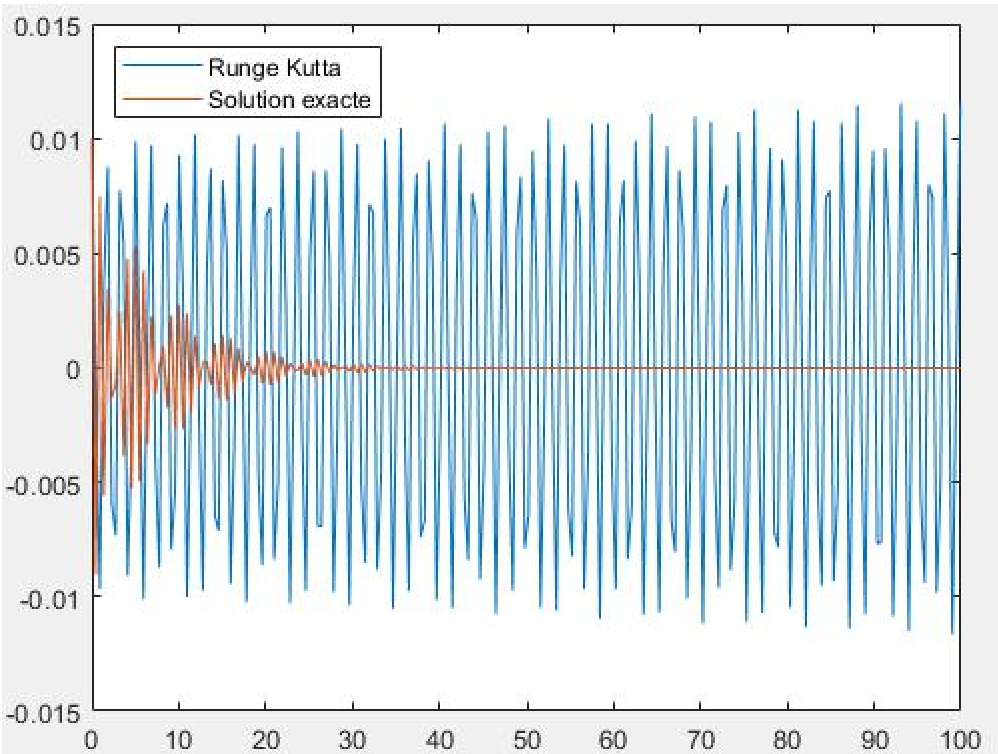


图 10 h=1.0136

Donc, on peut chiosir $h_c = 1.0135$ et $h_{\min} = 1.0134$ et $h_{\max} = 1.0136$.

Etude d ' un double pendule avec l ' hypoth`ese des petits mouvements

1.

1.1

$$ma^2\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}\begin{pmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{pmatrix}+mga\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix}=F_0\sin\omega t\begin{pmatrix}a\\ \frac{a}{\sqrt{2}}\end{pmatrix}$$

$$\ddot{q}=\omega_{02}q+F \quad \text{avec} \quad \omega_{02}=-\frac{g}{a}\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}^{-1}\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix} \quad , \quad F=\frac{F_0}{ma^2}\sin\omega t\begin{pmatrix}a\\ \frac{a}{\sqrt{2}}\end{pmatrix}\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}^{-1} \quad ,$$

$$q=\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix}$$

$$q_{n+1}=q_n+\Delta t\dot{q}_n+\Delta t^2(0.5-\beta)\ddot{q}_n+\Delta t^2\beta\ddot{q}_{n+1}$$
$$\dot{q}_{n+1}=\dot{q}_n+\Delta t(1-\gamma)\ddot{q}_n+\Delta t\gamma\ddot{q}_{n+1}$$

On a $\begin{pmatrix}1-\Delta t^2\beta\omega_{02} & 0\\ -\Delta t\gamma\omega_{02} & 1\end{pmatrix}\begin{pmatrix}q_{n+1}\\ \dot{q}_{n+1}\end{pmatrix}=\begin{pmatrix}1+\Delta t^2(0.5-\beta)\omega_{02} & \Delta t\\ \Delta t(1-\gamma)\omega_{02} & 1\end{pmatrix}\begin{pmatrix}q_n\\ \dot{q}_n\end{pmatrix}+\begin{pmatrix}0.5\Delta t^2\\ \Delta t\end{pmatrix}F$

Donc $B=\begin{pmatrix}1-\Delta t^2\beta\omega_{02} & 0\\ -\Delta t\gamma\omega_{02} & 1\end{pmatrix}, C=\begin{pmatrix}1+\Delta t^2(0.5-\beta)\omega_{02} & \Delta t\\ \Delta t(1-\gamma)\omega_{02} & 1\end{pmatrix}$

et $A=B^{-1}\times C$

Quand on choisi $\Delta t=0.01$

```
m=2;
a=0.5;
g=9.81;
F0=20;
w=2*pi;
theta10=0;
theta20=0;
dtheta10=-1.31519275;
dtheta20=-1.85996342;
q0=[theta10;theta20];
dq0=[dtheta10;dtheta20];
T0=8;
w02=-inv([2,1;1,1])*[2,0;0,1]*g/a;
F=F0*inv([2,1;1,1])*[a;a/sqrt(2)]/(m*a^2);
gama=0.5;
beta=0;
dt=0.01;
B=[[1,0;0,1]-dt^2*beta*w02,[0,0;0,0];-dt*gama*w02,[1,0;0,1]];
C=[[1,0;0,1]+dt^2*(0.5-beta)*w02,dt*[1,0;0,1];dt*(1-gama)*w02,[1,
```

```
0;0,1]];
    A=inv(B)*C;
```

On peut obtenir la matrice d'amplification

A =

9.9804e-01	9.8100e-04	1.0000e-02	0
1.9620e-03	9.9804e-01	0	1.0000e-02
-3.9182e-01	1.9582e-01	9.9804e-01	9.8100e-04
3.9163e-01	-3.9182e-01	1.9620e-03	9.9804e-01

1.2

On utilise la dichotomisation et trouver le pas de temps criticque. Quand $\Delta t \le 0.244$, la module des valeurs propres de la matrice d'amplification est 1.

Donc le pas de temps critique est 0.244.

1.3

$$\ddot{q} = \omega_{02} q + F \quad \text{avec}\ \omega_{02} = -\frac{g}{a}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, F = \frac{F_0}{ma^2}\sin\omega t\begin{pmatrix} a \\ \frac{a}{\sqrt{2}} \end{pmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1}$$

1.4

$$\begin{pmatrix} 1-\Delta t^2\beta\omega_{02} & 0 \\ -\Delta t\gamma\omega_{02} & 1 \end{pmatrix}\begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \end{pmatrix} = \begin{pmatrix} 1+\Delta t^2(0.5-\beta)\omega_{02} & \Delta t \\ \Delta t(1-\gamma)\omega_{02} & 1 \end{pmatrix}\begin{pmatrix} q_n \\ \dot{q}_n \end{pmatrix} + \begin{pmatrix} 0.5\Delta t^2 \\ \Delta t \end{pmatrix} F$$

$$\begin{bmatrix} 1 & 0 & -\Delta t^2\beta \\ 0 & 1 & -\Delta t\gamma \\ -\omega_{02} & 0 & 1 \end{bmatrix}\begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \\ \ddot{q}_{n+1} \end{pmatrix} = \begin{bmatrix} 1 & \Delta t & \Delta t^2(0.5-\beta) \\ 0 & 1 & \Delta t(1-\gamma) \\ 0 & 0 & 0 \end{bmatrix}\begin{pmatrix} q_n \\ \dot{q}_n \\ \ddot{q}_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ F \end{pmatrix}$$

1.5

```
    gama=0.5;
    beta=0;
    dt=0.02;
    B=[[1,0;0,1]-dt^2*beta*w02,[0,0;0,0];-dt*gama*w02,[1,0;0,1]];
    C=[[1,0;0,1]+dt^2*(0.5-beta)*w02,dt*[1,0;0,1];dt*(1-gama)*w02,[1,
0;0,1]];
    A=inv(B)*C;
    t=0:dt:T0;
    U(:,1)=[q0;dq0];
    for i=2:length(t)
        D=[dt*dt*0.5*F*sin(w*(j-1)*dt);dt*F*sin(w*(j-1)*dt)];
        U(:,i)=A*U(:,i-1)+inv(B)*D;
    end
    plot(t,U(1,:))
    hold on;
    plot(t,U(2,:))
    legend('q','dq')
```

1.6

T=0s

0

0

-1.3152e+00

-1.8600e+00

T=0.02s

-2.6452e-02 + 1.4645e-04i

-3.7409e-02 + 2.0711e-04i

-1.3270e+00 + 1.4628e-02i

-1.8766e+00 + 2.0687e-02i

T=0.04s

-5.3078e-02 + 5.8512e-04i

-7.5064e-02 + 8.2748e-04i

-1.3326e+00 + 2.9189e-02i

-1.8846e+00 + 4.1279e-02i

T=4s

-3.3678e-01 + 2.5572e-02i

-4.7628e-01 + 3.6165e-02i

-9.6205e-01 + 1.7292e-01i

-1.3605e+00 + 2.4454e-01i

2.

2.1

Le même résultat avec 1.1

$$\begin{pmatrix} 1-\Delta t^2\beta\omega_{02} & 0 \\ -\Delta t\gamma\omega_{02} & 1 \end{pmatrix}\begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \end{pmatrix} = \begin{pmatrix} 1+\Delta t^2(0.5-\beta)\omega_{02} & \Delta t \\ \Delta t(1-\gamma)\omega_{02} & 1 \end{pmatrix}\begin{pmatrix} q_n \\ \dot{q}_n \end{pmatrix} + \begin{pmatrix} 0.5\Delta t^2 \\ \Delta t \end{pmatrix} F$$

$$B = \begin{pmatrix} 1-\Delta t^2\beta\omega_{02} & 0 \\ -\Delta t\gamma\omega_{02} & 1 \end{pmatrix}, C = \begin{pmatrix} 1+\Delta t^2(0.5-\beta)\omega_{02} & \Delta t \\ \Delta t(1-\gamma)\omega_{02} & 1 \end{pmatrix}$$

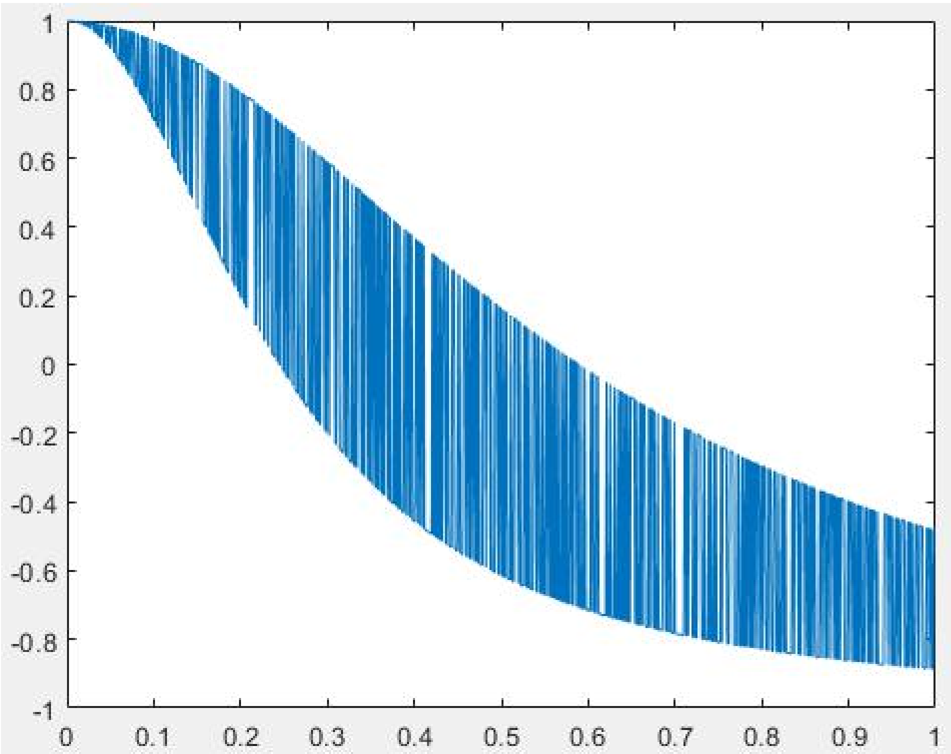
Et la matrice d'amplificatin $A = B^{-1} \times C$

2.2

```
clear all;
m=2;
a=0.5;
g=9.81;
F0=20;
w=2*pi;
theta10=0;
theta20=0;
```

```
dtheta10=-1.31519275;
dtheta20=-1.85996342;
q0=[theta10;theta20];
dq0=[dtheta10;dtheta20];
T0=8;
w02=-inv([2,1;1,1])*[2,0;0,1]*g/a;
gama=0.5;
beta=0.25;
pdt=0.001:0.001:1;
for j=1:length(pdt)
    dt=pdt(j);
    B=[[1,0;0,1]-dt^2*beta*w02,[0,0;0,0];-dt*gama*w02,[1,0;0,1]];

C=[[1,0;0,1]+dt^2*(0.5-beta)*w02,dt*[1,0;0,1];dt*(1-gama)*w02,[1,0;0,
1]];
    A=inv(B)*C;
    vp(:,j)=eig(A);
    vpmax(j)=real(max(vp(:,j)));
end
plot(pdt,vpmax)
```



Le plus grand valeur propre de cette matrice est toujourd entre -1et 1. Le module de valeur propre de la matrice d'amplification est toujours à inférieur à 1. Donc le résultat est toujours stable.

2.3

$$\ddot{q} = \omega_{02} q + F \quad \text{avec}\ \omega_{02} = -\frac{g}{a}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, F = \frac{F_0}{ma^2}\sin\omega t\begin{pmatrix} a \\ \frac{a}{\sqrt{2}} \end{pmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1}$$

2.4

$$\begin{pmatrix} 1\text{-}\Delta t^2\beta\omega_{02} & 0 \\ -\Delta t\gamma\omega_{02} & 1 \end{pmatrix}\begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \end{pmatrix} = \begin{pmatrix} 1+\Delta t^2(0.5-\beta)\omega_{02} & \Delta t \\ \Delta t(1-\gamma)\omega_{02} & 1 \end{pmatrix}\begin{pmatrix} q_n \\ \dot{q}_n \end{pmatrix} + \begin{pmatrix} 0.5\Delta t^2 \\ \Delta t \end{pmatrix} F$$

$$\begin{bmatrix} 1 & 0 & -\Delta t^2\beta \\ 0 & 1 & -\Delta t\gamma \\ -\omega_{02} & 0 & 1 \end{bmatrix}\begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \\ \ddot{q}_{n+1} \end{pmatrix} = \begin{bmatrix} 1 & \Delta t & \Delta t^2(0.5-\beta) \\ 0 & 1 & \Delta t(1-\gamma) \\ 0 & 0 & 0 \end{bmatrix}\begin{pmatrix} q_n \\ \dot{q}_n \\ \ddot{q}_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ F \end{pmatrix}$$

2.5

```
gama=0.5;
beta=0.25;
dt=0.02;
B=[[1,0;0,1]-dt^2*beta*w02,[0,0;0,0];-dt*gama*w02,[1,0;0,1]];
C=[[1,0;0,1]+dt^2*(0.5-beta)*w02,dt*[1,0;0,1];dt*(1-gama)*w02,[1,
0;0,1]];
A=inv(B)*C;
t=0:dt:T0;
U(:,1)=[q0;dq0];
for i=2:length(t)
    D=[dt*dt*0.5*F*sin(w*(j-1)*dt);dt*F*sin(w*(j-1)*dt)];
    U(:,i)=A*U(:,i-1)+inv(B)*D;
end
plot(t,U(1,:))
hold on;
plot(t,U(2,:))
legend('q','dq')
```

2.6

T=0s

0

0

-1.3152e+00

-1.8600e+00

T=0.02s

-2.6421e-02 + 1.4628e-04i

-3.7366e-02 + 2.0687e-04i

-1.3270e+00 + 1.4628e-02i

-1.8766e+00 + 2.0687e-02i

T=0.04s

-5.3017e-02 + 5.8445e-04i

-7.4978e-02 + 8.2653e-04i

-1.3326e+00 + 2.9189e-02i

-1.8846e+00 + 4.1279e-02i

T=4s

-3.3439e-01 + 2.5178e-02i

-4.7290e-01 + 3.5607e-02i

-9.6927e-01 + 1.7201e-01i

-1.3708e+00 + 2.4326e-01i

Oscillateur non linéaire à un degré de liberté

1.

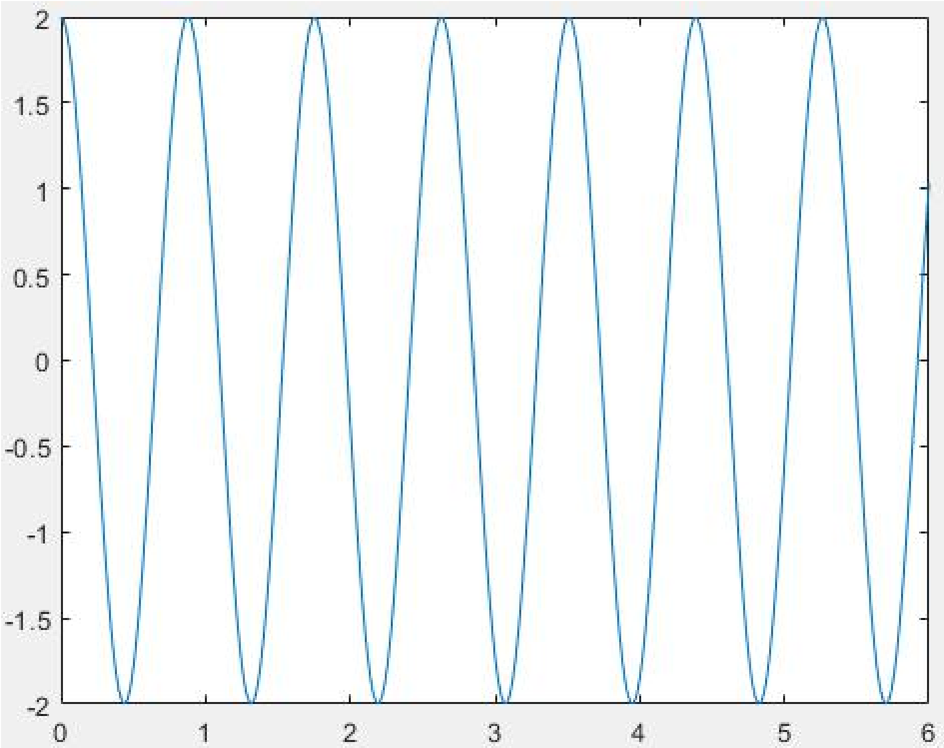
1.1

Quand $q_j, \dot{q}_j, \ddot{q}_j$ est connu, on peut obtenir

$$
\begin{cases}
\dot{q}_{j+1} = q_j + \Delta t \dot{q}_j + \Delta t^2 (0.5 - \beta) \ddot{q}_j \\
\ddot{q}_{j+1} = -\omega_0{}^2 q_{j+1} (1 + a q_{j+1}^2 \ ) \\
\dot{q}_{j+1} = \dot{q}_j + \Delta t (1 - \gamma) \ddot{q}_j + \gamma \Delta t \ddot{q}_{j+1}
\end{cases}
$$

1.2

```
clear all;
q0=2;
dq0=0;
w0=2*pi;
a=0.1;
T0=6;
m=1;
gama=0.5;
beta=0;
dt1=0.02;
t1=0:dt1:T0;
q1(1)=q0;
dq1(1)=dq0;
ddq1(1)=-w0^2*q1(1)*(1+a*q1(1)*q1(1));
for j = 2:length(t1)
    q1(j)=q1(j-1)+dt1*dq1(j-1)+dt1*dt1*0.5*ddq1(j-1);
    ddq1(j)=-w0^2*q1(j)*(1+a*q1(j)*q1(j));
    dq1(j)=dq1(j-1)+dt1*(1-gama)*ddq1(j-1)+gama*dt1*ddq1(j);
end
plot(t1,q1)
```



1.3

```
valeursdeq1=[q1(1),q1(2),q1(3),q1(length(t1))]
valeursdeq1 =
   2.0000    1.9779    1.9123    1.0329
```

Donc, q(0)=2.0000, q(0.02)=1.9779, q(0.04)=1.9123,q(6)=1.0329

2

2.1

$$\ddot{q}_{j+1}, \dot{q}_{j+1}, q_{j+1}$$

2.2

$$q_{j+1} = q^*_{j+1} + \Delta q_{j+1}$$

$$\dot{q}_{j+1} = \dot{q}^*_{j+1} + \Delta \dot{q}_{j+1}$$

$$\ddot{q}_{j+1} = \ddot{q}^*_{j+1} + \Delta \ddot{q}_{j+1}$$

2.3

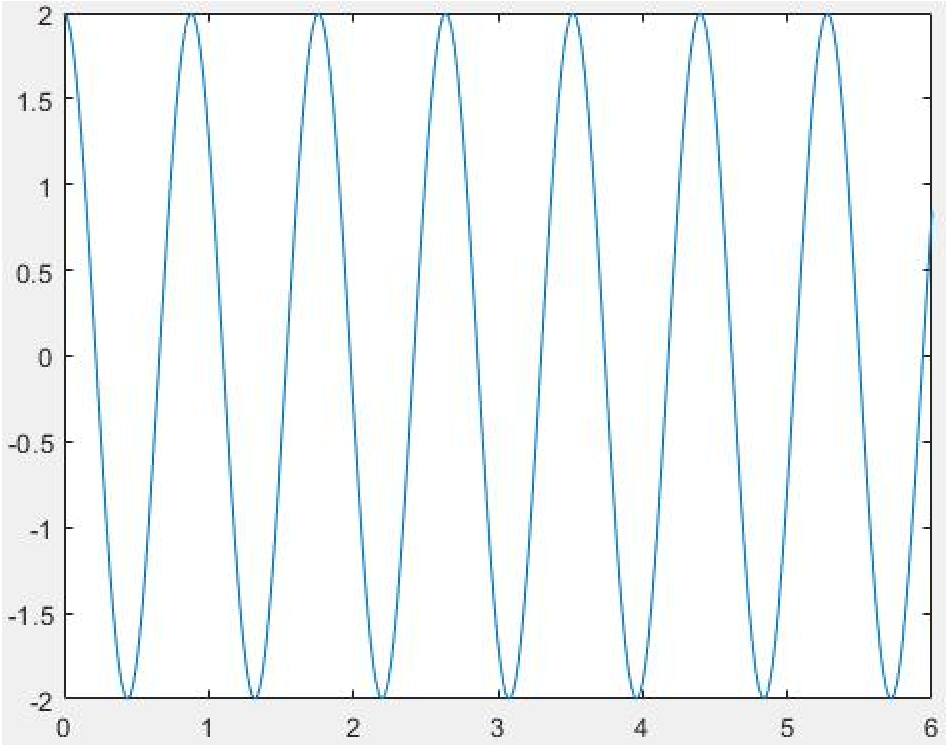
$$\Delta\ddot{q}_{n+1} = -\frac{f(\ddot{q}^*_{n+1}, \dot{q}^*_{n+1}, q^*_{n+1})}{\dfrac{\partial f}{\partial \ddot{q}^*_{n+1}} + \dfrac{\partial f}{\partial q^*_{n+1}} \beta \Delta t^2} \quad \text{avec } f(\ddot{q}^*_{n+1}, \dot{q}^*_{n+1}, q^*_{n+1}) = \ddot{q}^*_{n+1} + \omega_0^2 q^*_{n+1}(1 + a{q^*_{n+1}}^2)$$

Donc on a $\Delta\ddot{q}_{n+1} = -\dfrac{\ddot{q}^*_{n+1} + \omega_0^2 q^*_{n+1}(1 + a{q^*_{n+1}}^2)}{1 + (\omega_0^2 + 3a\omega_0^2 {q^*_{n+1}}^2)\beta\Delta t^2}$

$$\Delta\dot{q}_{n+1} = \dot{q}_n - \dot{q}^*_n = \gamma\Delta t \ddot{q}^*_{n+1} \quad \Delta q_{n+1} = q_{n+1} - q^*_{n+1} = \beta\Delta t^2 \ddot{q}^*_{n+1}$$

```
clear all;
q0=2;
dq0=0;
w0=2*pi;
a=0.1;
T0=6;
m=1;
gama=0.5;
beta=0.25;
dt=0.02;
t=0:dt:T0;
q(1)=q0;
dq(1)=dq0;
ddq(1)=-w0^2*q(1)*(1+a*q(1)*q(1));
sigma=0.01
qe(1)=q(1);
dqe(1)=dq(1);
ddqe(1)=ddq(1);
for j=2:length(t)
    ddqe(j)=0;
    dqe(j)=dqe(j-1)+dt*(1-gama)*ddqe(j-1);
    qe(j)=qe(j-1)+dt*dqe(j-1)+dt^2*(0.5-beta)*ddqe(j-1);
    erreur=ddqe(j)+w0^2*qe(j)*(1+a*qe(j)*qe(j));
    while abs(erreur)>=sigma

        deltaddq(j)=-(ddqe(j)+w0^2*qe(j)*(1+a*qe(j)*qe(j)))/(1+w0^2
        *beta*dt^2*(1+3*a*qe(j)*qe(j)));
        deltaq(j)=beta*dt*dt*deltaddq(j);
        deltadq(j)=gama*dt*deltaddq(j);
        qe(j)=qe(j)+deltaq(j);
        dqe(j)=dqe(j)+deltadq(j);
        ddqe(j)=ddqe(j)+deltaddq(j);
        erreur=ddqe(j)+w0*w0*qe(j)*(1+a*qe(j)*qe(j));
    end
    q(j)=qe(j);
    dq(j)=dqe(j);
    ddq(j)=ddqe(j);
end
plot(t,q)
```



2.4

```
valeurdeq=[q(1),q(2),q(3),q(length(t))]
valeurdeq =

    2.0000    1.9781    1.9131    0.8478
```

3.

3.1

L’énergie peut se diviser en 2 parties : l’énergie cinétique et l’énergie potentiel

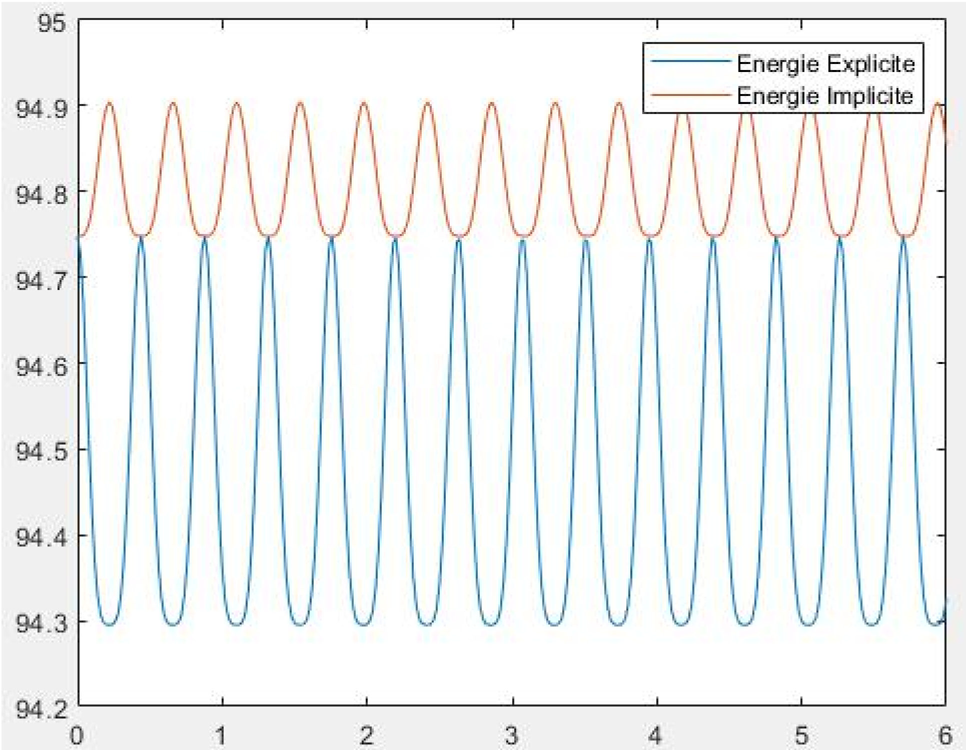
L’energie cinétique : $E_c = \frac{1}{2} m\dot{q}^2$

L’énergie potentiel : $E_p = \int \vec{F} dq = \frac{1}{2} kq^2 + \frac{1}{4} kaq^4$

3.2

```
k=w0^2*m;
for j=1:length(t)
    Ece(j)=0.5*m*dq1(j)^2;
    Epe(j)=0.5*k*q1(j)*q1(j)+0.25*k*a*q1(j)^4;
    Ee(j)=Ece(j)+Epe(j);
    Eci(j)=0.5*m*dq(j)^2;
    Epi(j)=0.5*k*q(j)*q(j)+0.25*k*a*q(j)^4;
    Ei(j)=Eci(j)+Epi(j);
end
plot(t,Ee)
```

```
hold on;
plot(t,Ei)
legend('Energie Explicite','Energie Implicite')
```



3.3

On trouve que l'énergie de Newmark implicite est plus stable que l'énergie de Newmark explicite.