Etude d’un oscillateur linéaire amorti à un degré de liberté

1.1 Résolution avec un schéma d’ EULER explicite

On le fait sous forme matricielle:

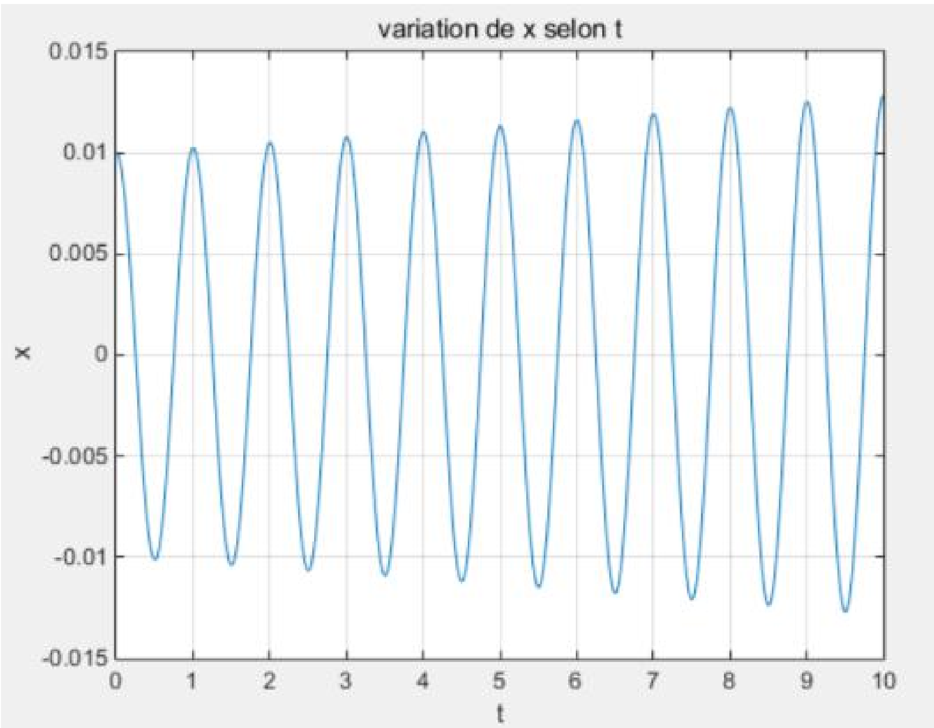
$$\begin{vmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{vmatrix} = \begin{vmatrix} q_j \\ \dot{q}_j \end{vmatrix} + \Delta t \begin{vmatrix} \dot{q}_j \\ \ddot{q}_j \end{vmatrix} = \begin{vmatrix} q_j \\ \dot{q}_j \end{vmatrix} + \Delta t \begin{vmatrix} \dot{q}_j \\ -2\varepsilon w_0 \dot{q}_j - w_0^2 q_j \end{vmatrix}$$

$$= \begin{pmatrix} 1 & \Delta t \\ -w_0^2 \Delta t & 1-2\Delta t \varepsilon w_0 \end{pmatrix} \begin{vmatrix} q_j \\ \dot{q}_j \end{vmatrix} \quad \text{donc} \quad \begin{vmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{vmatrix} = A \begin{vmatrix} q_j \\ \dot{q}_j \end{vmatrix}$$

$$\text{avec } A = \begin{pmatrix} 1 & \Delta t \\ -w_0^2 \Delta t & 1-2\Delta t \varepsilon w_0 \end{pmatrix}$$

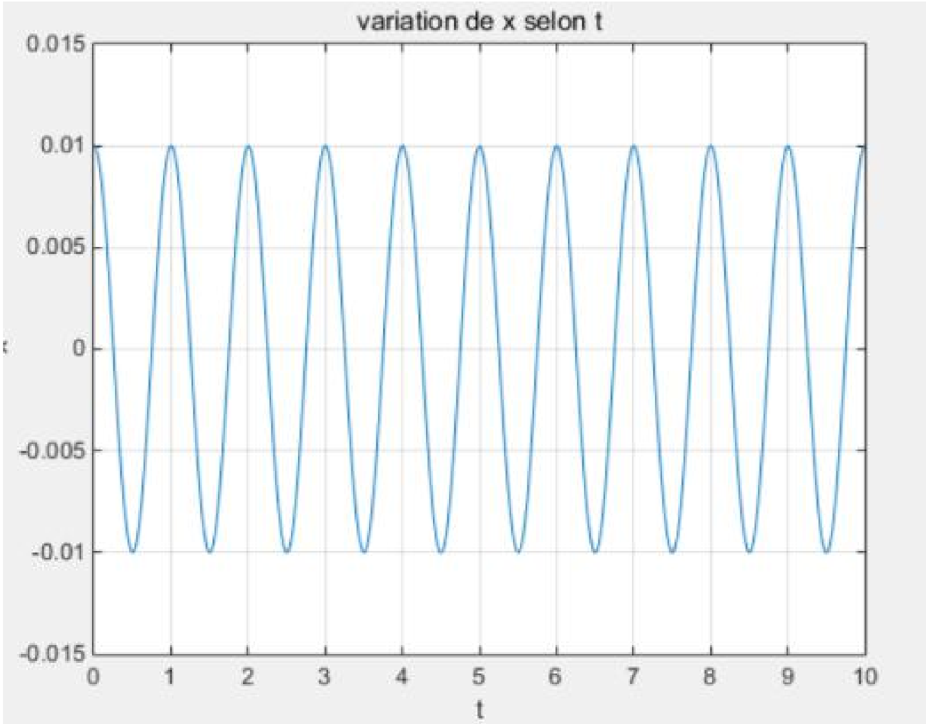
```
%% Resolution avec un schema d'Euler explicite
Aex = [1 dt;-w0^2*dt 1-2*dt*eps*w0];
```

A) Si dt > 2*eps/w0



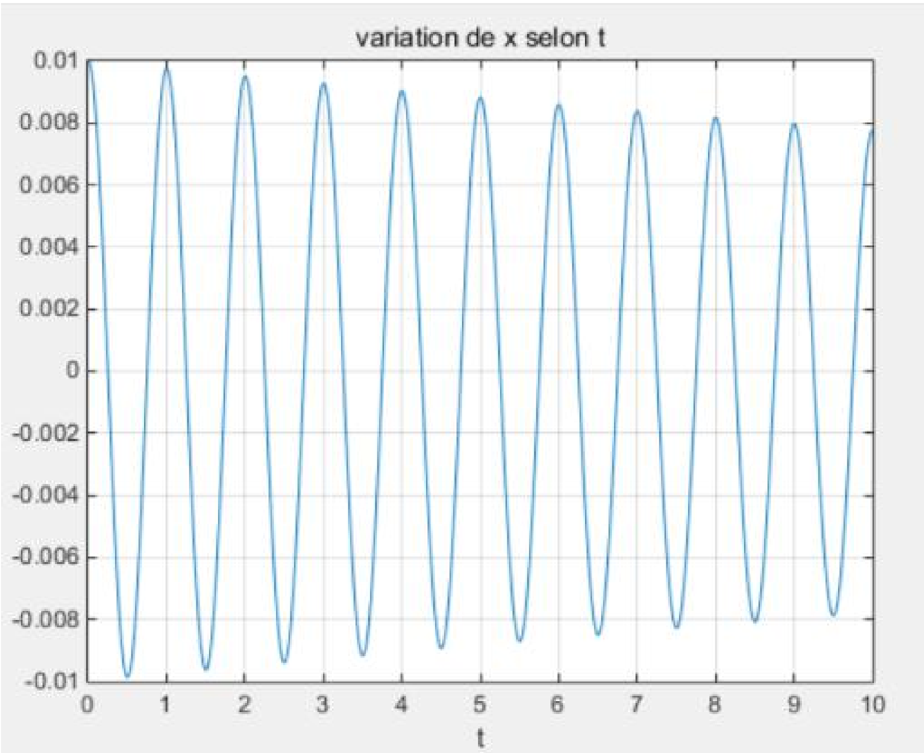
Il est instable

B) Si dt = 2*eps/w0



Il est de regime critique

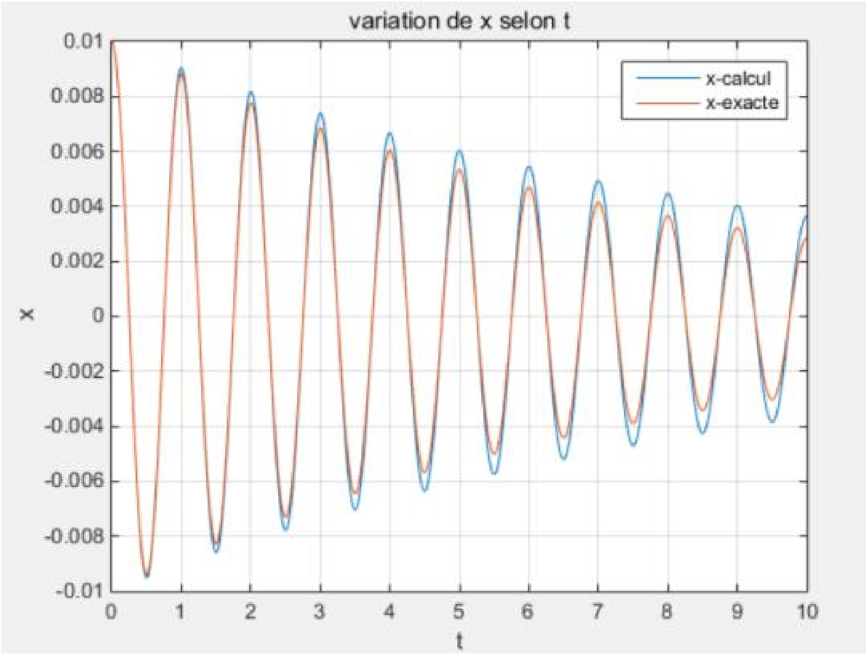
C) Si dt = 2*eps/w0*0.8



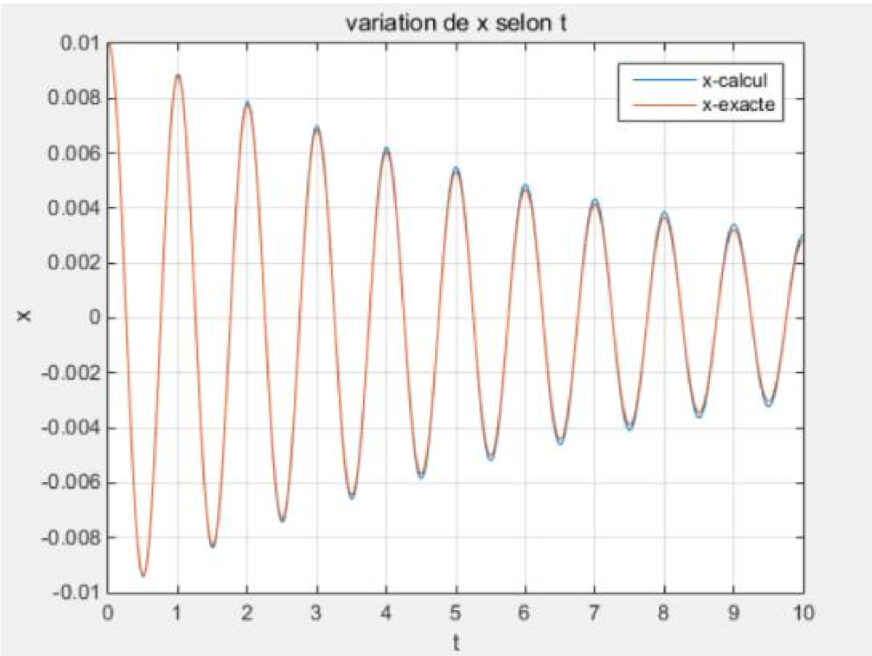
Il a tendance de converger

D)

dt = 0.2 * 2*eps/w0



dt = 0.05 * 2*eps/w0



Critère de la précision: x doit converger comme x-exact, et puis, la différence entre x-calcul et x-exacte soit être petite.

Code de 1.1

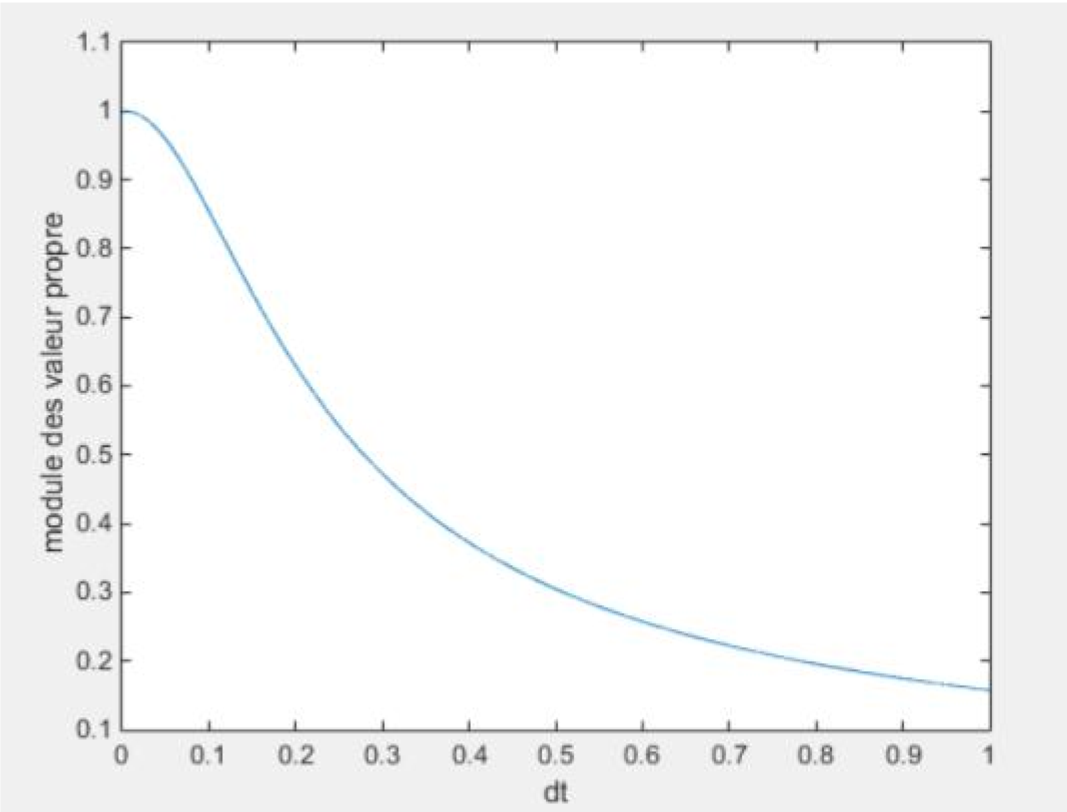
```
%% Etude de l'oscillateur lineaire amorti a un degre de liberte
clc
clear all
close all
  %% Initialisation
  T0 = 1;
  w0 = 2*pi/T0;
  eps = 0.02;
  %b = 2*eps*w0*m;
  x0 = 0.01;
  xp0 = 0;
  %dt = 2*eps/w0;
  %dt = 2*eps/w0*1.2;
  dt = 2*eps/w0*0.05;
  U(1,1) = x0;
  U(2,1) = xp0;
  T = [0:dt:10*T0];
  omega = w0* power(1-eps^2,1/2);
  xexacte = exp(-eps*w0*T).*(x0*cos(omega*T) +
(eps*w0*x0+xp0)/omega*sin(omega*T));
  %% Resolution avec un schema d'Euler explicite
  Aex = [1 dt;-w0^2*dt 1-2*dt*eps*w0];
  Uex = U;
  %% solution explicite
  for j = 0:dt:10*T0
      a = round(j/dt)-156
      Uex(:,round(j/dt)+2) = Aex * Uex(:,round(j/dt)+1);
  end
  %% plot
  Uex(:,end) = [];
  figure(1)
  plot([0:dt:10*T0],Uex(1,:))
  grid on
  xlabel('t')
  ylabel('x')
  title('variation de x selon t');

  figure(2)
  plot([0:dt:10*T0],Uex(1,:),[0:dt:10*T0],xexacte)
  grid on
```

```
xlabel('t')
ylabel('x')
title('variation de x selon t');
legend('x-calcul','x-exacte')
```

1.2 Résolution avec un schéma d'EULER implicite

Si on fait avec un schéma d'Euler implicite, on calcule les module des valeur propres de la matrice d'amplification en dt différente



On a vu que la fonction de module de valeur propre de la matrice d'amplification est decroissante, donc il n'y a pas de pas de temps critique.

Code de 1.2

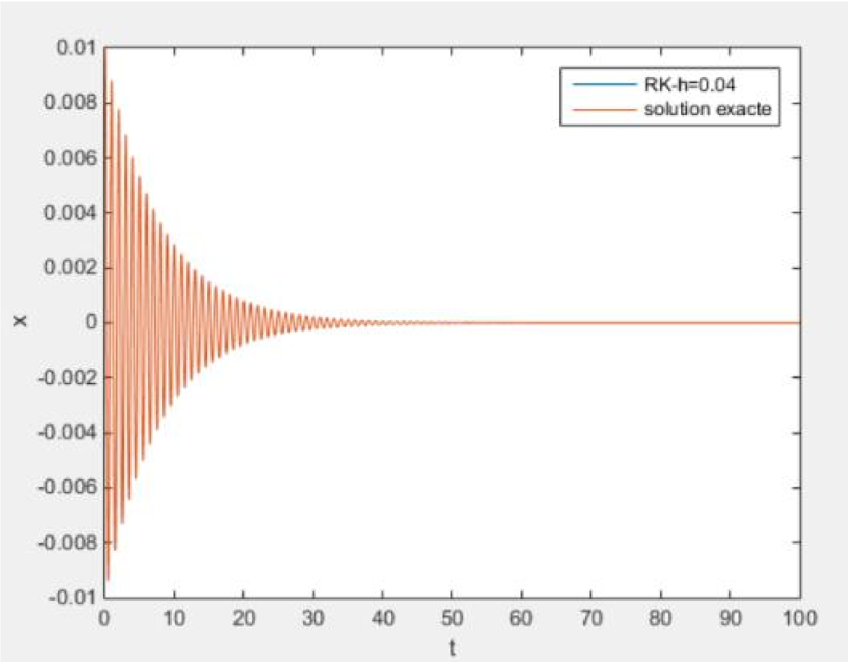
```
%% Implicite
Aim = inv(Aex);% determiner la matrice d'amplification Aim
dt_tt = 0:0.0001:1;
%Chercher le pas de temps critique
for i = 1:length(dt_tt)
    Aex_tt{1,i} = [1 dt_tt(i);-w0^2*dt_tt(i) 1-2*dt_tt(i)*eps*w0];
```

```
    Aim_tt{1,i} = inv(Aex_tt{1,i});
    vpAim_tt(:,i) = eig(Aim_tt{1,i});
    module_tt(i) = sqrt(real(vpAim_tt(1,i))^2+imag(vpAim_tt(1,i))^2);
end
plot(dt_tt,module_tt);
xlabel('dt');
ylabel('module des valeur propre');
```

1.3a

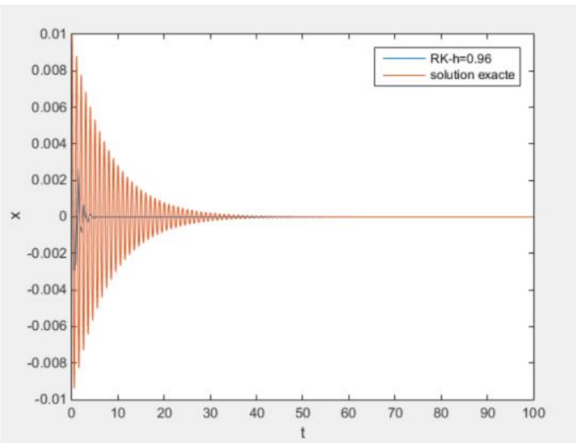
```
h = 0.04;
%h = 0.96;
%h = 1.04;
```

Quand h = 0.04



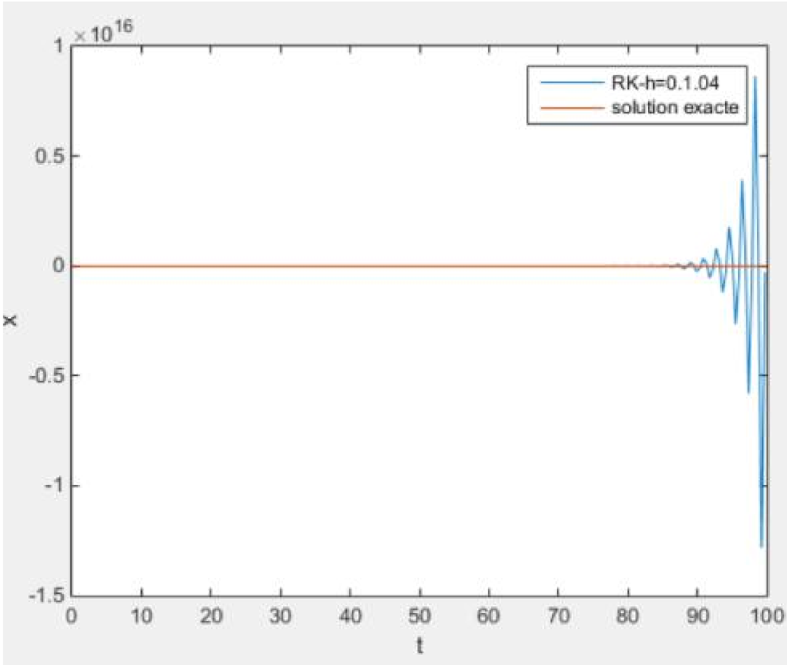
Le résultat est très proche à la solution exacte, stable et précis.

Quand h = 0.96



Le résultat est stable mais pas précis

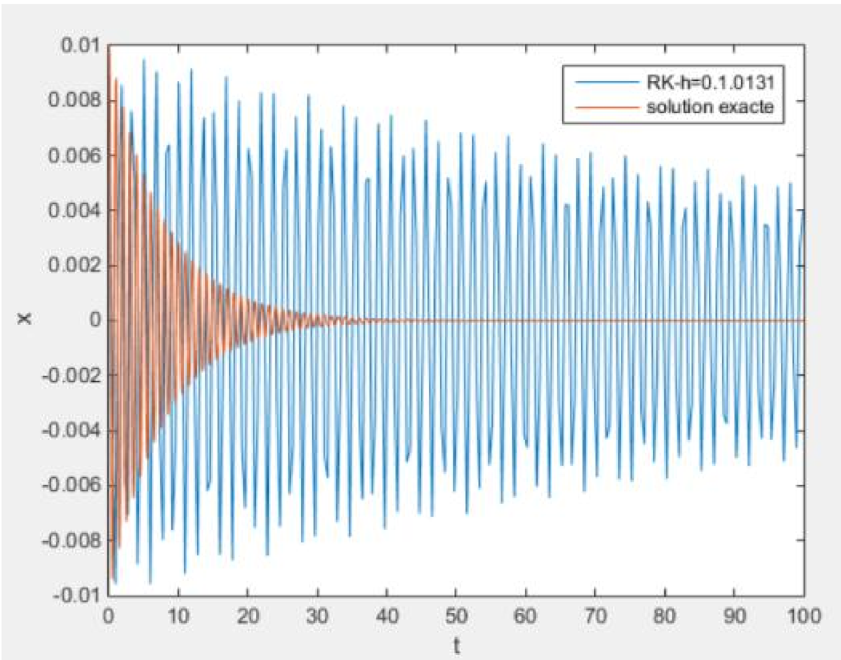
Quand h = 1.04



Le résultat n'est pas stable ni précis, il deverge

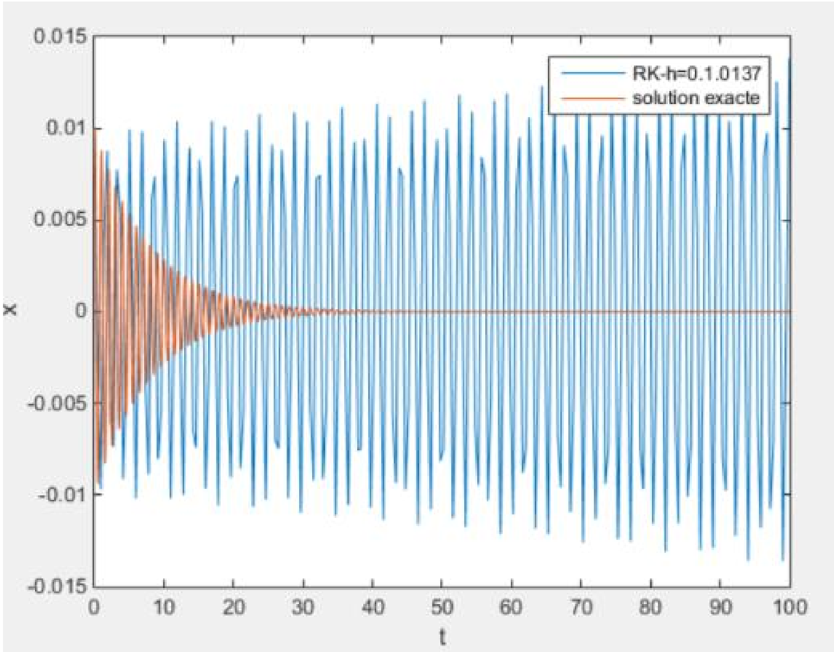
1.3b

On essaye comme suivant
Quand h = 1.0131



Convergente

Quand h = 1.0137



Divergent

Donc

et on a

$: \mathrm{dt} = h_c \times \frac{2\sqrt{2}}{\omega_0}$,

1.0131< hc<1.0137

Code de 1.3

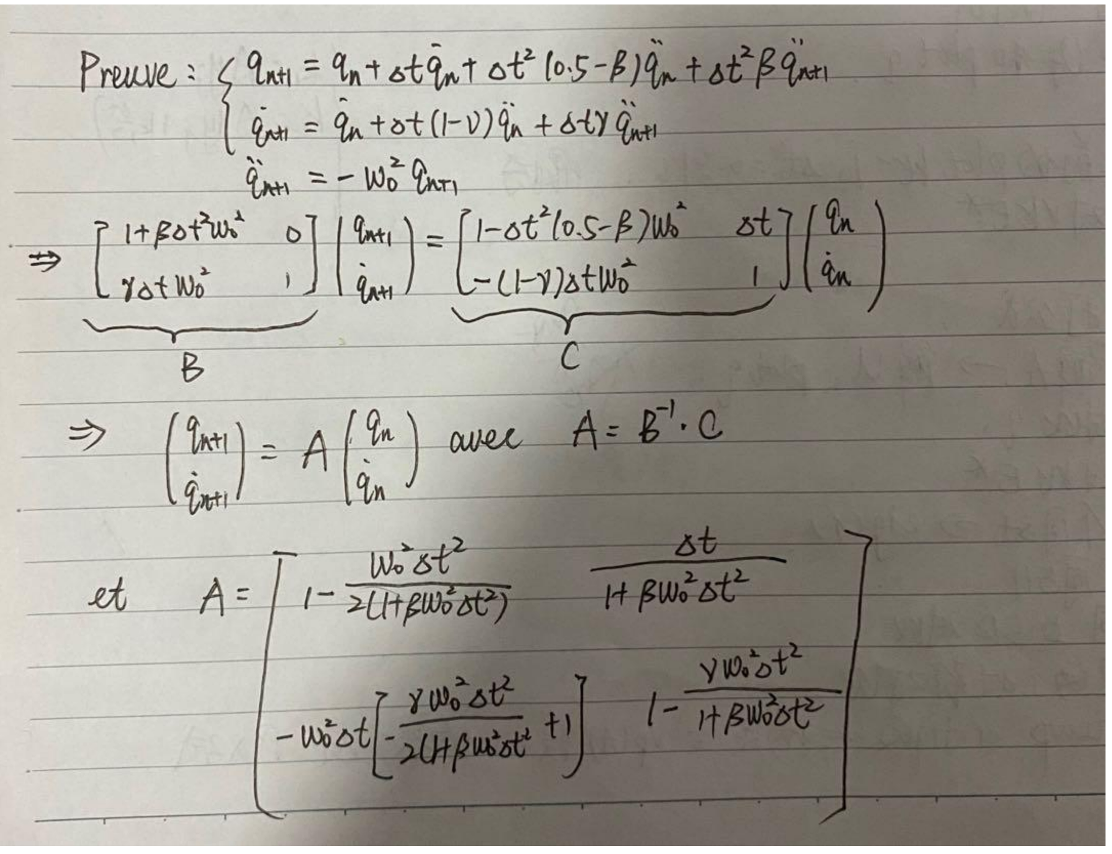
```
%% resolution avec un schema RUNGE KUTTA
% 1.3a
%h = 0.04;
% h = 0.96;
h = 1.0131
%h = 1.04;
dt_rk = h*2*sqrt(2)/w0;
t_rk = 0:dt_rk:100*T0;
Ark = [0,1;-w0^2,-2*eps*w0];
Urk(:,1) = [x0;xp0];
for i = 1:length(t_rk)-1
    k1 = Ark * Urk(:,i);
    k2 = Ark * (Urk(:,i) + k1*dt_rk/2);
    k3 = Ark * (Urk(:,i) + k2*dt_rk/2);
    k4 = Ark * (Urk(:,i) + k3*dt_rk);
    K = (k1+2*k2+2*k3+k4)/6;
```

```
    Urk(:,i+1) = Urk(:,i) + K*dt_rk;
end
figure(2)
plot(t_rk,Urk(1,:),T,xexacte)
%legend('RK-h=0.04','solution exacte')
%legend('RK-h=0.96','solution exacte')
legend('RK-h=0.1.0131','solution exacte')
xlabel('t');
ylabel('x');
```

Etude d'un double pendule avec l'hypothèse des petits mouvements

1. Newmark explicite, gamma=0.5 beta=0

1.1 On en deduit la matrice d'amplification:



Preuve :

$$\begin{cases} q_{n+1} = q_n + \Delta t\,\dot{q}_n + \Delta t^2 (0.5-\beta)\ddot{q}_n + \Delta t^2 \beta\, \ddot{q}_{n+1} \\ \dot{q}_{n+1} = \dot{q}_n + \Delta t (1-\gamma)\ddot{q}_n + \Delta t\, \gamma\, \ddot{q}_{n+1} \\ \ddot{q}_{n+1} = -\omega_0^2 q_{n+1} \end{cases}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 1+\beta\Delta t^2\omega_0^2 & 0 \\ \gamma\Delta t\,\omega_0^2 & 1 \end{bmatrix}}_{B} \begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \end{pmatrix} = \underbrace{\begin{bmatrix} 1-\Delta t^2(0.5-\beta)\omega_0^2 & \Delta t \\ -(1-\gamma)\Delta t\,\omega_0^2 & 1 \end{bmatrix}}_{C} \begin{pmatrix} q_n \\ \dot{q}_n \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \end{pmatrix} = A \begin{pmatrix} q_n \\ \dot{q}_n \end{pmatrix} \quad \text{avec} \quad A = B^{-1} \cdot C$$

$$\text{et} \quad A = \begin{bmatrix} 1 - \dfrac{\omega_0^2\Delta t^2}{2(1+\beta\omega_0^2\Delta t^2)} & \dfrac{\Delta t}{1+\beta\omega_0^2\Delta t^2} \\ -\omega_0^2\Delta t\left[-\dfrac{\gamma\omega_0^2\Delta t^2}{2(1+\beta\omega_0^2\Delta t^2)} + 1\right] & 1 - \dfrac{\gamma\omega_0^2\Delta t^2}{1+\beta\omega_0^2\Delta t^2} \end{bmatrix}$$

1.2

D'apres le cour quand gamma = 0.5, il faut que :

$$\left(\gamma + \frac{1}{2}\right)^2 - 4\beta \le \frac{4}{\omega_0{}^2\Delta t^2}$$

On a maintenant gamma = 0.5 et beta = 0, donc dt<2/w0, alors le pas de temps critique est **0.4515**

1.3

$$ma^2 \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \ddot{q}_0^2 + mga \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} q_0 = 0$$

$$\Leftrightarrow \quad \ddot{q}_0^2 = -\frac{g}{a} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} q_0$$

$$= -\frac{g}{a} \begin{bmatrix} 2 & -1 \\ -2 & 2 \end{bmatrix} q_0$$

1.4

Si on ajoute la force qui est dépendante du temps

$$q_{n+1} = q_n + \Delta t \dot{q}_n + \Delta t^2 (0.5-\beta) \ddot{q}_n + \Delta t^2 \beta \ddot{q}_{n+1}$$

et $$\dot{q}_{n+1} = \dot{q}_n + \Delta t (1-\gamma) \ddot{q}_n + \Delta t \gamma \ddot{q}_{n+1}$$

$$\Rightarrow ma^2 \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \ddot{q}_n + mga \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} q_n = F_0 \sin\omega t \begin{vmatrix} a \\ \frac{a}{\sqrt{2}} \end{vmatrix}$$

1.5

```
%% premier cas Q1

gamma1 = 0.5;
beta1 = 0;
dt = 0.02;
T = 0:dt:T0;
M = inv([2,1;1,1])*m*g*a*[2,0;0,1]/(m*a^2);
M1 = F0*inv([2,1;1,1])*[a;a/sqrt(2)]/(m*a^2);

B = [[1 0;0 1]+beta1*dt^2*M 0*[1 0;0 1];gamma1*dt*M [1 0;0 1]];
C = [[1 0;0 1]-(0.5-beta1)*dt^2*M dt*[1 0;0 1];-(1-gamma1)*dt*M [1 0;0
1] ];
A = inv(B)*C;

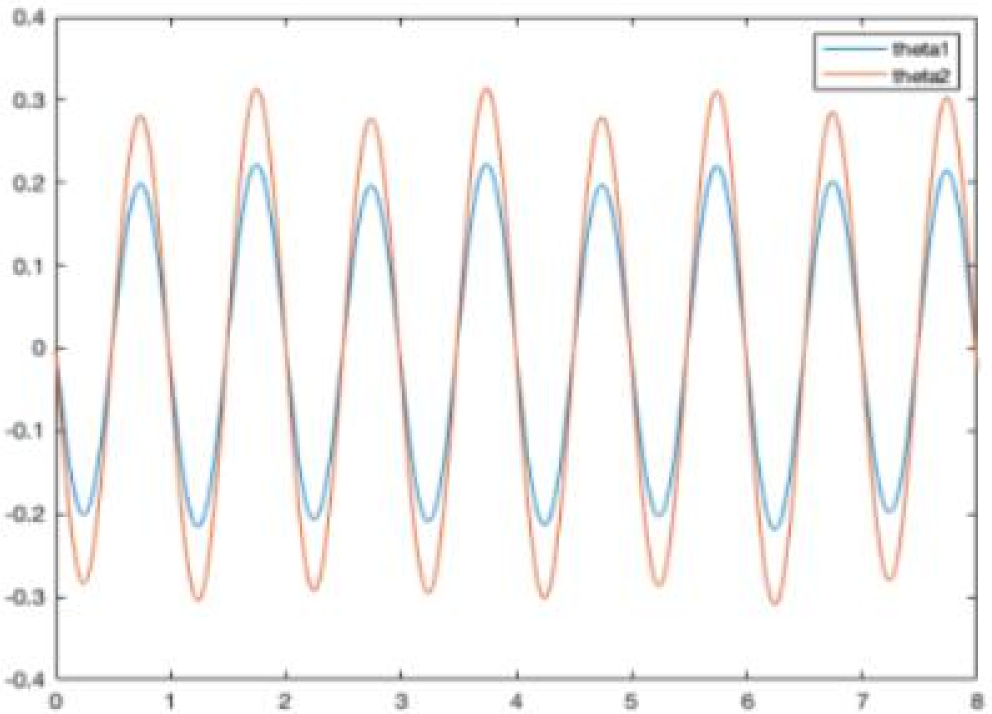
vp = eig(A)

rl = [real(vp(2));real(vp(4))];
```

```
ig = [imag(vp(2));imag(vp(4))];
nm = [abs(vp(2));abs(vp(4))];
U(:,1) = [theta0;thetap0];
for i = 1:length(T)-1
    F =
[dt^2*(0.5-beta1)*M1*sin(w*i*dt);dt*(1-gamma1)*M1*sin(w*i*dt)];
    U(:,i+1) = A*U(:,i)+inv(B)*F;
end

plot(T,U(1,:),T,U(2,:))
grid on
legend('theta1','theta1')
```

On obtient le resultat comme suivant



1.6 S

Quand t = 0

```
theta =

     0
     0

thetap =

   -1.3152
   -1.8600

thetapp =

   1.0e-14 *

    0.0717
    0.1015
```

Quand t = dt

```
theta =

   -0.0262
   -0.0370
```

```
thetap =

   -1.3048
   -1.8453
```

```
thetapp =

    0.3006
    0.4251
```

Quand t = 2dt

```
theta =

   -0.0519
   -0.0734
```

```
thetap =

   -1.2813
   -1.8120
```

```
thetapp =

    0.5965
    0.8436
```

Quand t = 0.5

```
theta =

   -0.1817
   -0.2569
```

```
thetap =

    0.6934
    0.9806
```

```
thetapp =

    2.0879
    2.9527
```

2. Newmark implicite, gamma=0.5 beta=0.25

2.1 De meme avec partie 1

Preuve :

$$\begin{cases} q_{n+1} = q_n + \Delta t\,\dot{q}_n + \Delta t^2 (0.5-\beta)\ddot{q}_n + \Delta t^2 \beta \ddot{q}_{n+1} \\ \dot{q}_{n+1} = \dot{q}_n + \Delta t (1-\gamma)\ddot{q}_n + \Delta t \gamma \ddot{q}_{n+1} \\ \ddot{q}_{n+1} = -\omega_0^2 q_{n+1} \end{cases}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 1+\beta\Delta t^2\omega_0^2 & 0 \\ \gamma\Delta t\,\omega_0^2 & 1 \end{bmatrix}}_{B} \begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \end{pmatrix} = \underbrace{\begin{bmatrix} 1-\Delta t^2(0.5-\beta)\omega_0^2 & \Delta t \\ -(1-\gamma)\Delta t\,\omega_0^2 & 1 \end{bmatrix}}_{C} \begin{pmatrix} q_n \\ \dot{q}_n \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \end{pmatrix} = A \begin{pmatrix} q_n \\ \dot{q}_n \end{pmatrix} \quad \text{avec} \quad A = B^{-1}\cdot C$$

$$\text{et} \quad A = \begin{bmatrix} 1 - \dfrac{\omega_0^2\Delta t^2}{2(1+\beta\omega_0^2\Delta t^2)} & \dfrac{\Delta t}{1+\beta\omega_0^2\Delta t^2} \\ -\omega_0^2\Delta t\left[-\dfrac{\gamma\omega_0^2\Delta t^2}{2(1+\beta\omega_0^2\Delta t^2)} + 1\right] & 1 - \dfrac{\gamma\omega_0^2\Delta t^2}{1+\beta\omega_0^2\Delta t^2} \end{bmatrix}$$

2.2

A partir de partie 1, il faut juste changer la valeur de beta = 0.25

2.3

$$ma^2\begin{bmatrix}2&1\\1&1\end{bmatrix}\ddot{q}_0^2 + mga\begin{bmatrix}2&0\\0&1\end{bmatrix}q_0 = 0$$

$$\Leftrightarrow \quad \ddot{q}_0^2 = -\frac{g}{a}\begin{bmatrix}1&-1\\-1&2\end{bmatrix}\begin{bmatrix}2&0\\0&1\end{bmatrix}q_0$$

$$= -\frac{g}{a}\begin{bmatrix}2&-1\\-2&2\end{bmatrix}q_0$$

2.4

Si on ajoute la force qui est dépendante du temps

$$q_{n+1} = q_n + \Delta t\,\dot{q}_n + \Delta t^2(0.5-\beta)\ddot{q}_n + \Delta t^2\beta\,\ddot{q}_{n+1}$$

et $$\dot{q}_{n+1} = \dot{q}_n + \Delta t(1-\gamma)\ddot{q}_n + \Delta t\,\gamma\,\ddot{q}_{n+1}$$

$$\Rightarrow \quad ma^2\begin{bmatrix}2&1\\1&1\end{bmatrix}\ddot{q}_n + mga\begin{bmatrix}2&0\\0&1\end{bmatrix}q_n = F_0\sin\omega t\begin{vmatrix}a\\ \frac{a}{\sqrt{2}}\end{vmatrix}$$

2.5

```
%% premier cas Q1

  gamma1 = 0.5;
  beta1 = 0.25;
  dt = 0.02;
  T = 0:dt:T0;
  M = inv([2,1;1,1])*m*g*a*[2,0;0,1]/(m*a^2);
  M1 = F0*inv([2,1;1,1])*[a;a/sqrt(2)]/(m*a^2);

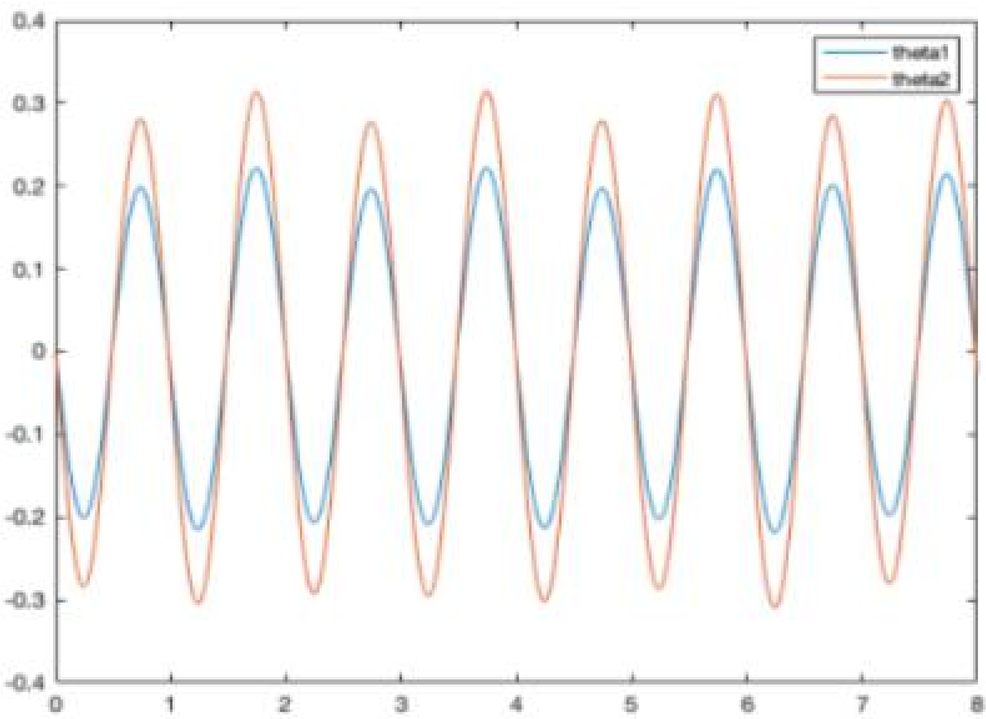
  B = [[1 0;0 1]+beta1*dt^2*M 0*[1 0;0 1];gamma1*dt*M [1 0;0 1]];
  C = [[1 0;0 1]-(0.5-beta1)*dt^2*M dt*[1 0;0 1];-(1-gamma1)*dt*M [1 0;0
1] ];
  A = inv(B)*C;
```

```
vp = eig(A)

rl = [real(vp(2));real(vp(4))];
ig = [imag(vp(2));imag(vp(4))];
nm = [abs(vp(2));abs(vp(4))];
U(:,1) = [theta0;thetap0];
for i = 1:length(T)-1
    F =
[dt^2*(0.5-beta1)*M1*sin(w*i*dt);dt*(1-gamma1)*M1*sin(w*i*dt)];
    U(:,i+1) = A*U(:,i)+inv(B)*F;
end

plot(T,U(1,:),T,U(2,:))
grid on
legend('theta1','theta1')
```

On obtient le resultat comme suivant



Quand t = 0

```
theta =

     0
     0

thetap =

   -1.3152
   -1.8600

thetapp =

   1.0e-14 *

    0.0717
    0.1015
```

Quand t = dt

```
theta =

   -0.0262
   -0.0370
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```
thetap =

   -1.3048
   -1.8453
```

```
thetapp =

    0.3006
    0.4251
```

Quand t = 2dt

```
theta =

   -0.0519
   -0.0734
```

```
thetap =

   -1.2813
   -1.8120
```

```
thetapp =

    0.5965
    0.8436
```

Quand t = 0.5

```
theta =

   -0.1817
   -0.2569
```

```
thetap =

    0.6934
    0.9806
```

```
thetapp =

    2.0879
    2.9527
```

Etude d’un oscillateur non linéaire à un degré de Liberté

1. NEWMARK explicite

1.1

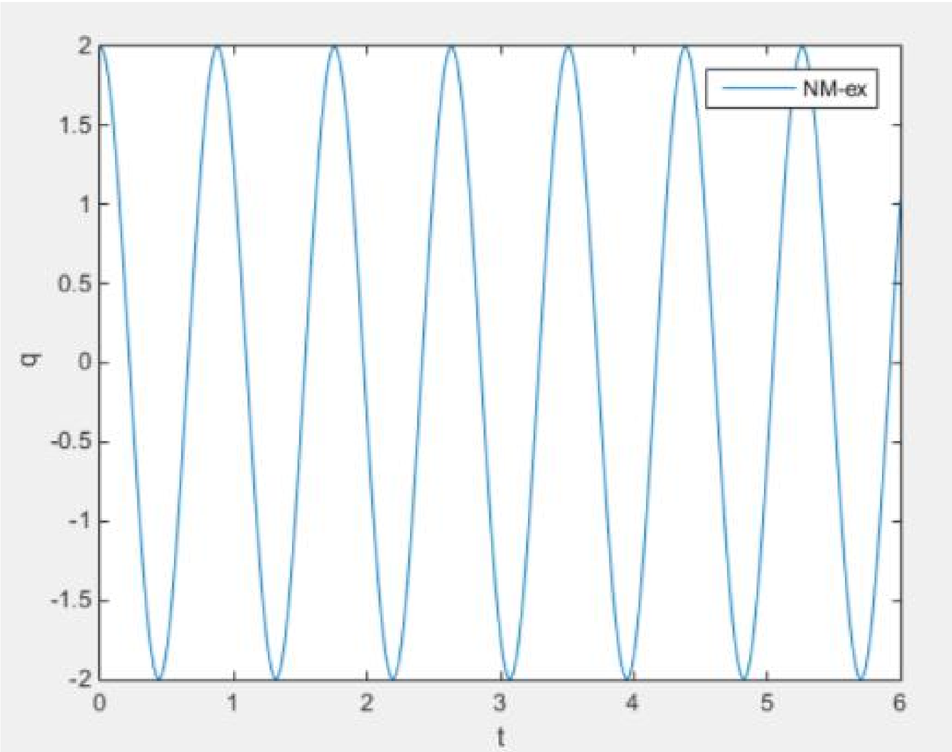
$$
\begin{cases}
q_{j+1} = q_j + \Delta t\,\dot{q}_j + \Delta t^2 (0.5-\beta)\,\ddot{q}_j + \beta \Delta t^2\,\ddot{q}_{j+1} \\
\dot{q}_{j+1} = \dot{q}_j + \Delta t\,(1-\gamma)\,\ddot{q}_j + \gamma \Delta t\,\ddot{q}_{j+1} \\
\ddot{q}_j + w_0^2\, q_j (1+a q_j^2) = 0
\end{cases}
$$

1.2

Code de 1.2

```
%% Oscillateur non lineaire a 1 DDL
clc
clear all
close all
  %% Initialisation global
  q0 = 2;
  qp0 = 0;
  q(1) = q0;
  qp(1) = qp0;
  w0 = 2*pi;
  a = 0.1;
  T0 = 6;
  %% NEWMARK explicite
  gamma1 = 0.5;
  beta1 = 0;
  dt = 0.02
  T = 0:dt:T0;
  for j = 1:length(T) - 1
      qpp(j) = -w0^2*q(j)*(1+a*q(j)*q(j));
      q(j+1) = q(j)+dt*qp(j)+dt^2*(0.5-beta1)*qpp(j);
      qp(j+1) =
qp(j)+dt*(1-gamma1)*qpp(j)+gamma1*dt*(-w0^2*q(j+1)*(1+a*q(j+1)*q(j+1))
);
  end
```

```
plot(T,q);
legend('NM-ex')
xlabel('t');
ylabel('q');
```



1.3

Code de 1.3

```
Ans1 = [q(1) q(2) q(3) q(length(T))]
```

```
Ans1 =

    2.0000    1.9779    1.9123    1.0329
```

2. NEWMARK inplicite

2.1

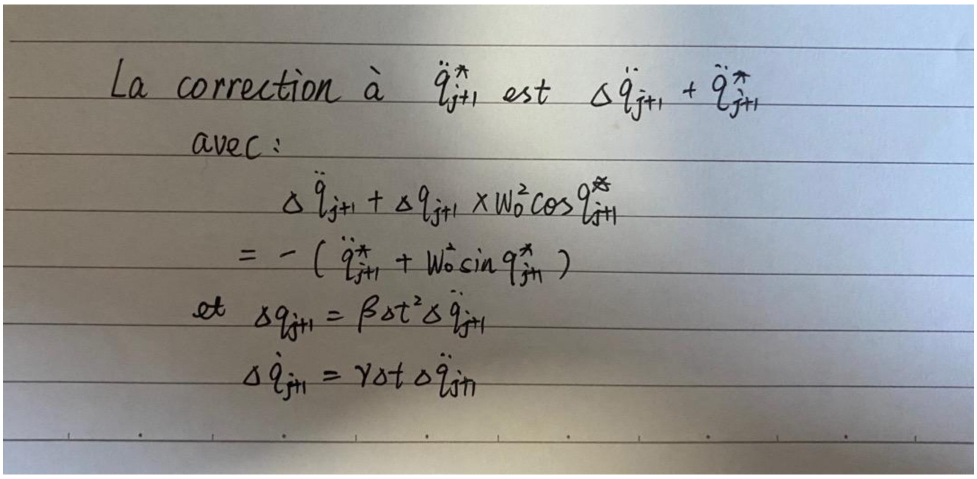
```
%% NEWMARK implicite
gamma2 = 0.5;
beta2 = 0.25;
```

On cherche à minimiser la valeur absolue de qpp+w0^2*q*(1+a*q^2)

$$\ddot{q} + \omega_0^2 q \left(1 + a q^2\right)$$

Pour que les résultat soient plus proche de solution réel

2.2



2.3

Code de 2.3

```
%% NEWMARK implicite
gamma2 = 0.5;
beta2 = 0.25;

%si on choisit une precision de 0.01
d = 0.01;
qe(1) = q(1);%estimation de q
qpe(1) = qp(1); %estimation de qp
qppe(1) = qpp(1);%estimation de qpp

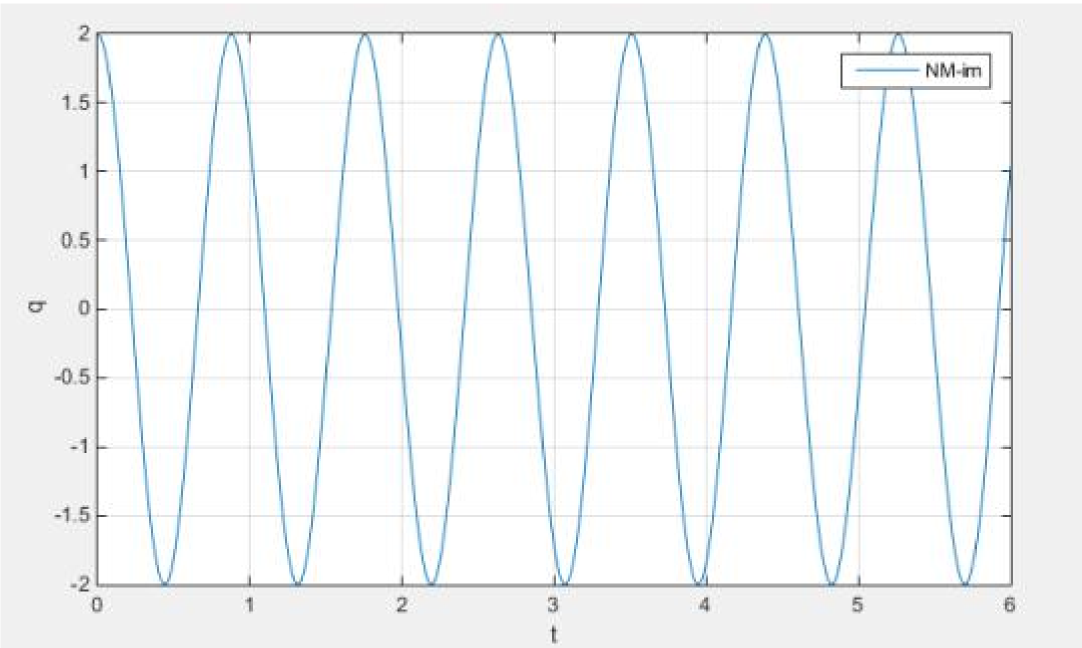
for i = 1:length(T)-1
    qe(i+1) = q(i)+dt*qp(i)+dt^2*(0.5-beta2)*qpp(i);
    qpe(i+1) = qp(i)+dt*(1-gamma2)*qpp(i);
    qppe(i+1) = 0;

    h = qppe(i+1) +w0^2*qe(i+1)*(1+a*qe(i+1)*qe(i+1)); %fonction de estimation
    while abs(h)>=d
        delta_qppe = -(qppe(i+1) +w0^2*qe(i+1)*(1+a*qe(i+1)*qe(i+1)))/(1+w0^2*beta2*dt^2*(1+3*a*qe(i+1)*qe(i+1)));
```

```
        delta_qe = beta2*dt^2*delta_qppe;
        delta_qpe = gamma2*dt*delta_qppe;

        %correction
        qe(i+1) = qe(i+1)+delta_qe;
        qpe(i+1) = qpe(i+1)+delta_qpe;
        qppe(i+1) = qppe(i+1)+delta_qppe;
        h = qppe(i+1) +w0^2*qe(i+1)*(1+a*qe(i+1)*qe(i+1));
    end
    q(i+1) = qe(i+1);
    qp(i+1) = qpe(i+1);
    qpp(i+1) = qppe(i+1);
end
 figure(2)
 plot(T,q)   ;
 legend('NM-im')
 xlabel('t');
 ylabel('q');
 grid on
```

On obtient le resultat;



2.4 on print le résultat

```
Ans2 = [q(1) q(2) q(3) q(length(t)) ]
```

```
Ans2 =

    2.0000    1.9781    1.9131    0.8478
```

3. énergie Mécanique

3.1

L'énergie mécanique pour cet oscillateur non linéaire est définie comme la somme de l'énergie cinétique et l'énergie potentiel.

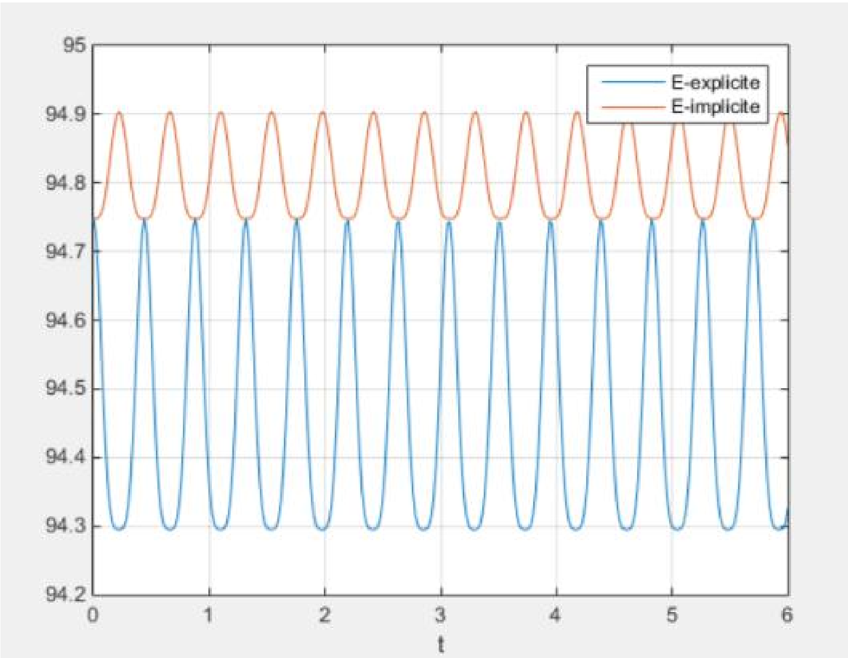
```
%pour l'energie cinetique, c'est 0.5*dq^2

%pour l'energie potentiel,on fait un integrale,

%c'est 0.5*w0*w0*q*q+0.25*alpha*w0*w0*q^4
```

3.2

On compare les deux énergies obtenues par différentes intégrations:



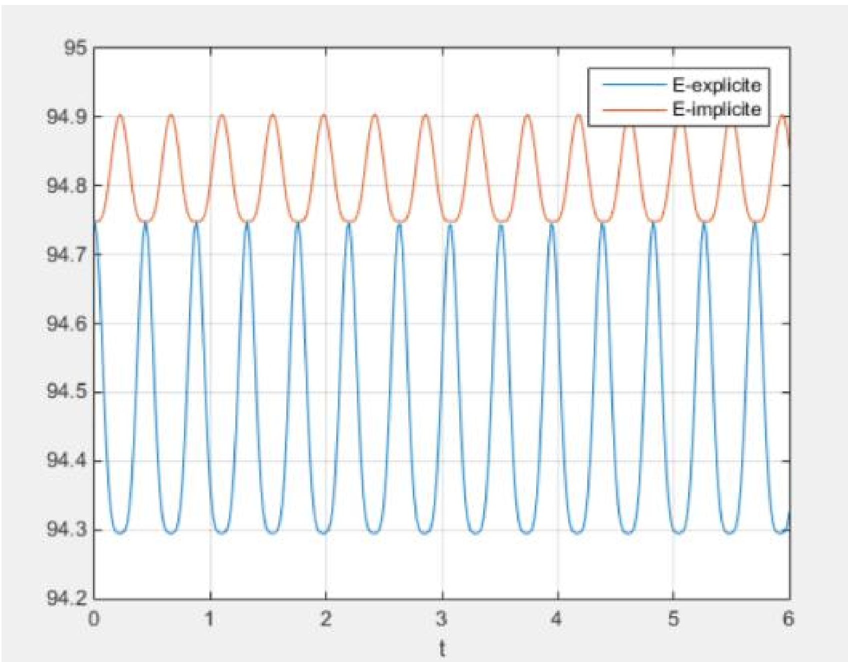
Code de 3.2

```
%% Energie mécanique
    for n = 1:length(T)
```

```
    %explicite
    Ec_ex(n) = 0.5*qp_ex(n)^2;
    Ep_ex(n) = 0.5*w0^2*q_ex(n)^2+0.25*w0^2*a*q_ex(n)^4;
    Etot_ex(n) =  Ec_ex(n)+Ep_ex(n);
    %implicite
    Ec_im(n) = 0.5*qp(n)^2;
    Ep_im(n) = 0.5*w0^2*q(n)^2+0.25*w0^2*a*q(n)^4;
    Etot_im(n) =  Ec_im(n)+Ep_im(n);
end
figure(3)
plot(T,Etot_ex,T,Etot_im)
grid on
xlabel('t')
legend('E-explicite','E-implicite')
```

3.3

Comme on a pris dt = 0.02, on peut conclure directement selon le résultat de question 3.2



l'énergie implicite est toujours plus grande de l'énergie explicite

mais, quelque fois, ils ont la meme l'énergie, et l'énergie calcule par méthode implicite est moin oscillée

Tous les code

```
%% Oscillateur non lineaire a 1 DDL
clc
clear all
close all
  %% Initialisation global
  q0 = 2;
  qp0 = 0;
  q(1) = q0;
  qp(1) = qp0;
  w0 = 2*pi;
  a = 0.1;
  T0 = 6;
  qpp(1) = -w0^2*q(1)*(1+a*q(1)*q(1));

  %% NEWMARK explicite
  gamma1 = 0.5;
  beta1 = 0;
  dt = 0.02
  T = 0:dt:T0;
  for j = 1:length(T)
      qpp(j) = -w0^2*q(j)*(1+a*q(j)*q(j));
      q(j+1) = q(j)+dt*qp(j)+dt^2*(0.5-beta1)*qpp(j);
      qp(j+1) =
qp(j)+dt*(1-gamma1)*qpp(j)+gamma1*dt*(-w0^2*q(j+1)*(1+a*q(j+1)*q(j+1))
);
  end
  figure(1)
  q(302)=[];
  plot(T,q);
  hold on
  legend('NM-ex')
  xlabel('t');
  ylabel('q');
  q_ex = q;
  qp_ex = qp;
  qpp1_ex = qpp;

  Ans1 = [q(1) q(2) q(3) q(length(T))]
  %% NEWMARK implicite
  gamma2 = 0.5;
  beta2 = 0.25;
```

```
%si on choisit une precision de 0.0001
d = 0.01;
qe(1) = q(1);%estimation de q
qpe(1) = qp(1); %estimation de qp
qppe(1) = qpp(1);%estimation de qpp

for i = 1:length(T)-1
    qe(i+1) = q(i)+dt*qp(i)+dt^2*(0.5-beta2)*qpp(i);
    qpe(i+1) = qp(i)+dt*(1-gamma2)*qpp(i);
    qppe(i+1) = 0;

    h = qppe(i+1) +w0^2*qe(i+1)*(1+a*qe(i+1)*qe(i+1)); %fonction de
estimation
    while abs(h)>=d
        delta_qppe = -(qppe(i+1)
+w0^2*qe(i+1)*(1+a*qe(i+1)*qe(i+1)))/(1+w0^2*beta2*dt^2*(1+3*a*qe(i+1)
*qe(i+1)));
        delta_qe = beta2*dt^2*delta_qppe;
        delta_qpe = gamma2*dt*delta_qppe;

        %correction
        qe(i+1) = qe(i+1)+delta_qe;
        qpe(i+1) = qpe(i+1)+delta_qpe;
        qppe(i+1) = qppe(i+1)+delta_qppe;
        h = qppe(i+1) +w0^2*qe(i+1)*(1+a*qe(i+1)*qe(i+1));
    end
    q(i+1) = qe(i+1);
    qp(i+1) = qpe(i+1);
    qpp(i+1) = qppe(i+1);
end
 figure(2)
 plot(T,q)   ;
 legend('NM-im')
 xlabel('t');
 ylabel('q');
 grid on

 Ans2 = [q(1) q(2) q(3) q(length(T)) ]
 %% Energie mecanique
   for n = 1:length(T)
       %explicite
       Ec_ex(n) = 0.5*qp_ex(n)^2;
       Ep_ex(n) = 0.5*w0^2*q_ex(n)^2+0.25*w0^2*a*q_ex(n)^4;
       Etot_ex(n) =  Ec_ex(n)+Ep_ex(n);
```

```
    %implicite
    Ec_im(n) = 0.5*qp(n)^2;
    Ep_im(n) = 0.5*w0^2*q(n)^2+0.25*w0^2*a*q(n)^4;
    Etot_im(n) =  Ec_im(n)+Ep_im(n);
end
figure(3)
plot(T,Etot_ex,T,Etot_im)
grid on
xlabel('t')
legend('E-explicite','E-implicite')
```