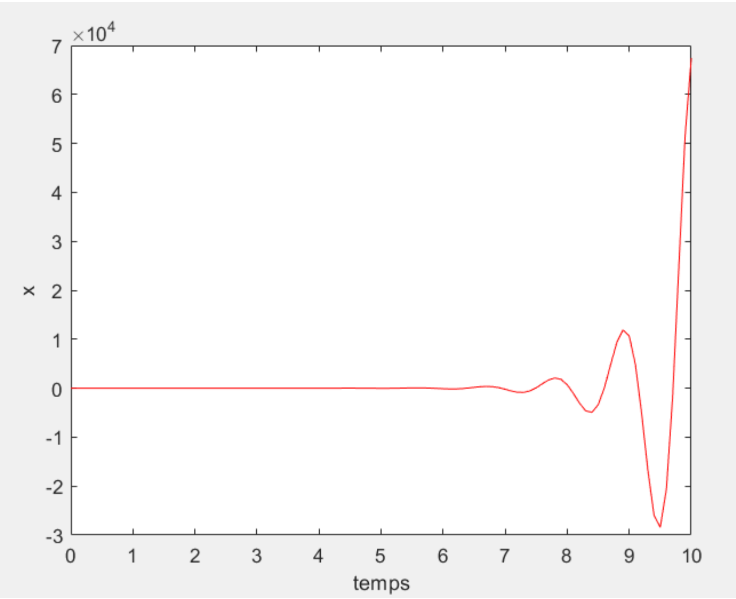
Etude d'un oscillateur linéaire amorti à un degré de liberté

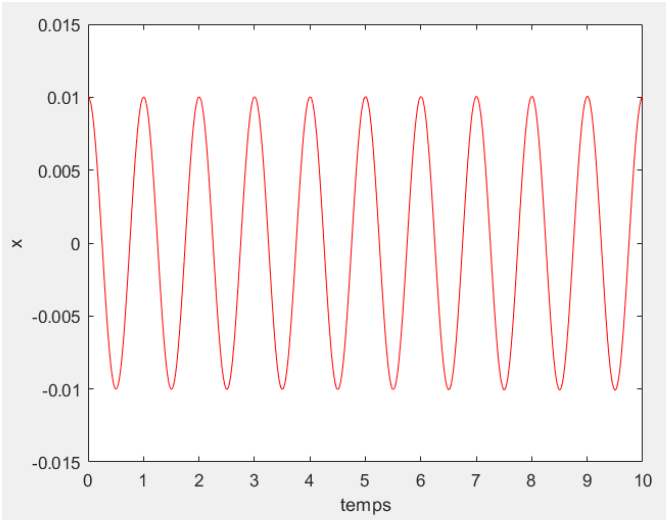
1.1Euler explicite

```
clear;
clc
w=2*pi;
T=1;
h=0.001;
eps=0.02;
t=0:h:10;
x(1)=0.01;
v(1)=0;
for i=1:length(t)-1
    x(i+1)=x(i)+h*v(i);
    v(i)=1/h*(x(i+1)-x(i));
    a(i)=-w^2*x(i)-2*eps*w*v(i);
    v(i+1)=v(i)+h*a(i);
end
figure(1);
plot(t,x,'r');
xlabel('temps');
ylabel('x');
```

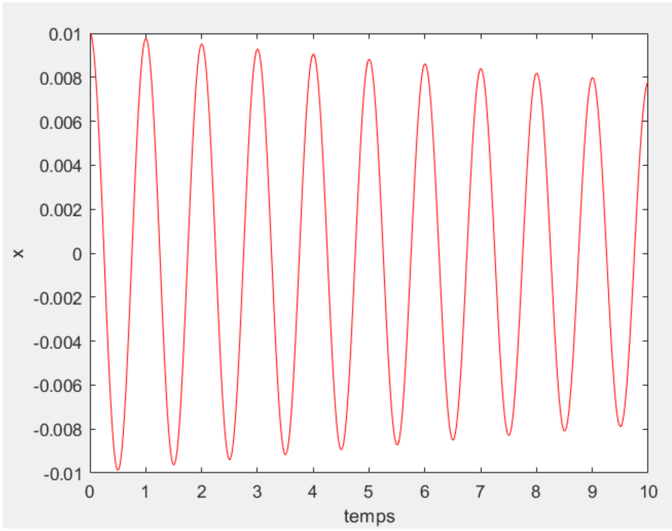
1.1a) si $\Delta t = 0.1s > \frac{2\varepsilon}{\omega_0} = 0.0064s$:



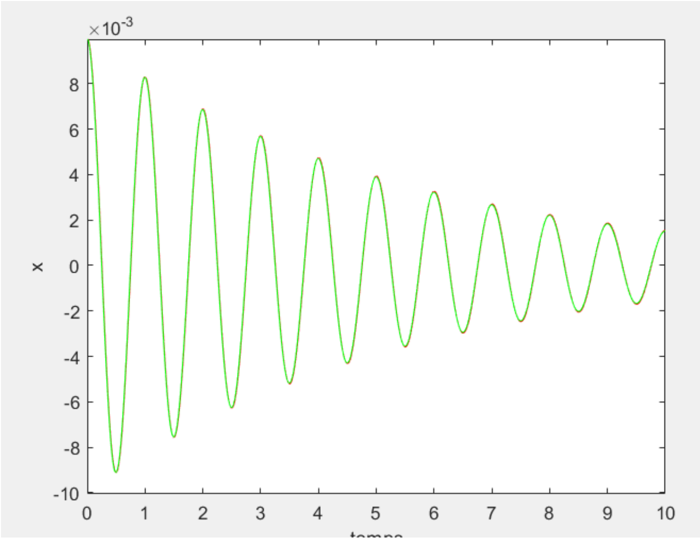
b) si $\Delta t = \frac{2\varepsilon}{\omega_0} = 0.0064s$:



c) si $\Delta t = 0.8 * \frac{2\varepsilon}{\omega_0} = 0.0051s$:



d) après plusieurs d'essai, je trouve que quand $\Delta t = 0.0001s$, la solution numérique(courbe rouge) est presque la même que la solution analytique(courbe verte) :



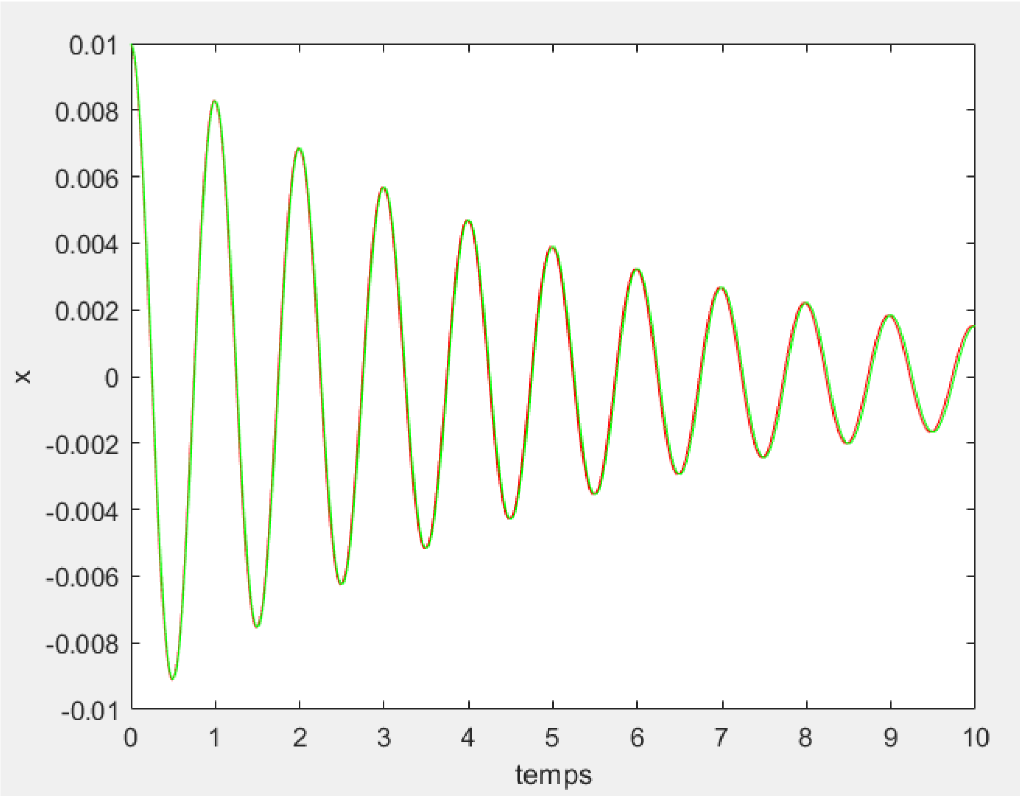
Par calcule, quand la valeur du rapport est plus petit que $\Delta t / \frac{2\varepsilon}{\omega_0} = 0.0156$, la solution numérique est précise suffisamment.

1.2Euler implicite

```
clear;
clc
w=2*pi;
T=1;
h=0.1;
eps=0.03;
t=0:h:10;
x(1)=0.01;
v(1)=0;
omg=w*(1-eps^2)^(1/2)
ans=exp(-eps*w*t).*(0.01*cos(t*omg)+(eps*w*0.0)/omg*sin(omg*t));
for i=1:length(t)-1
    a(i+1)=-w^2*x(i)-2*eps*w*v(i);
    v(i+1)=v(i)+h*a(i+1);
    x(i+1)=x(i)+h*v(i+1);
end

figure(1);
plot(t,x,'r');
xlabel('temps');
ylabel('x');
hold on
plot(t,ans,'g');
hold off
```

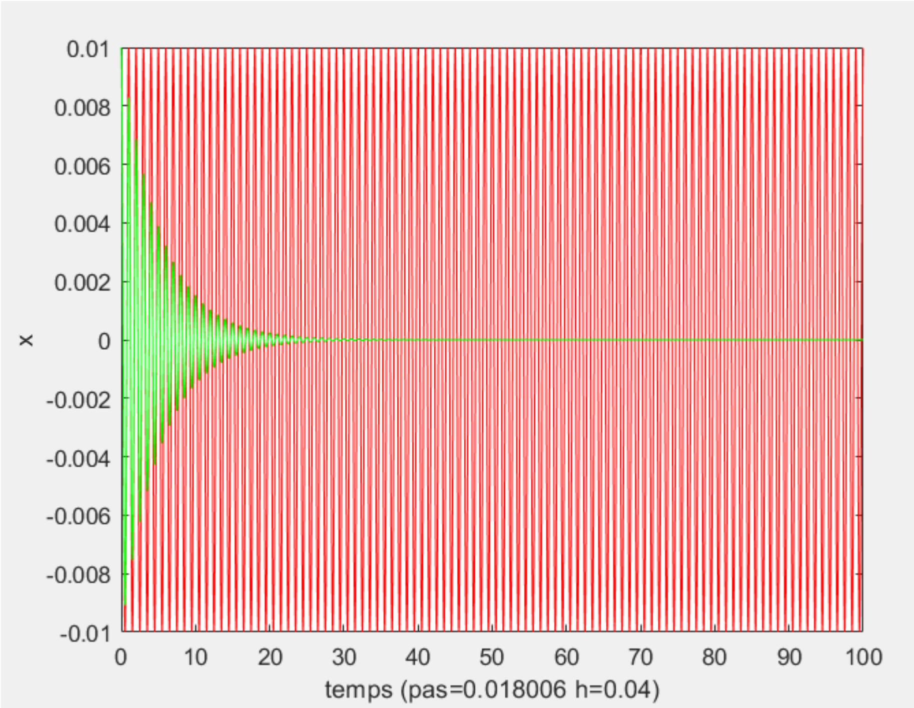
Quand $\Delta t = 0.02s$, la solution numérique(courbe rouge) est presque la même que la solution analytique(courbe verte) :



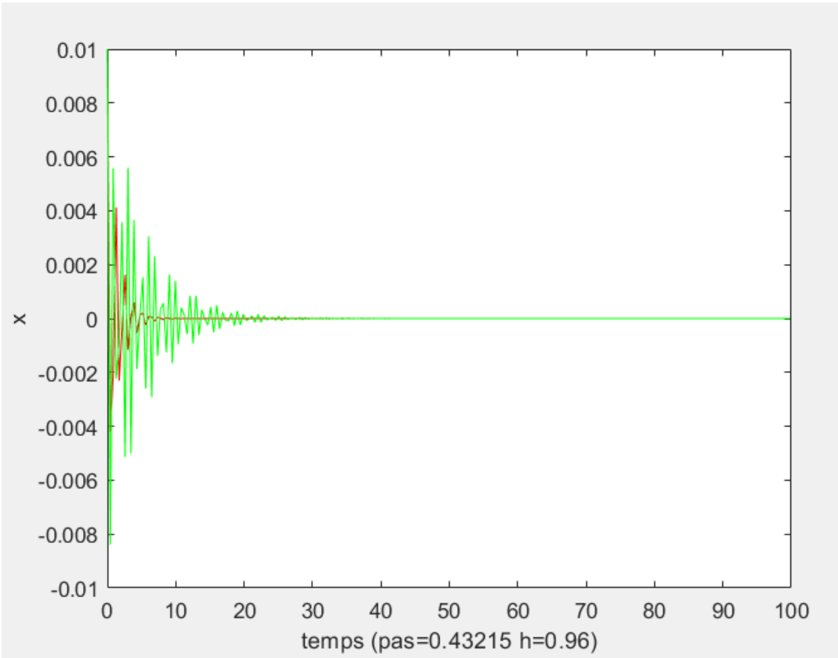
1.3RUNGE KUTTA

```
clear;
clc
T=1;
h=1.04;
w0=2*pi;
pas=h*2*sqrt(2)/w0
eps=0.03;
t=0:pas:100;
omg=w0*(1-eps^2)^(1/2)
x=zeros(2,length(t));
x(:,1)=[0.01;0];
for i=1:length(t)-1
    a=x(:,i);
    K1=cal_f2(a,i);
    K2=cal_f2(a+K1*pas/2,i+pas/2);
    K3=cal_f2(a+K2*pas/2,i+pas/2);
    K4=cal_f2(a+K3*pas,i+pas);
    K=(K1+2*K2+2*K3+K4)/6;
    x(:,i+1)=a+K*pas;
end
ans=exp(-eps*w0*t).*(0.01*cos(t*omg)+(eps*w0*0.0)/omg*sin(omg*t));
figure(1);
plot(t,x(1,:),'r');
xlabel(['temps (pas=',num2str(pas),' h=',num2str(h),')']);
ylabel('x');
hold on
plot(t,ans,'g');
```

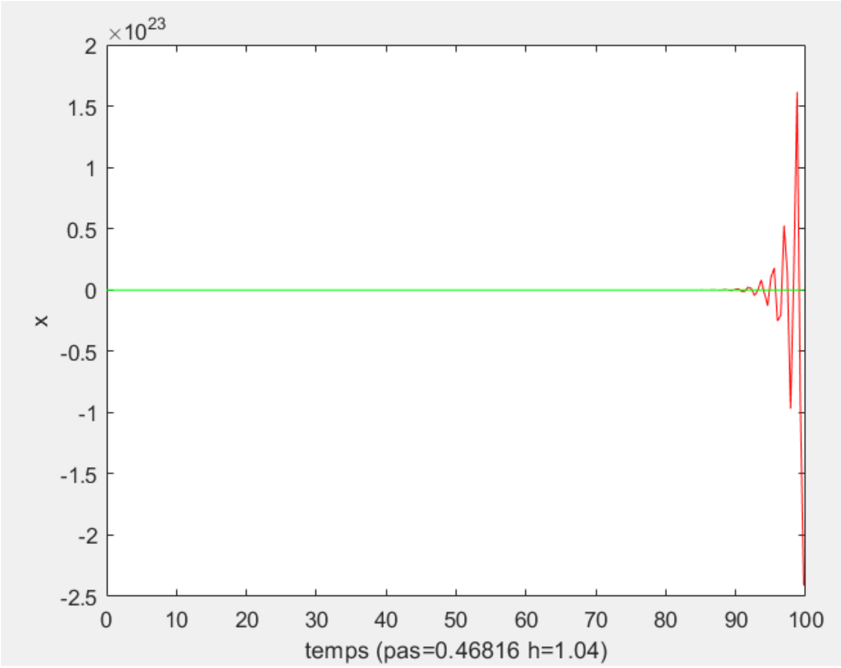
1.3a) h=0.04



h=0.96



h=1.04



Conclusion : le pas de temps très grand ou très petit cause de l'absence de stabilité et précision. Le pas de temps critique est donc dans un petit intervalle.

1.3b) le temps critique :

$$\Delta t_c = h_c \times \frac{2\sqrt{2}}{\omega_0} \quad \text{où} \quad 0.9883 < h_c < 0.9890$$

Etude d'un double pendule avec l'hypothèse des petits mouvements

1. Newmark explicite

1.1matrice d'amplification :

$$ma^2 \begin{bmatrix}2&1\\1&1\end{bmatrix}\begin{vmatrix}\ddot\theta_1\\\ddot\theta_2\end{vmatrix} + mga\begin{bmatrix}2&0\\0&1\end{bmatrix}\begin{matrix}\theta_1\\\theta_2\end{matrix} = F_0\sin wt \begin{vmatrix} a \\ \frac{a}{\sqrt2}\end{vmatrix}$$

on pose $q_n = \begin{vmatrix}\theta_{1n}\\\theta_{2n}\end{vmatrix}$ alors $\dot q_n = \begin{vmatrix}\dot\theta_{1n}\\\dot\theta_{2n}\end{vmatrix}$

① $q_{n+1} = q_n + \Delta t\,\dot q_n + \Delta t^2(0.5-\beta)\,\ddot q_n + \Delta t^2\beta\,\ddot q_{n+1}$

$$q_{n+1} = q_n + \Delta t\,\dot q_n + \begin{bmatrix}2&1\\1&1\end{bmatrix}^{-1}\cdot\frac{\Delta t^2(0.5-\beta)}{ma^2}\left(F_0\sin wt_n\begin{vmatrix}a\\\frac{a}{\sqrt2}\end{vmatrix} - mga\begin{bmatrix}2&0\\0&1\end{bmatrix}q_n\right) + \begin{bmatrix}2&1\\1&1\end{bmatrix}^{-1}\cdot\frac{\Delta t^2\beta}{ma^2}\left(F_0\sin wt_{n+1}\begin{vmatrix}a\\\frac{a}{\sqrt2}\end{vmatrix} - mga\,q_{n+1}\right)$$

② $\dot q_{n+1} = \dot q_n + \Delta t(1-\gamma)\,\ddot q_n + \Delta t\gamma\,\ddot q_{n+1}$

$$\dot q_{n+1} = \dot q_n + \begin{bmatrix}2&1\\1&1\end{bmatrix}^{-1}\cdot\frac{\Delta t(1-\gamma)}{ma^2}\left(F_0\sin wt_n\begin{vmatrix}a\\\frac{a}{\sqrt2}\end{vmatrix} - mga\begin{bmatrix}2&0\\0&1\end{bmatrix}q_n\right) + \begin{bmatrix}2&1\\1&1\end{bmatrix}^{-1}\cdot\frac{\Delta t\,\gamma}{ma^2}\left(F_0\sin wt_{n+1}\begin{vmatrix}a\\\frac{a}{\sqrt2}\end{vmatrix} - mga\begin{bmatrix}2&0\\0&1\end{bmatrix}q_{n+1}\right)$$

$M_1 = \begin{bmatrix}2&1\\1&1\end{bmatrix}$ $\quad M_2 = \begin{bmatrix}2&0\\0&1\end{bmatrix}$ $\quad M_3 = \begin{vmatrix}a\\\frac{a}{\sqrt2}\end{vmatrix}$

$$\underbrace{\begin{bmatrix}I + \Delta t^2\beta\,\mathrm{inv}(M_1)M_2 & 0\\ \Delta t\,\gamma\,\mathrm{inv}(M_1)M_2 & I\end{bmatrix}}_{M_8}\begin{vmatrix}q_{n+1}\\\dot q_{n+1}\end{vmatrix} = \underbrace{\begin{bmatrix}I-\Delta t^2(0.5-\beta)M_1^{-1}M_2 & \Delta t\\ -\Delta t(1-\gamma)M_1^{-1}M_2 & I\end{bmatrix}}_{M_9}\begin{vmatrix}q_n\\\dot q_n\end{vmatrix} + \underbrace{\begin{bmatrix}\Delta t^2(0.5-\beta)M_1^{-1}M_3 & \Delta t^2\beta\,M_1^{-1}M_3\\ \Delta t(1-\gamma)M_1^{-1}M_3 & \Delta t\gamma M_1^{-1}M_3\end{bmatrix}}_{M_{10}}\begin{vmatrix}F_0\sin wt_n\\F_0\sin wt_{n+1}\end{vmatrix}$$

$$A = M_8^{-1}\cdot M_9$$

Par calcule sur Matlab, j'obtiens la matrice d'amplitude A :

```
clear all;
m=2;
a=0.5;
g=9.8;
F0=20;
w=2*pi;
beta=0;
gamma=0.5;
syms dt;
I=[1,0;0,1];
M1=m*a^2*[2,1;1,1];
M2=m*g*a*[2,0;0,1];
M3=[a;a/sqrt(2)];
M4=I+dt*beta*inv(M1)*M2;
M5=dt*gamma*inv(M1)*M2;
M6=I-dt^2*(0.5-beta)*inv(M1)*M2;
M7=-dt*(1-gamma)*inv(M1)*M2;

M8=[M4,[0,0;0,0];M5,I];
M9=[M6,dt*I;M7,I];
A=inv(M8)*M9
eigmax=[];
for dt=0:0.001:1
    eigmax=[eigmax,max(abs(eig(eval(A))))];
end
dt=0:0.001:1;
plot(dt,eigmax,'r')
```

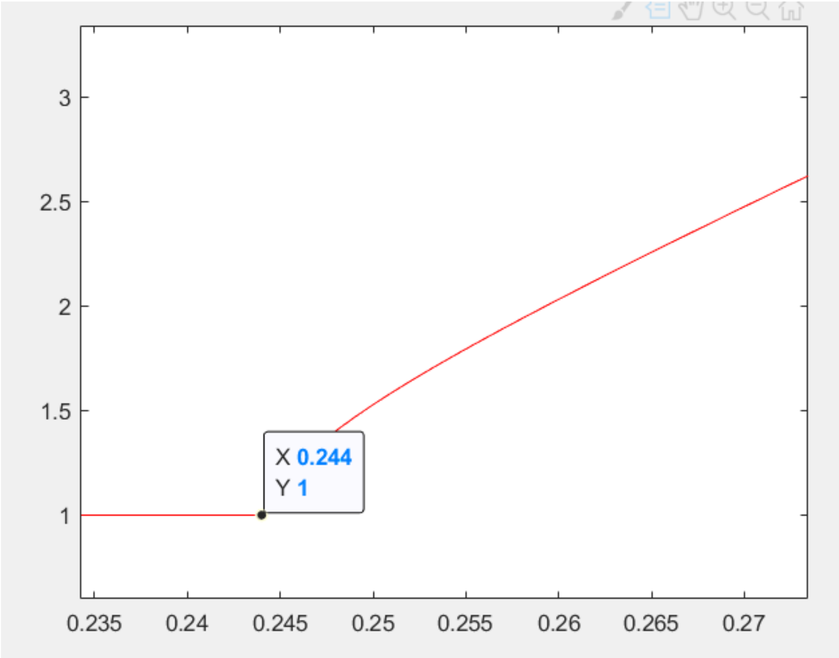
```
A =

[                                 1 - (98*dt^2)/5,                                    (49*dt^2)/5,              dt,                0]
[                                     (98*dt^2)/5,                                1 - (98*dt^2)/5,               0,               dt]
[ (98*dt*((98*dt^2)/5 - 1))/5 - (98*dt)/5 + (4802*dt^3)/25, (49*dt)/5 - (49*dt*((98*dt^2)/5 - 1))/5 - (4802*dt^3)/25, 1 - (98*dt^2)/5,      (49*dt^2)/5]
[ (98*dt)/5 - (98*dt*((98*dt^2)/5 - 1))/5 - (9604*dt^3)/25, (98*dt*((98*dt^2)/5 - 1))/5 - (98*dt)/5 + (4802*dt^3)/25,     (98*dt^2)/5, 1 - (98*dt^2)/5]
```

1.2 pas de temps critique :

Je plot la norme maximale des valeurs propres d'A en fonction de dt, et le schéma est ci-dessous :

Lorsque dt est plus grand que 0.244s, au moins une norme de valeur propre d'A est plus grande que 1. Donc, le pas de temps critique est 0.244s.



1.3 La relation entre les conditions initiales :

$$\begin{cases} \ddot{q}_0 = -\frac{mga}{ma^2}\begin{bmatrix}2&1\\1&1\end{bmatrix}^{-1}\cdot\begin{bmatrix}2&0\\0&1\end{bmatrix} q_0 \\ q_0 = 0 \\ \dot{q}_0 = \begin{vmatrix} -1.31519275 \\ -1.85996342\end{vmatrix}\end{cases}$$

1.4

$$q_{n+1} = q_n + \Delta t\,\dot{q}_n + \Delta t^2 (1-\beta)\,\ddot{q}_n$$

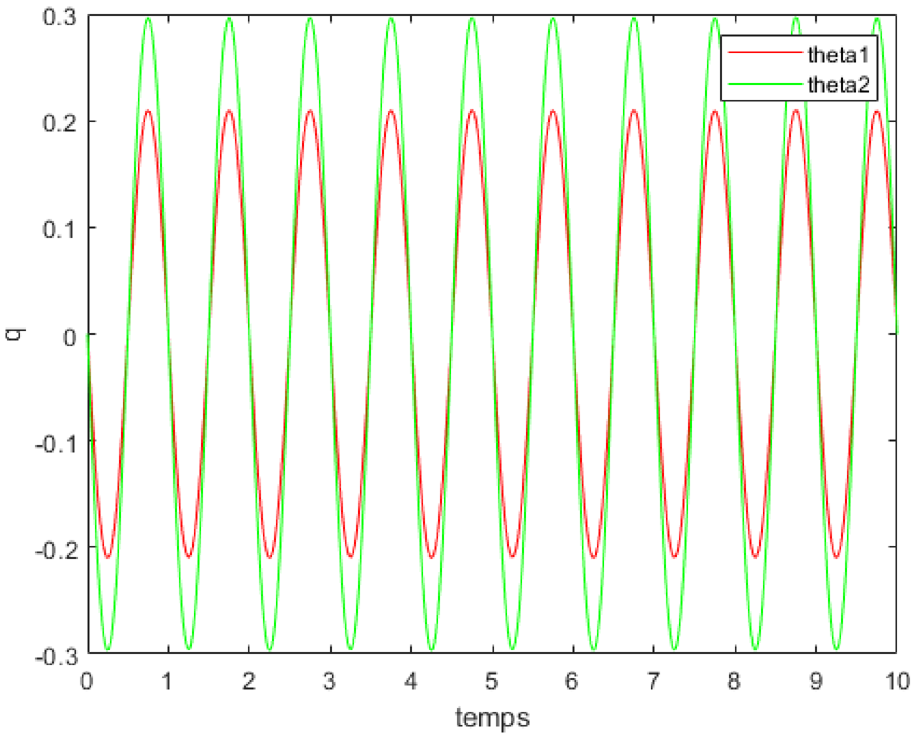
$$\ddot{q}_{n+1} = -\frac{g}{a} M_1^{-1} M_2\, q_{n+1} + \frac{F_0}{ma^2} \sin w t_{n+1} \cdot M_3$$

$$\dot{q}_{n+1} = \dot{q}_n + \Delta t (1-\gamma)\,\ddot{q}_n + \Delta t\,\gamma\,\ddot{q}_{n+1}$$

1.5

```
clear all;
close all;
m=2;
a=0.5;
g=9.81;
F0=20;
w=2*pi;
beta=0;
gamma=0.5;
dt=0.02;
I=[1,0;0,1];
M1=m*a^2*[2,1;1,1];
M2=m*g*a*[2,0;0,1];
M3=[a;a/sqrt(2)];
M4=I+dt^2*beta*inv(M1)*M2;
M5=dt*gamma*inv(M1)*M2;
M6=I-dt^2*(0.5-beta)*inv(M1)*M2;
M7=-dt*(1-gamma)*inv(M1)*M2;

M8=[M4,[0,0;0,0];M5,I];
M9=[M6,dt*I;M7,I];
A=inv(M8)*M9
M10=inv(M1)*M3;
B=inv(M8)*[dt^2*(0.5-beta)*M10,dt^2*beta*M10;dt*(1-gamma)*M10,dt*gamma*(M10)]
dt=0.02;
t=0:dt:10;
q=zeros(2,length(t));
dq=zeros(2,length(t));
ddq=zeros(2,length(t));
q(:,1)=[0;0];
dq(:,1)=[-1.31519275;-1.85996342];
ddq(:,1)=-inv(M1)*M2*[0;0];
for i=1:length(t)-1
    Q1=[q(:,i);dq(:,i)];
    Q2=A*Q1+inv(M8)*([dt^2*(0.5-beta)*M10,dt^2*beta*M10;dt*(1-gamma)*M10,dt*gamma*(M10)]*[F0*sin(w*(i-1)*dt);F0*sin(w*(i)*dt)]);
    q(:,i+1)=Q2(1:2,1);
    dq(:,i+1)=Q2(3:4,1);
end
figure(1);
plot(t,q(1,:),'r');
xlabel('temps');
ylabel('q');
hold on
plot(t,q(2,:),'g');
legend('theta1','theta2')
hold off
```



	0s	dt	2dt	0.5s
q	0 0	-0.0263 -0.0372	-0.0522 -0.0738	-0.000299 -0.000423
dq	-1.32 -1.09	-1.30 -1.85	-1.28 -1.808	1.31 1.86
ddq	0 0	1.04 1.47	2.06 2.91	0.00343 0.00486

2.1 matrice d'amplification A :

A[:,1]

```
                       (2401*dt^4)/(2401*dt^4 + 980*dt^2 + 50) - (10*(49*dt^2 + 5)*((49*dt^2)/5 - 1))/(2401*dt^4 + 980*dt^2 + 50),
                    (98*dt^2*(49*dt^2 + 5))/(2401*dt^4 + 980*dt^2 + 50) - (490*dt^2*((49*dt^2)/5 - 1))/(2401*dt^4 + 980*dt^2 + 50),
         (4802*dt^3)/(2401*dt^4 + 980*dt^2 + 50) - (98*dt)/5 + (98*(49*dt^3 + 10*dt)*((49*dt^2)/5 - 1))/(2401*dt^4 + 980*dt^2 + 50),
(98*dt)/5 - (980*dt*((49*dt^2)/5 - 1))/(2401*dt^4 + 980*dt^2 + 50) - (4802*dt^2*(49*dt^3 + 10*dt))/(5*(2401*dt^4 + 980*dt^2 + 50)),
```

A[:,2]

```
                    (49*dt^2*(49*dt^2 + 5))/(2401*dt^4 + 980*dt^2 + 50) - (245*dt^2*((49*dt^2)/5 - 1))/(2401*dt^4 + 980*dt^2 + 50),
                       (2401*dt^4)/(2401*dt^4 + 980*dt^2 + 50) - (10*(49*dt^2 + 5)*((49*dt^2)/5 - 1))/(2401*dt^4 + 980*dt^2 + 50),
(49*dt)/5 - (490*dt*((49*dt^2)/5 - 1))/(2401*dt^4 + 980*dt^2 + 50) - (2401*dt^2*(49*dt^3 + 10*dt))/(5*(2401*dt^4 + 980*dt^2 + 50)),
         (4802*dt^3)/(2401*dt^4 + 980*dt^2 + 50) - (98*dt)/5 + (98*(49*dt^3 + 10*dt)*((49*dt^2)/5 - 1))/(2401*dt^4 + 980*dt^2 + 50),
```

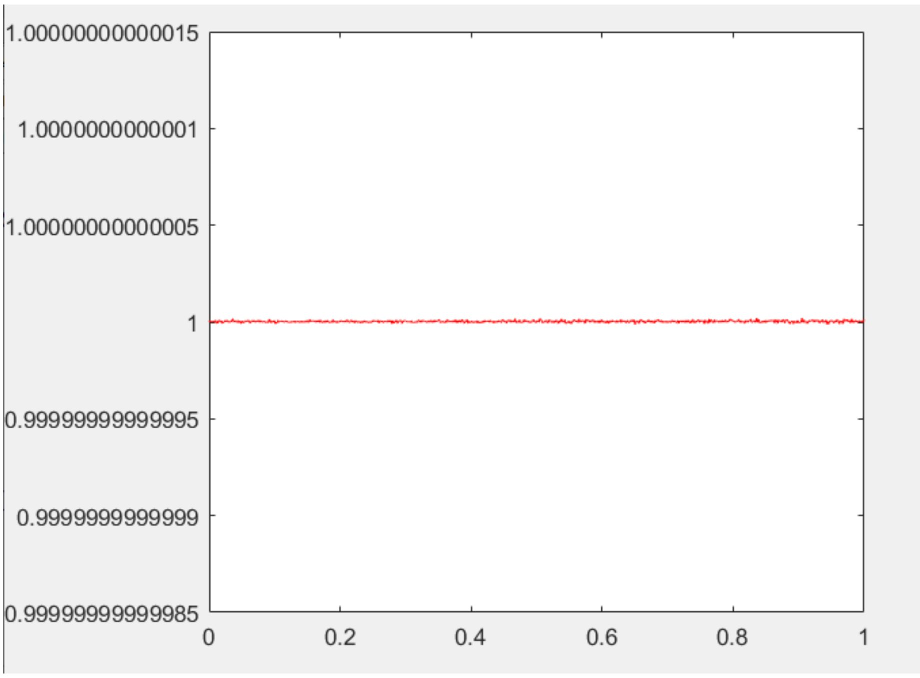
A[:,3]

```
         (10*dt*(49*dt^2 + 5))/(2401*dt^4 + 980*dt^2 + 50),
                     (490*dt^3)/(2401*dt^4 + 980*dt^2 + 50),
1 - (98*dt*(49*dt^3 + 10*dt))/(2401*dt^4 + 980*dt^2 + 50),
                     (980*dt^2)/(2401*dt^4 + 980*dt^2 + 50),
```

A[:,4]

```
                     (245*dt^3)/(2401*dt^4 + 980*dt^2 + 50)]
         (10*dt*(49*dt^2 + 5))/(2401*dt^4 + 980*dt^2 + 50)]
                     (490*dt^2)/(2401*dt^4 + 980*dt^2 + 50)]
1 - (98*dt*(49*dt^3 + 10*dt))/(2401*dt^4 + 980*dt^2 + 50)]
```

2.2 La plus grande valeur propre de cette matrice est presque 1 pour tous les pas de temps dans intervalle [0,1].



2.3

$$\begin{cases} \ddot{q}_0 = -\frac{mga}{ma^2}\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}^{-1}\cdot\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix} q_0 \\ q_0 = 0 \\ \dot{q}_0 = \begin{vmatrix} -1.31519275 \\ -1.85996342 \end{vmatrix} \end{cases}$$

2.4

$$q_{j+1} = q_j + \Delta t\,\dot{q}_j + \Delta t^2 \times 0.25 \times \ddot{q}_j + \Delta t^2 \times 0.25 \times \ddot{q}_{j+1}$$

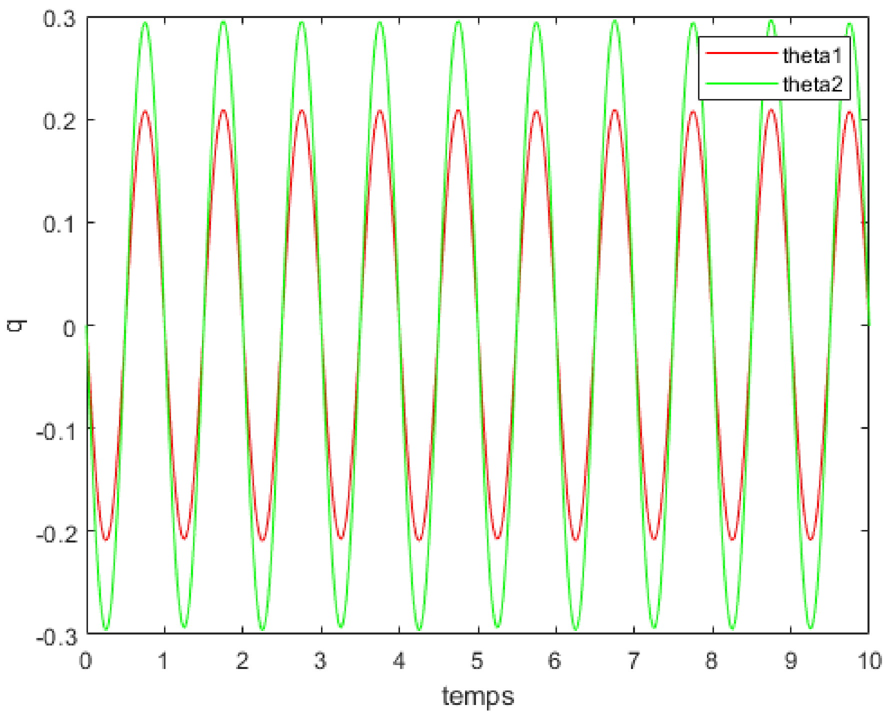
$$ma^2\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}\ddot{q}_{j+1} + mga\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix} q_j = F_0 \sin \omega t \begin{vmatrix} a \\ \frac{a}{\sqrt{2}} \end{vmatrix}$$

$$\dot{q}_{j+1} = \dot{q}_j + dt \times 0.5 \times \ddot{q}_j + dt \times 0.5 \times \ddot{q}_{j+1}$$

2.5

```
clear all;
close all;
m=2;
a=0.5;
g=9.81;
F0=20;
w=2*pi;
beta=0.25;
gamma=0.5;
dt=0.02;
I=[1,0;0,1];
M1=m*a^2*[2,1;1,1];
M2=m*g*a*[2,0;0,1];
M3=[a;a/sqrt(2)];
M4=I+dt^2*beta*inv(M1)*M2;
M5=dt*gamma*inv(M1)*M2;
M6=I-dt^2*(0.5-beta)*inv(M1)*M2;
M7=-dt*(1-gamma)*inv(M1)*M2;

M8=[M4,[0,0;0,0];M5,I];
M9=[M6,dt*I;M7,I];
A=inv(M8)*M9
M10=inv(M1)*M3;
B=inv(M8)*[dt^2*(0.5-beta)*M10,dt^2*beta*M10;dt*(1-gamma)*M10,dt*gamma*(M10)]
dt=0.02;
t=0:dt:10;
q=zeros(2,length(t));
dq=zeros(2,length(t));
ddq=zeros(2,length(t));
q(:,1)=[0;0];
dq(:,1)=[-1.31519275;-1.85996342];
ddq(:,1)=-inv(M1)*M2*[0;0];
for i=1:length(t)-1
    Q1=[q(:,i);dq(:,i)];
    Q2=A*Q1+inv(M8)*([dt^2*(0.5-beta)*M10,dt^2*beta*M10;dt*(1-gamma)*M10,dt*gamma*(M10)]*[F0*sin(w*(i-1)*dt);F0*sin(w*(i)*dt)]);
    q(:,i+1)=Q2(1:2,1);
    dq(:,i+1)=Q2(3:4,1);
    ddq(:,i+1)=-inv(M1)*M2*q(:,i+1)+inv(M1)*F0*sin(w*(i)*dt)*M3;
end
figure(1);
plot(t,q(1,:),'r');
xlabel('temps');
ylabel('q');
hold on
plot(t,q(2,:),'g');
legend('theta1','theta2')
hold off
```



2.6

	0s	dt	2dt	0.5s
q	0 0	-0.0262 -0.0371	-0.0520 -0.0735	-0.000922 -0.00130
dq	-1.32 -1.86	-1.30 -1.85	-1.27 -1.80	1.31 1.86

Oscillateur non linéaire a un degré de liberté

1.1

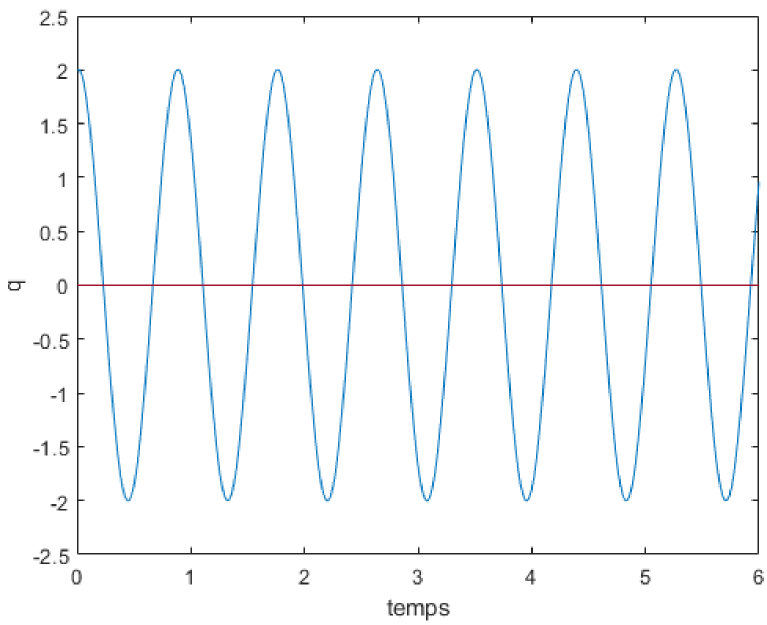
$$q_{j+1} = q_j + \Delta t\, \dot{q}_j + \Delta t^2 (0.5-\beta)\, \ddot{q}_j$$

$$\ddot{q}_{j+1} = -W_0^2\, q_{j+1} \left(1 + a\, q_{j+1}^2\right)$$

$$\dot{q}_{j+1} = \dot{q}_j + \Delta t (1-\gamma)\, \ddot{q}_j + \gamma \Delta t\, \ddot{q}_{j+1}$$

1.2

```
clear all;
w0=2*pi;
a=0.1;
beta=0;
gamma=0.5;
dt=0.02;
t=0:dt:6;
q=zeros(length(t));
dq=zeros(length(t));
ddq=zeros(length(t));
q(1)=2;
dq(1)=0;
for i=1:length(t)-1
    q(i+1)=q(i)+dt*dq(i)+dt^2*(0.5-beta)*ddq(i);
    ddq(i+1)=-w0^2*q(i+1)*(1+a*q(i+1)^2);
    dq(i+1)=dq(i)+dt*(1-gamma)*ddq(i)+gamma*dt*ddq(i+1);
end
figure(1)
plot(t,q)
xlabel('temps');
ylabel('q');
```



1.3

t	0	dt	2dt	T0=6s
q	2	2	1.9558	0.9599

2.1 on cherche à minimiser la valeur absolue de ddq+w0^2*q*(1+a*q^2)

2.2

$$\ddot{q} + \omega_0^2 q(1+aq^2)=0$$

Calcul de la correction :

$$f(\ddot{q}^*_{j+1} + \Delta\ddot{q}_{j+1},\ q^*_{j+1} + \Delta q_{j+1}) = 0 = f(\ddot{q}^*_{j+1},\ q^*_{j+1}) + \frac{\partial f}{\partial q^*_{j+1}}\Delta q_{j+1} + \frac{\partial f}{\partial \ddot{q}^*_{j+1}}\Delta\ddot{q}_{j+1}$$

$$\ddot{q}^*_{j+1} + \Delta\ddot{q}_{j+1} + \omega_0^2(q^*_{j+1} + \Delta q_{j+1})(1 + a(q^*_{n+1} + \Delta q_{n+1})^2) = \ddot{q}^*_{j+1} + \omega_0^2 q^*_{j+1}(1 + a {q^*_{j+1}}^2) + \frac{\partial f}{\partial q^*_{j+1}} \cdot \beta\Delta t^2 \Delta\ddot{q}_{j+1} + \frac{\partial f}{\partial \ddot{q}^*_{j+1}}\Delta\ddot{q}_{j+1}$$

$$\Delta\ddot{q}_{j+1} = -\frac{f(\ddot{q}^*_{j+1},\ \dot{q}^*_{j+1},\ q^*_{j+1})}{\frac{\partial f}{\partial \ddot{q}^*_{j+1}} + \frac{\partial f}{\partial q^*_{j+1}}\beta\Delta t^2}$$

2.3

```
clear all;
w0=2*pi;
a=0.1;
beta=0.25;
gamma=0.5;
dt=0.02;
t=0:dt:6;
q=zeros(1,length(t));
dq=zeros(1,length(t));
ddq=zeros(1,length(t));
energ=zeros(1,length(t));
q(1)=2;
dq(1)=0;
for i=1:length(t)-1
    q(i+1)=q(i)+dt*dq(i)+dt^2*(0.5-beta)*ddq(i);
    ddq(i+1)=0;
    dq(i+1)=dq(i)+dt*(1-gamma)*ddq(i);
    cddq = (-(ddq(i+1)+w0*w0*q(i+1)*(1+a*q(i+1)*q(i+1))))/(1+beta*dt*dt*(w0*w0+3*w0*w0*a*q(i+1)*q(i+1)));
    cdq=gamma*dt* cddq;
    cq=beta*dt*dt* cddq;
    q(i+1)=q(i+1)+cq;
    dq(i+1)=dq(i+1)+cdq;
    ddq(i+1)=ddq(i+1)+cddq;
    energ(i)= 0.5*dq(i)^2 + 0.5*w0*w0*q(i)*q(i)+0.25*a*w0*w0*q(i)^4;
end
figure(1)
plot(t,q)
xlabel('temps');
ylabel('q');
figure(2)
plot(t,energ,'r')
```

2.4

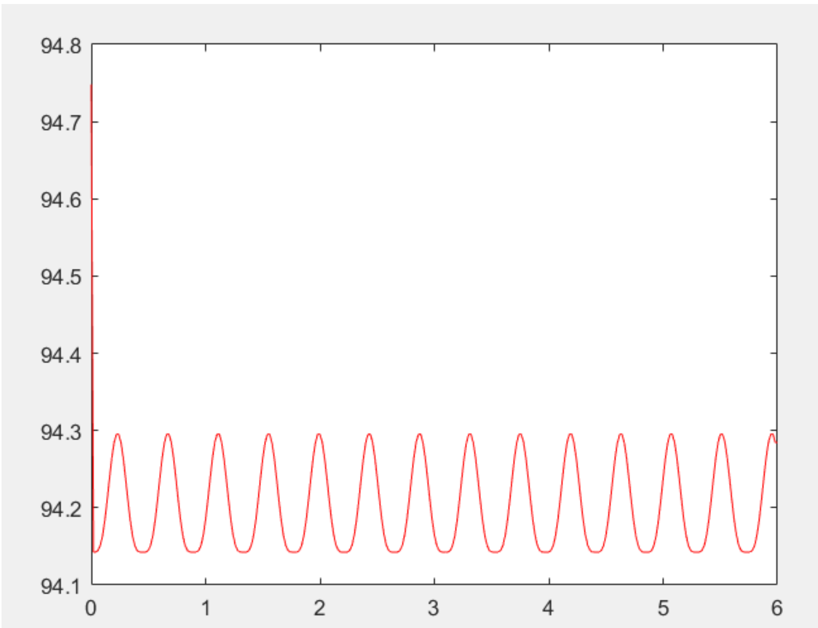
t	0	dt	2dt	T0=6s
q	2	1.989	1.946	0.6694

3.1

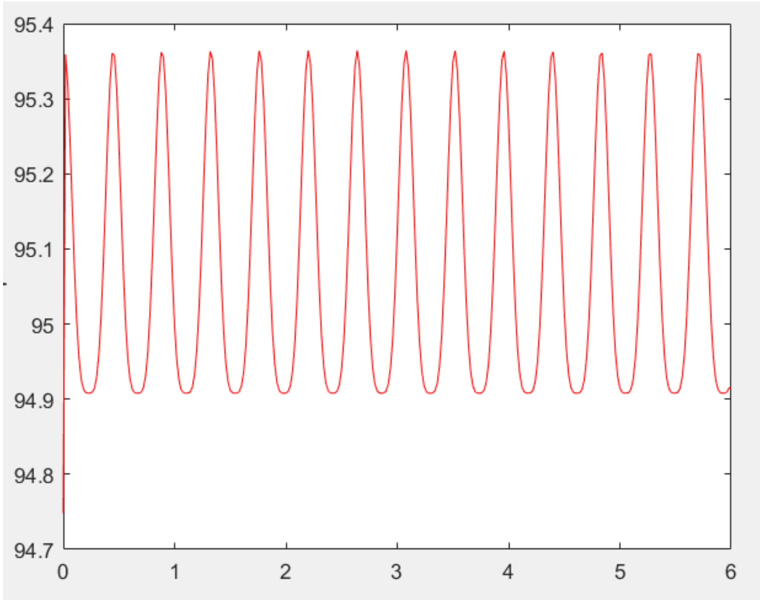
$$E_m = E_c + E_p = \frac{1}{2}\dot{q}^2 + \frac{1}{2}w_0^2 q^2 + \frac{a}{4} w_0^2 q^4$$

3.2

Pour NEWMARK implicite, l’énergie diminue premièrement et après, il varie presque périodiquement dans un intervalle très petit. Sa valeur est toute petite que la valeur initiale :



Pour NEWMARK explicite, l’énergie augmente premièrement et après, il varie presque périodiquement dans un intervalle très petit. Sa valeur est toute grande que la valeur initiale :



La comparaison entre eux :