
Oscillateur conservatif linéaire à un degré de liberté

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0

```
%Retrouver l'équation du mouvement du pendule simple avec les  
équations de Lagrange  
A = imread('IMG_0324.jpg');  
imshow(A);
```

On sait que l'équation de Lagrange

$$L = E_c - E_p \quad \frac{d}{dt} \left[\frac{\partial L}{\partial \dot{x}_i} \right] - \frac{\partial L}{\partial x_i} = Q_i \quad \text{avec} \quad \frac{\partial E_p}{\partial \dot{x}_i} = 0$$

Et pour pendule simple, on a

$$\begin{cases} E_c = \frac{1}{2} \dot{\theta}^2 \\ E_p = -mgd \cos \theta + \text{cte.} \end{cases}$$

Donc $L = E_c - E_p = \frac{1}{2} \dot{\theta}^2 + mgd \cos \theta + \text{cte}$ $\delta W = 0$.

$$\delta W = \sum_{i=1}^N Q_i \delta Q_i = 0 \Rightarrow Q_i = 0$$

Selon l'équation de Lagrange, $\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{\theta}} \right] - \frac{\partial L}{\partial \theta} = 0$.

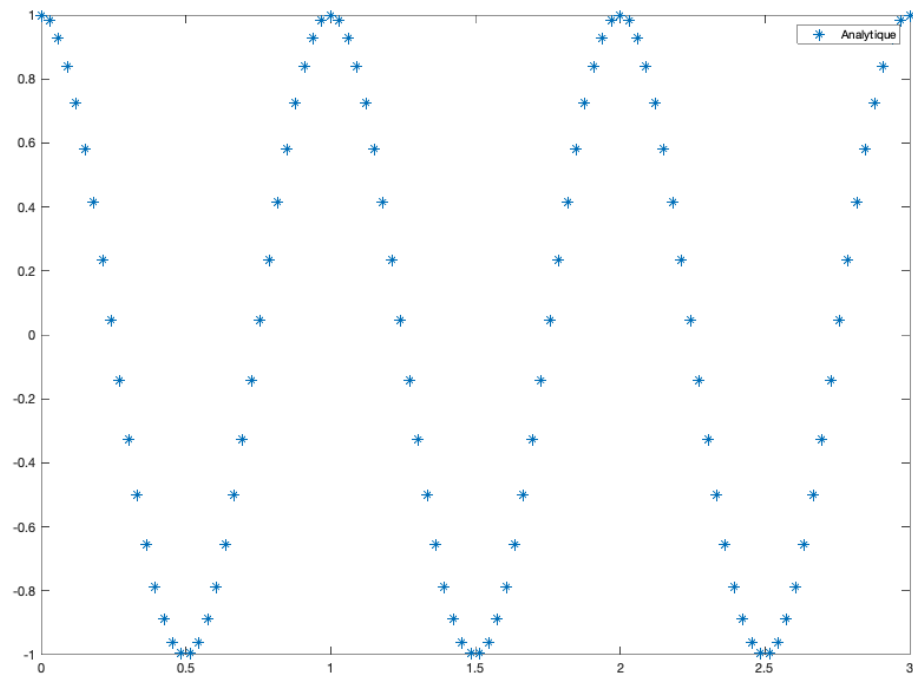
On a l'équation du mouvement du pendule simple

$$\Rightarrow I \ddot{\theta} + mgd \sin \theta = 0$$

1.1

```
clear all
clf;
w0=2*pi;
syms q t ;
b='D2q=-(2*pi)^2*q';
q = simplify(dsolve(b,'q(0)=1','Dq(0)=0'));
% on trouve q =cos(2*pi*t)
T0 = 3;
t = linspace(0,T0,100);
q= cos(2*pi*t); %on a déjà trouver
plot(t,q,'*')
legend('Analytique');
```

##: Support of character vectors and strings will be removed in a future release. Use sym objects to define differential equations instead.



1.2

```
%dq = simplify(diff(q,t));  
%E =1/2*(dq^2+(2*pi)^2*q^2);  
%E = simplify(E);  
E = 2*pi^2;  
% E est une constante
```

2.1

```
B = imread('IMG_0325.jpg');  
imshow(B);
```

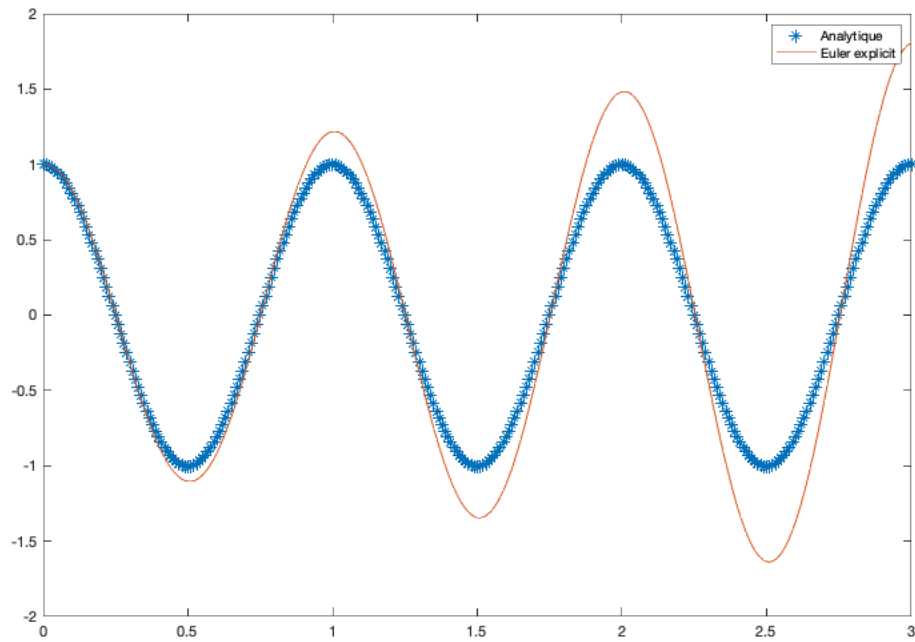
$$\text{On a } \begin{cases} \begin{pmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{pmatrix} = \begin{pmatrix} q_j \\ \dot{q}_j \end{pmatrix} + \Delta t \begin{pmatrix} \dot{q}_j \\ \ddot{q}_j \end{pmatrix} \\ \ddot{q}_j = -\omega_0^2 q_j \end{cases}$$

$$\begin{pmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{pmatrix} = \begin{pmatrix} q_j + \Delta t \dot{q}_j \\ \dot{q}_j + \Delta t \ddot{q}_j \end{pmatrix} = \begin{pmatrix} q_j + \Delta t \dot{q}_j \\ \dot{q}_j - \omega_0^2 \Delta t q_j \end{pmatrix}$$

$$\text{Donc } \begin{pmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{pmatrix} = \begin{bmatrix} 1 & \Delta t \\ -\omega_0^2 \Delta t & 1 \end{bmatrix} \begin{pmatrix} q_j \\ \dot{q}_j \end{pmatrix}$$

2.2

```
clf;
dt=0.01;
t=0:dt:T0;
q0=1;
dq0=0;
A=[1,dt; -W0^2*dt,1];
U(:,1)=[q0;dq0];
for j=1:(length(t)-1)
    U(:,j+1)= A*U(:,j);
end
q = cos(W0*t);
plot(t,q,'*',t,U(1,:));
legend('Analytique','Euler explicite');
```



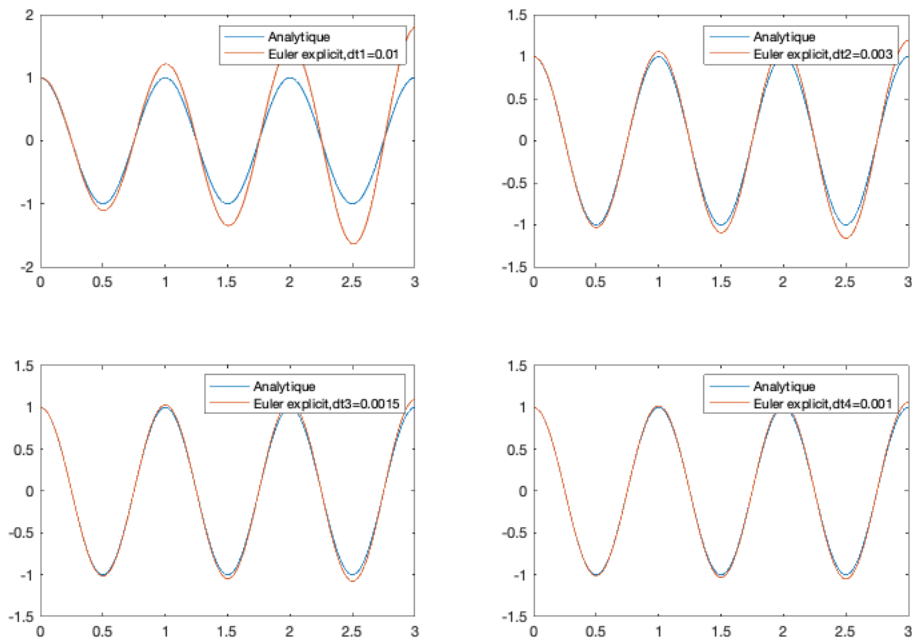
2.3

```
clf;
dt1=0.01;
dt2=0.003;
dt3=0.0015;
dt4=0.001;
t1=0:dt1:T0;
t2=0:dt2:T0;
t3=0:dt3:T0;
t4=0:dt4:T0;
q0=1;
dq0=0;
A1=[1,dt1; -w0^2*dt1,1];
U1(:,1)=[q0;dq0];
for j=1:(length(t1)-1)
    U1(:,j+1)= A1*U1(:,j);
end
q1 = cos(w0*t1);
A2=[1,dt2; -w0^2*dt2,1];
U2(:,1)=[q0;dq0];
for j=1:(length(t2)-1)
    U2(:,j+1)= A2*U2(:,j);
end
q2 = cos(w0*t2);
A3=[1,dt3; -w0^2*dt3,1];
U3(:,1)=[q0;dq0];
for j=1:(length(t3)-1)
    U3(:,j+1)= A3*U3(:,j);
```

```

end
q3 = cos(W0*t3);
A4=[1,dt4; -W0^2*dt4,1];
U4(:,1)=[q0;dq0];
for j=1:(length(t4)-1)
    U4(:,j+1)= A4*U4(:,j);
end
q4 = cos(W0*t4);
subplot(2,2,1)
plot(t1,q1,t1,U1(1,:))
legend('Analytique','Euler explicite,dt1=0.01')
subplot(2,2,2)
plot(t2,q2,t2,U2(1,:))
legend('Analytique','Euler explicite,dt2=0.003')
subplot(2,2,3)
plot(t3,q3,t3,U3(1,:))
legend('Analytique','Euler explicite,dt3=0.0015')
subplot(2,2,4)
plot(t4,q4,t4,U4(1,:))
legend('Analytique','Euler explicite,dt4=0.001')
%On peut voir que plus le pas de temps  $\Delta t$  est petit, plus la
  divergence est lente

```



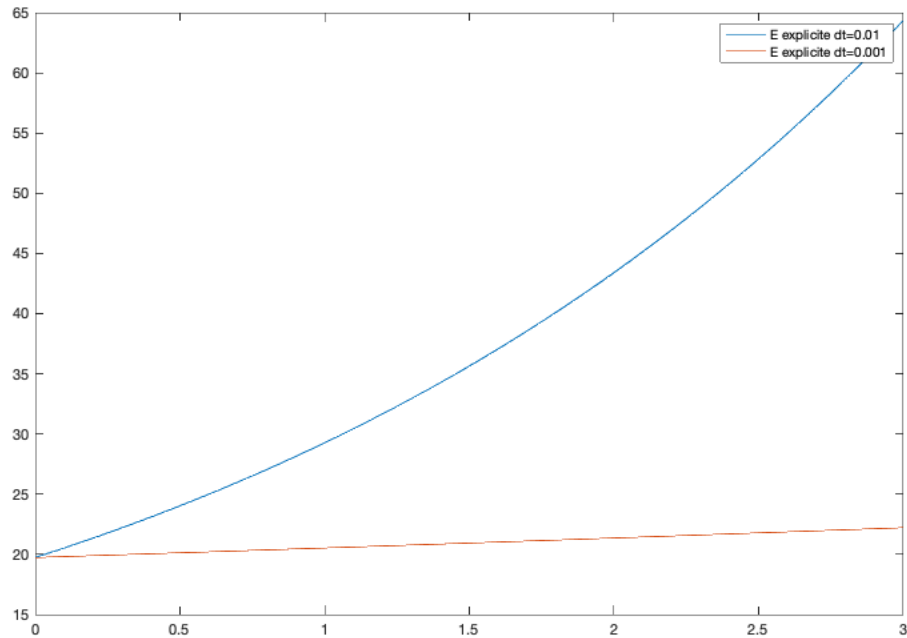
2.4

```

clf
for j=1:length(t)
    E1(j)=1/2*(U(2,j)*U(2,j)+4*pi*pi*U(1,j)*U(1,j));
end

```

```
for j=1:length(t4)
    E4(j)=1/2*(U4(2,j)*U4(2,j)+4*pi*pi*U4(1,j)*U4(1,j));
end
plot(t,E1,t4,E4)
legend('E explicite dt=0.01','E explicite dt=0.001')
%On peut voir que E explicite est plus grande que celle calculée à
% partir de la solution exacte. Et elle n'est pas une constante
%E augment moins vite si dt est plus petit et devient plus près avec E
% exacte.
```



2.5

```
val1=eig(A1)
val2=eig(A2)
val3=eig(A3)
val4=eig(A4)
% On voit que le schema d'Euler explicite est toujours instable
% les valeurs absolues des valeurs propres sont plus grandes si dt est
% plus grandes
```

val1 =

```
1.0000 + 0.0628i
1.0000 - 0.0628i
```

val2 =

```
1.0000 + 0.0188i  
1.0000 - 0.0188i
```

```
val3 =
```

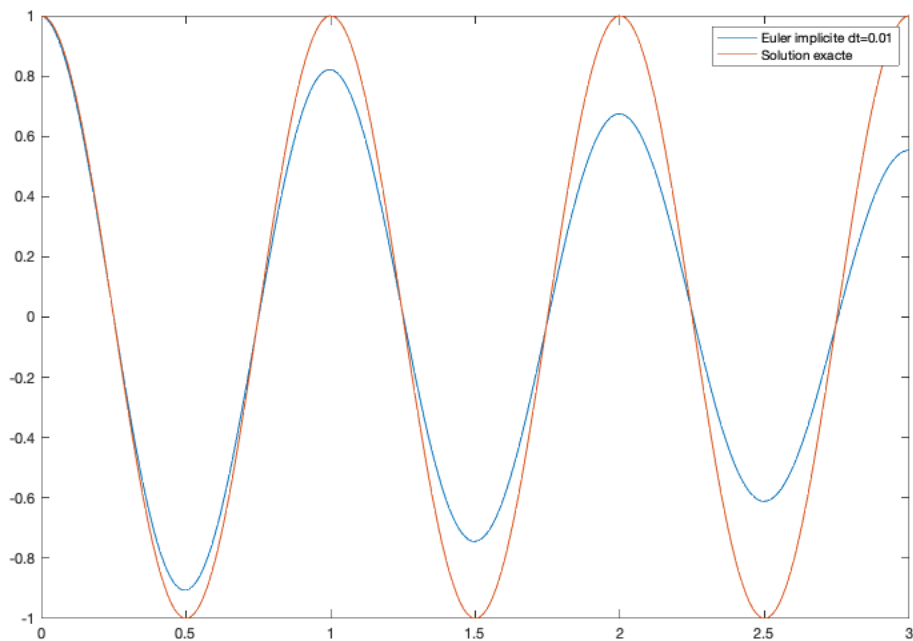
```
1.0000 + 0.0094i  
1.0000 - 0.0094i
```

```
val4 =
```

```
1.0000 + 0.0063i  
1.0000 - 0.0063i
```

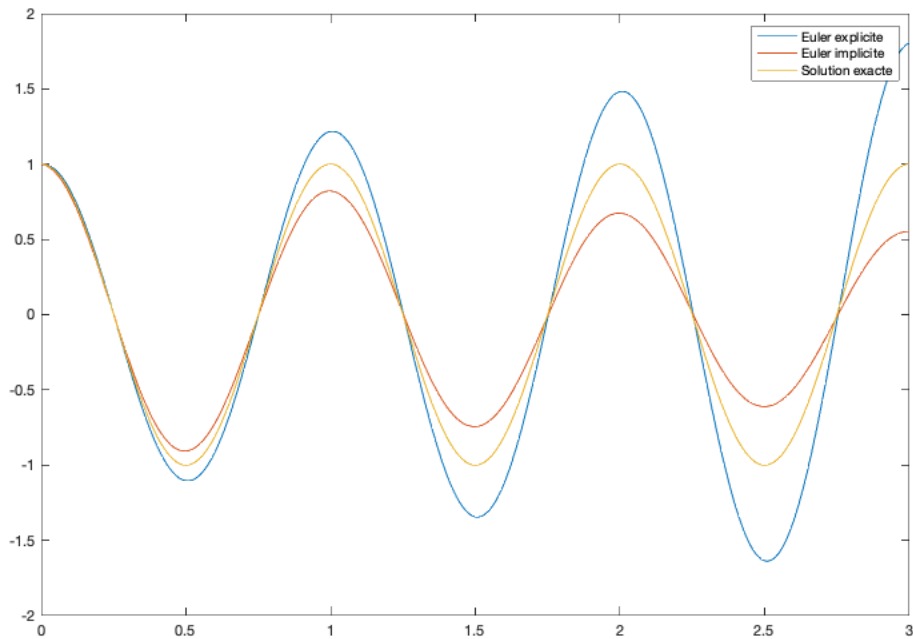
3.1

```
dt=0.01;  
t=0:dt:T0;  
B=[1, -dt; W0^2*dt, 1];  
B=inv(B);  
Uim(:,1)=[q0;dq0];  
for j=1:(length(t)-1)  
    Uim(:,j+1)= B*Uim(:,j);  
end  
plot(t,Uim(1,:),t,q)  
hold on;  
legend('Euler implicite dt=0.01','Solution exacte')
```



3.2

```
clf;  
plot(t,U(1,:),t,Uim(1,:),t,q)  
legend('Euler explicite','Euler implicite','Solution exacte')
```

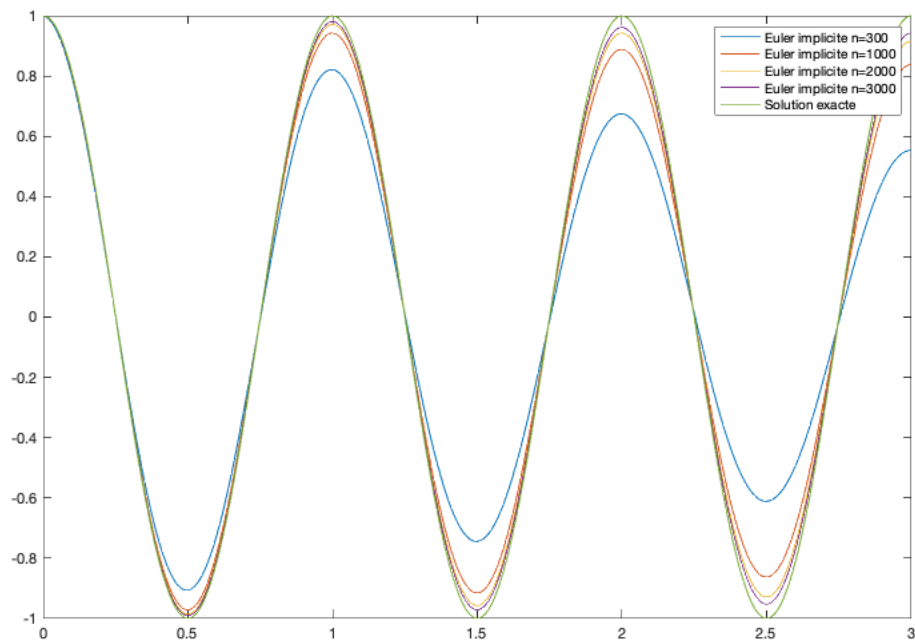


3.3

```
clf;  
plot(t,Uim(1,:))  
hold on;  
  
n1=1000;  
dtim1=T0/n1;  
tim1=0:dtim1:T0;  
Uim1(:,1)=[q0;dq0];  
B1=[1,-dtim1; w0^2*dtim1,1];  
B1=inv(B1);  
for j=1:(length(tim1)-1)  
    Uim1(:,j+1)= B1*Uim1(:,j);  
end  
plot(tim1,Uim1(1,:))  
  
hold on;  
n2=2000;  
dtim2=T0/n2;  
tim2=0:dtim2:T0;  
Uim2(:,1)=[q0;dq0];
```

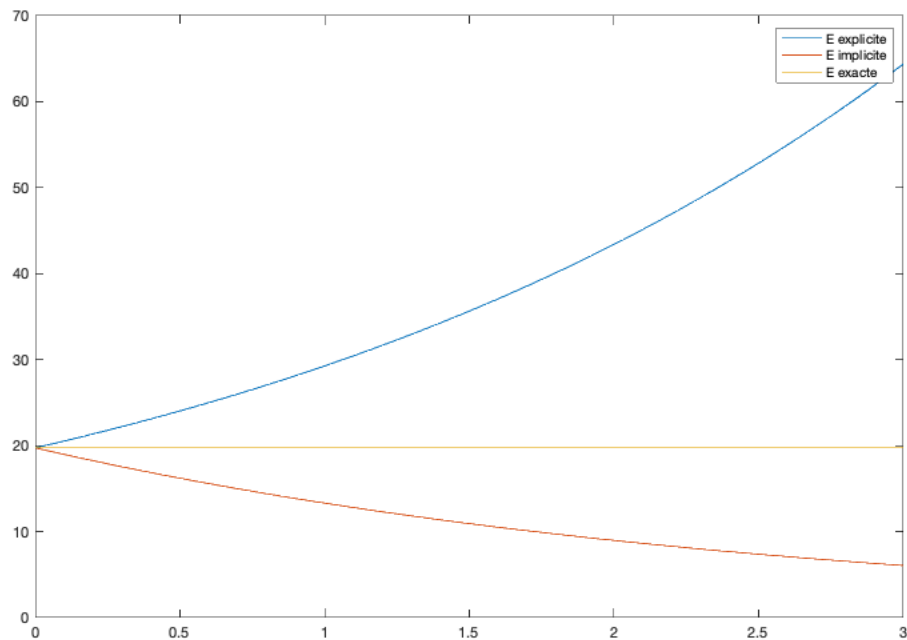
```
B2=[1, -dtim2; W0^2*dtim2, 1];
B2=inv(B2);
for j=1:(length(tim2)-1)
    Uim2(:,j+1)= B2*Uim2(:,j);
end
plot(tim2,Uim2(1,:))
hold on;

n3=3000;
dtim3=T0/n3;
tim3=0:dtim3:T0;
Uim3(:,1)=[q0;dq0];
B3=[1, -dtim3; W0^2*dtim3, 1];
B3=inv(B3);
for j=1:(length(tim3)-1)
    Uim3(:,j+1)= B3*Uim3(:,j);
end
plot(tim3,Uim3(1,:))
hold on;
plot(t,q)
legend('Euler implicite n=300','Euler implicite n=1000','Euler
implicite n=2000','Euler implicite n=3000','Solution exacte')
%En testant différents pas de temps, on peut voir que un amortissement
numérique
%On pourra remarquer cependant que plus le pas de temps  $\Delta t$  est petit,
plus l'atténuation des oscillations est faible.
```



3.4

```
clf;
for j=1:length(t)
    Eimp(j)=1/2*(Uim(2,j)*Uim(2,j)+4*pi*pi*Uim(1,j)*Uim(1,j));
end
plot(t,E1)
hold on;
plot(t,Eimp)
hold on;
Eexa(1:length(t))=E;
plot(t,Eexa)
legend('E explicite','E implicite','E exacte')
```



3.5

```
valim=eig(B)
valim1=eig(B1)
valim2=eig(B2)
valim3=eig(B3)
%Quand dt est plus de 0.0015 On voit que le schema d'Euler implicite
    est toujours stable
%mais quand dt est moins de 0.001, le schema d'Euler implicite est
    toujours instable

valim =
```

```
0.9961 + 0.0626i  
0.9961 - 0.0626i
```

```
valim1 =
```

```
0.9996 + 0.0188i  
0.9996 - 0.0188i
```

```
valim2 =
```

```
0.9999 + 0.0094i  
0.9999 - 0.0094i
```

```
valim3 =
```

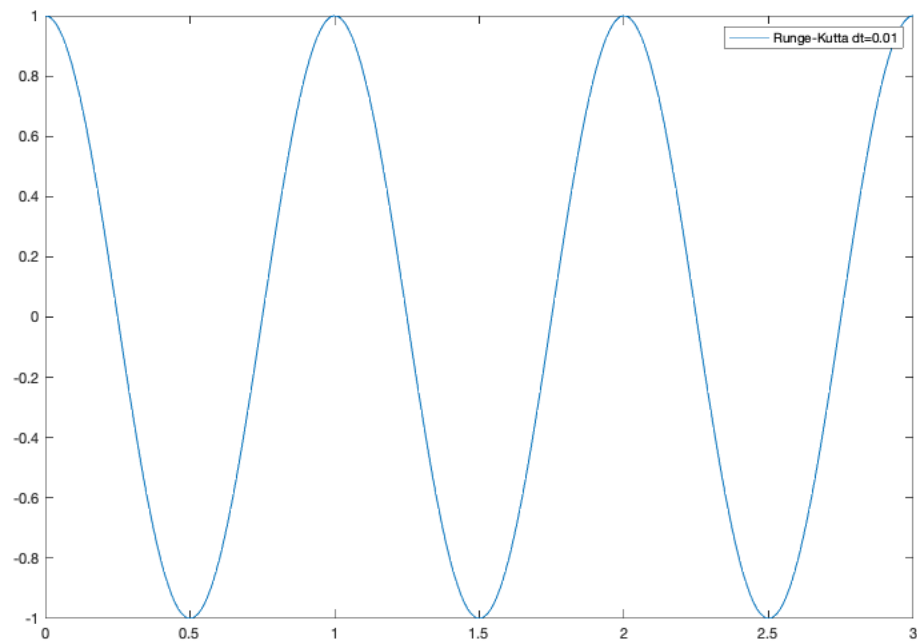
```
1.0000 + 0.0063i  
1.0000 - 0.0063i
```

4.1

```
%x=q dx=dq  
%X=[x;dx]  
%M = [0 , 1; -W0^2, 0]  
%dX = M * X;  
%Afin d'obtenir une formulation adaptée aux schémas du premier ordre.
```

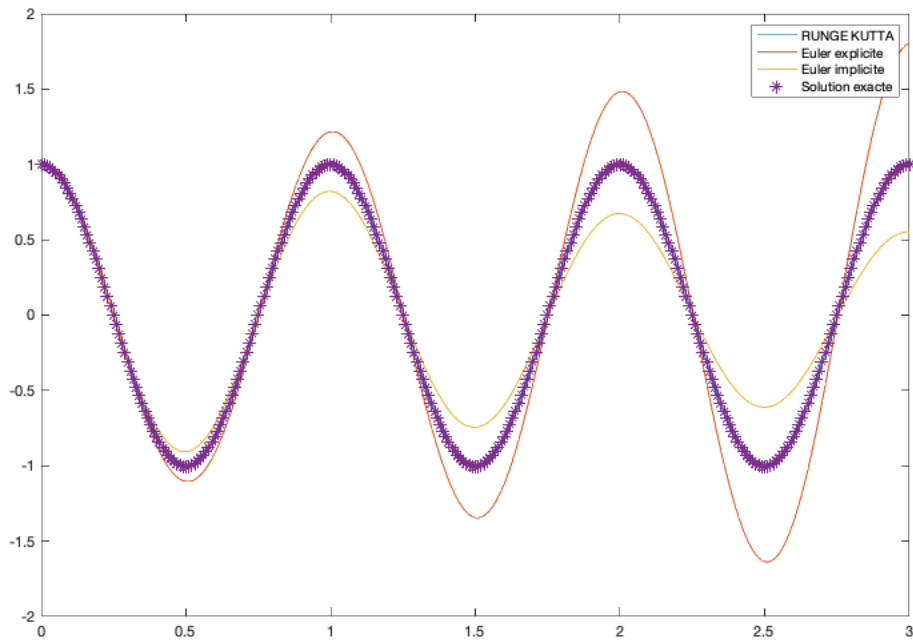
4.2

```
clf;  
dt = 0.01;  
t=0:dt:T0;  
C=[0,1; -W0^2,0];  
Urk(:,1)=[q0;dq0];  
for j=1:(length(t)-1)  
    k1=C*Urk(:,j);  
    k2=C*(Urk(:,j)+1/2*k1*dt);  
    k3=C*(Urk(:,j)+1/2*k2*dt);  
    k4=C*(Urk(:,j)+k3*dt);  
    Urk(:,j+1)= Urk(:,j)+1/6*dt*(k1+2*k2+2*k3+k4);  
end  
plot(t,Urk(1,:));  
legend('Runge-Kutta dt=0.01')
```



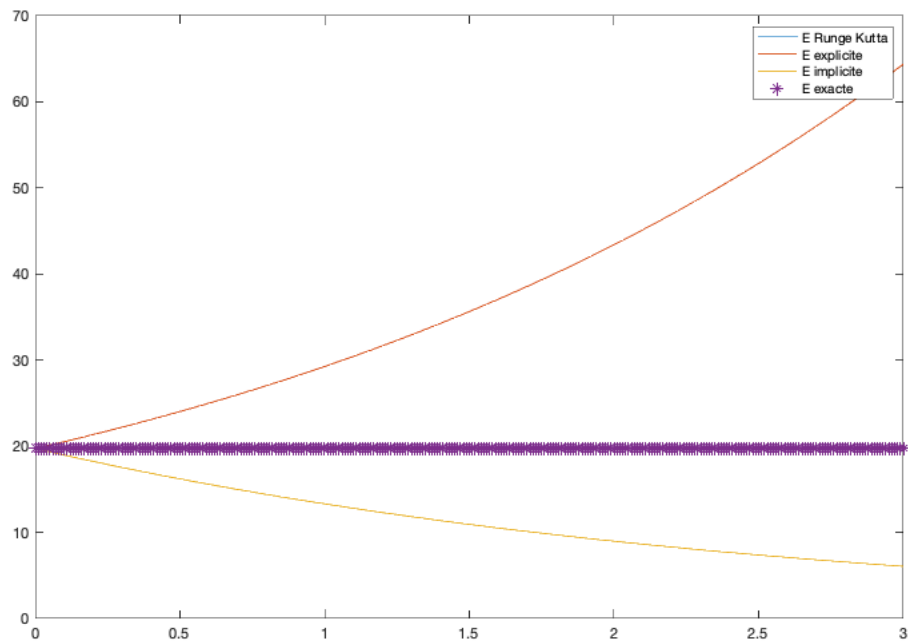
4.3

```
clf;
plot(t,Urk(1,:))
hold on;
plot(t,U(1,:))
hold on;
plot(t,Uim(1,:))
hold on;
plot(t,q,'*')
legend('RUNGE KUTTA','Euler explicite','Euler implicite','Solution
exacte')
%on peut voir que Runge Kutta est plus pres du solution exacte
```



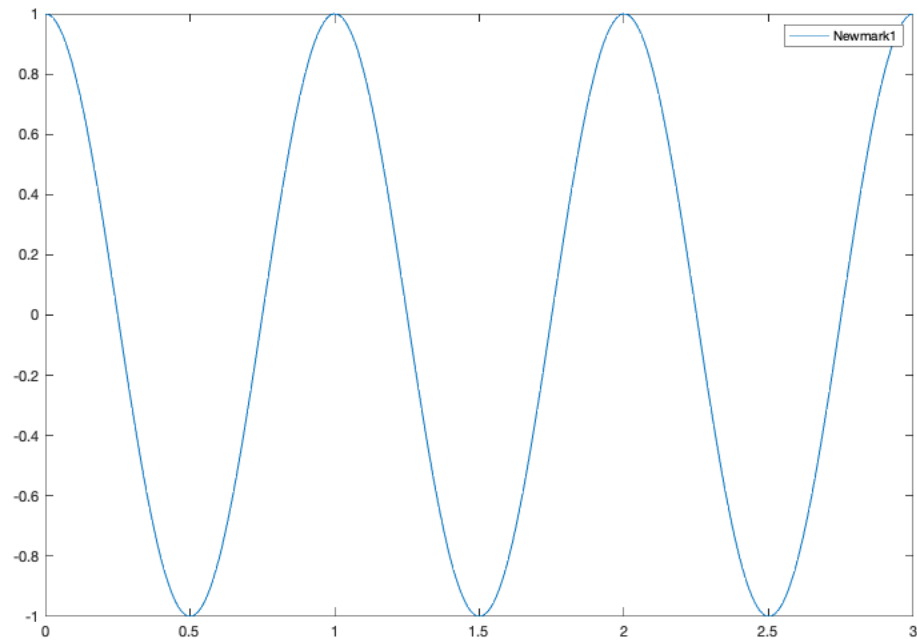
4.4

```
clf;
for j=1:length(t)
    Erk(j)=1/2*(Urk(2,j)*Urk(2,j)+4*pi*pi*Urk(1,j)*Urk(1,j));
end
plot(t,Erk)
hold on;
plot(t,E1)
hold on;
plot(t,Eimp)
hold on;
plot(t,Eexa, '*')
legend('E Runge Kutta', 'E explicite', 'E implicite', 'E exacte')
%on peut voir E de Runge Kutta est plus pres du solution exacte
```



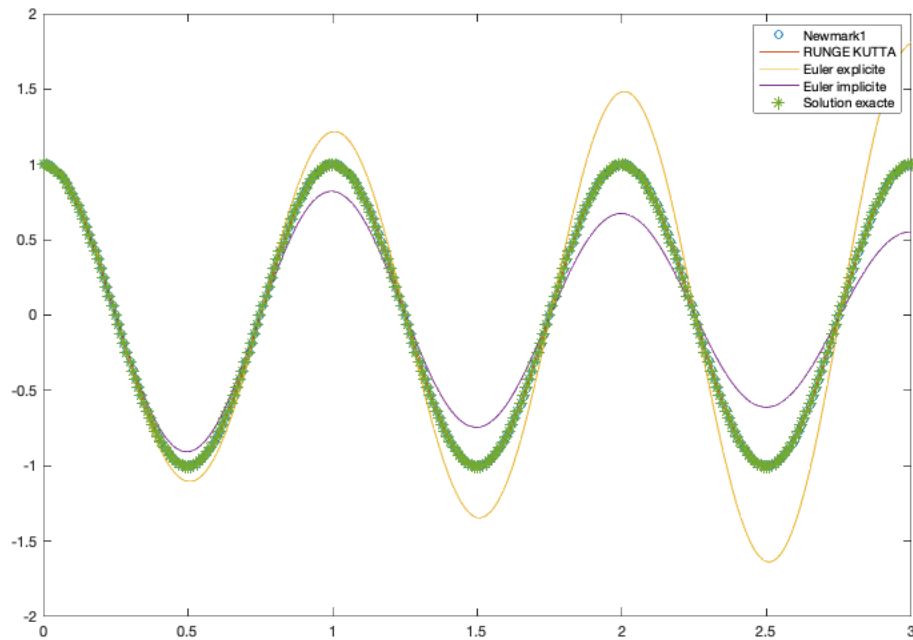
5.1.1

```
clf;
w0 = 2*pi;
n=300;dt=T0/n;
t=0:dt:T0;
gama1=0.5;beta1=0.25;
Bnewm1=[1+beta1*dt*dt*w0*w0,0;gama1*dt*w0*w0,1];
Cnewm1=[1-(0.5-beta1)*dt*dt*w0*w0,dt; -(1-gama1)*dt*w0*w0,1];
Anewm1=inv(Bnewm1)*Cnewm1; % La matrice d'amplification
Unewm1(:,1)=[q0;dq0];
for j=1:(length(t)-1)
    Unewm1(:,j+1)=Anewm1*Unewm1(:,j);
end
plot(t,Unewm1(1,:))
legend('Newmark1')
```



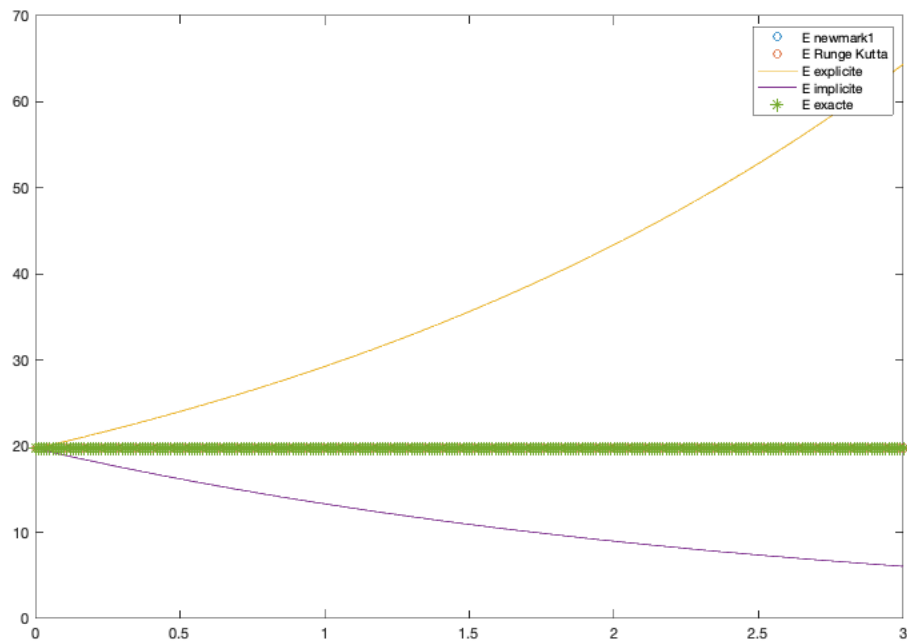
5.1.2

```
clf;
plot(t,Unewm1(1,:), 'o')
hold on;
plot(t,Urk(1,:))
hold on;
plot(t,U(1,:))
hold on;
plot(t,Uim(1,:))
hold on;
plot(t,q, '*')
legend('Newmark1','RUNGE KUTTA','Euler explicite','Euler
implicite','Solution exacte')
```

5.1.3

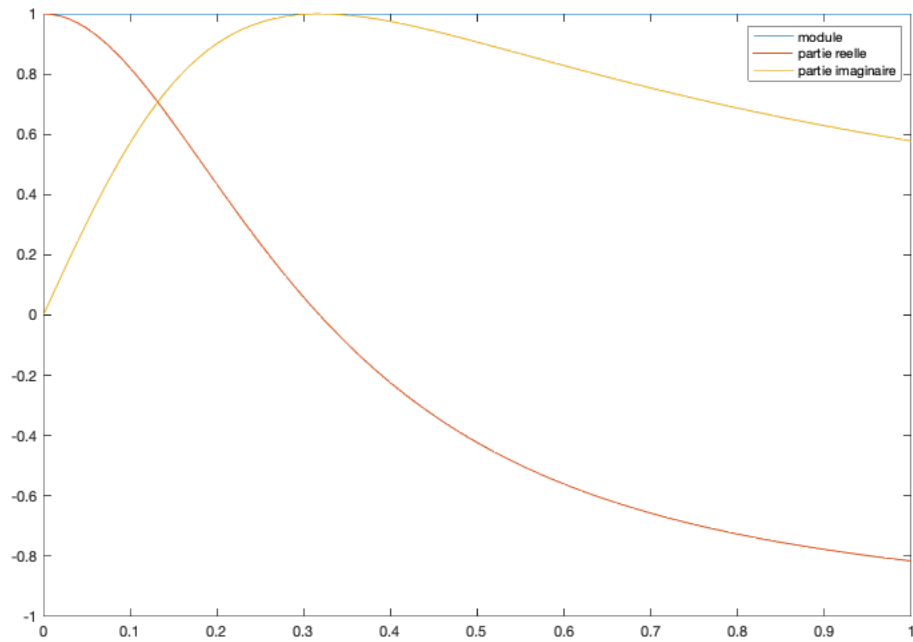
```
for j=1:length(t)
    Enewm1(j)=1/2*(Unewm1(2,j)*Unewm1(2,j)+4*pi*pi*Unewm1(1,j)*Unewm1(1,j));
end
clf;
plot(t,Enewm1,'o')
hold on;
plot(t,Erk,'o')
hold on;
plot(t,E1)
hold on;
plot(t,Eimp)
hold on;
plot(t,Eexa,'*')
legend('E newmark1','E Runge Kutta','E explicite','E implicite','E
exacte')
%le schema de NEWMARK gama=0.5 beta=0.25 et le schema de RUNGE KUTTA
%ces deux converge vers la solution exacte
```



5.1.4

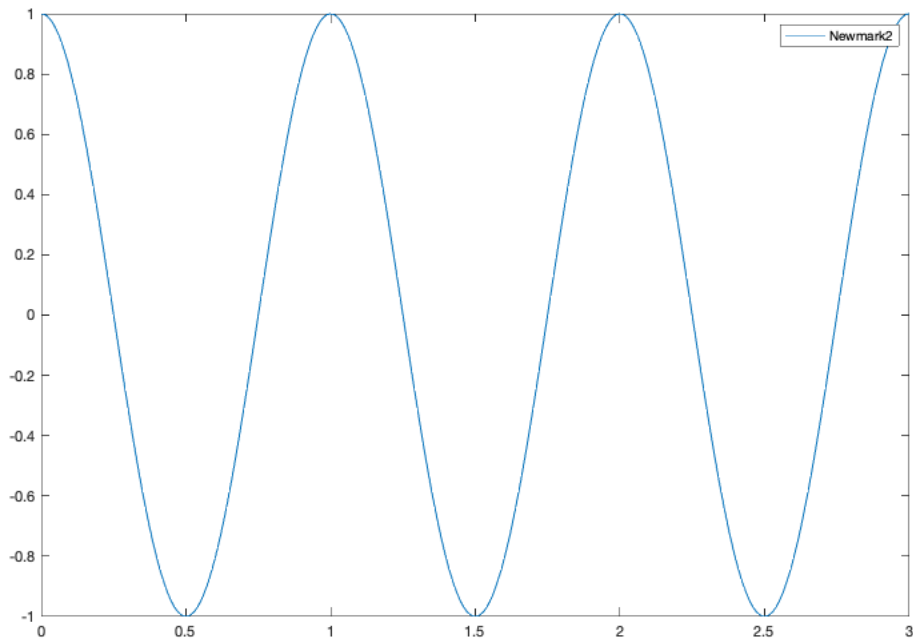
```
clf;
dt514=0:0.01:1;
for j=1:length(dt514)

    Bnewm514{1,j}=[1+beta1*dt514(j)*dt514(j)*w0*w0,0;gama1*dt514(j)*w0*w0,1];
    Cnewm514{1,j}=[1-(0.5-beta1)*dt514(j)*dt514(j)*w0*w0,dt514(j); -
(1-gama1)*dt514(j)*w0*w0,1];
    Anewm514{1,j}=inv(Bnewm514{1,j})*Cnewm514{1,j};
    VPA514(:,j)=eig(Anewm514{1,j});
    moduleVP(j)=sqrt(real(VPA514(1,j))^2+imag(VPA514(1,j))^2);
end
plot(dt514,moduleVP)
hold on;
plot(dt514,real(VPA514(1,:)))
hold on;
plot(dt514,imag(VPA514(1,:)))
legend('module','partie reelle','partie imaginaire')
%plus le pas de temps dt est petit, plus la partie reelle est petit
%la module ne change pas avec dt
```



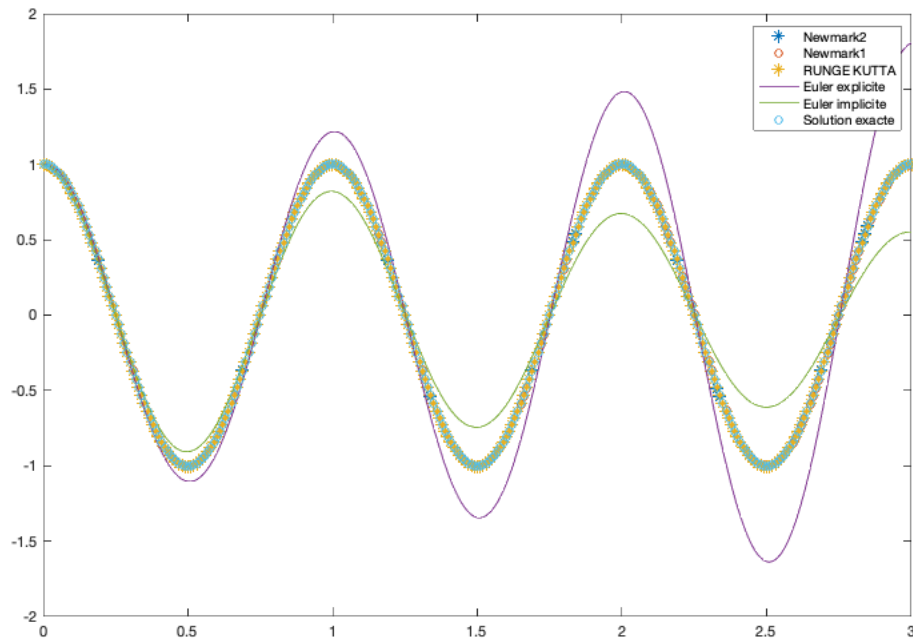
5.2.1

```
clf;
n=300;dt=T0/n;
t=0:dt:T0;
gama2=0.5;beta2=0;
Bnewm2=[1+beta2*dt*dt*w0*w0,0;gama2*dt*w0*w0,1];
Cnewm2=[1-(0.5-beta2)*dt*dt*w0*w0,dt; -(1-gama2)*dt*w0*w0,1];
% La matrice d'amplification
Anewm2=inv(Bnewm2)*Cnewm2;
Unewm2(:,1)=[q0;dq0];
for j=1:(length(t)-1)
    Unewm2(:,j+1)=Anewm2*Unewm2(:,j);
end
plot(t,Unewm2(1,:))
legend('Newmark2')
```



5.2.2

```
clf;
plot(t,Unewm2(1,:), '*')
hold on;
plot(t,Unewm1(1,:), 'o')
hold on;
plot(t,Urk(1,:), '*')
hold on;
plot(t,U(1,:))
hold on;
plot(t,Uim(1,:))
hold on;
plot(t,q, 'o')
legend('Newmark2', 'Newmark1', 'RUNGE KUTTA', 'Euler explicite', 'Euler
implicite', 'Solution exacte')
% On peut voir que les solutions a partie des schemas de NEWMARK et
RUNGE-KUTTA beaucoup plus pres avec la solution exacte
```

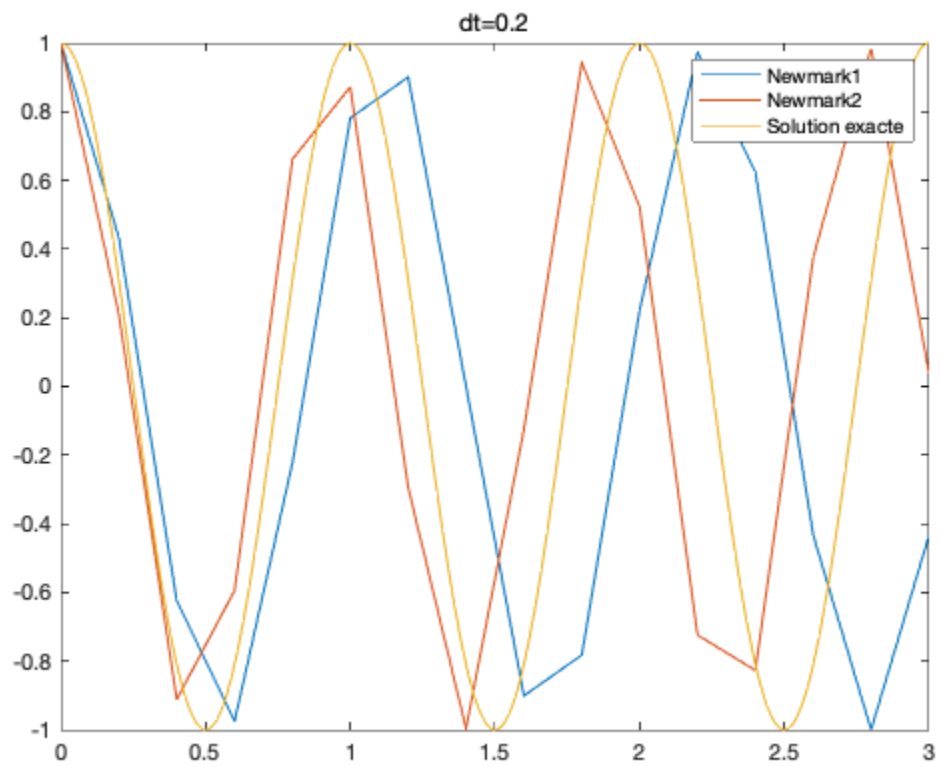


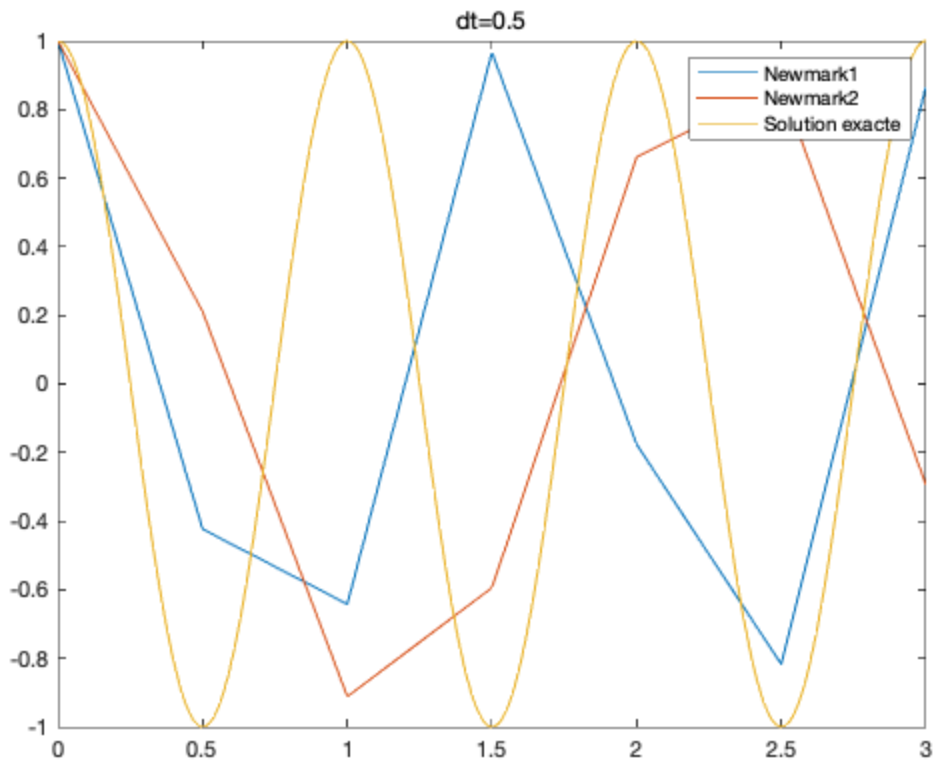
5.2.3

```
%dt=0.2
clf;
dt1=0.2;
t1=0:dt1:T0;
gama1=0.5;beta1=0.25;
Bnewm1=[1+beta1*dt1*dt1*w0*w0,0;gama1*dt1*w0*w0,1];
Cnewm1=[1-(0.5-beta1)*dt1*dt1*w0*w0,dt1; -(1-gama1)*dt1*w0*w0,1];
% La matrice d'amplification
Anewm1=inv(Bnewm1)*Cnewm1;
Unewm15231(:,1)=[q0;dq0];
for j=1:length(t1)
    Unewm15231(:,j+1)=Anewm1*Unewm15231(:,j);
end
Unewm15231(:,length(t1)+1)=[];
gama2=0.5;beta2=0;
Bnewm2=[1+beta2*dt1*dt1*w0*w0,0;gama2*dt1*w0*w0,1];
Cnewm2=[1-(0.5-beta2)*dt1*dt1*w0*w0,dt1; -(1-gama2)*dt1*w0*w0,1];
% La matrice d'amplification
Anewm2=inv(Bnewm2)*Cnewm2;
Unewm25231(:,1)=[q0;dq0];
for j=1:length(t1)
    Unewm25231(:,j+1)=Anewm2*Unewm25231(:,j);
end
Unewm25231(:,length(t1)+1)=[];
figure(1)
plot(t1,Unewm15231(1,:))
hold on;
```

```
plot(t1,Unewm25231(1,:))
hold on;
plot(t,q)
legend('Newmark1','Newmark2','Solution exacte')
title('dt=0.2')

%dt=0.5
dt2=0.5;
t2=0:dt2:T0;
gama1=0.5;beta1=0.25;
Bnewm1=[1+beta1*dt2*dt2*w0*w0,0;gama1*dt2*w0*w0,1];
Cnewm1=[1-(0.5-beta1)*dt2*dt2*w0*w0,dt2; -(1-gama1)*dt2*w0*w0,1];
% La matrice d'amplification
Anewm1=inv(Bnewm1)*Cnewm1;
Unewm15232(:,1)=[q0;dq0];
for j=1:length(t2)
    Unewm15232(:,j+1)=Anewm1*Unewm15232(:,j);
end
Unewm15232(:,length(t2)+1)=[];
gama2=0.5;beta2=0;
Bnewm25=[1+beta2*dt2*dt2*w0*w0,0;gama2*dt2*w0*w0,1];
Cnewm2=[1-(0.5-beta2)*dt2*dt2*w0*w0,dt2; -(1-gama2)*dt2*w0*w0,1];
% La matrice d'amplification
Unewm25232(:,1)=[q0;dq0];
for j=1:length(t2)
    Unewm25232(:,j+1)=Anewm2*Unewm25232(:,j);
end
Unewm25232(:,length(t2)+1)=[];
figure(2)
plot(t2,Unewm15232(1,:))
hold on;
plot(t2,Unewm25232(1,:))
hold on;
plot(t,q)
legend('Newmark1','Newmark2','Solution exacte')
title('dt=0.5')
%On peut voir que si le pas de temps est tres grand, les solutions ne
converge pas bien avec la solution exacte.
```





5.2.4

```
clf
close all
dt524=0:0.0001:1;
for j=1:length(dt524)

    Bnewm524{1,j}=[1+beta2*dt524(j)*dt524(j)*w0*w0,0;gama2*dt524(j)*w0*w0,1];
    Cnewm524{1,j}=[1-(0.5-beta2)*dt524(j)*dt524(j)*w0*w0,dt524(j); -
(1-gama2)*dt524(j)*w0*w0,1];
    Anewm524{1,j}=inv(Bnewm524{1,j})*Cnewm524{1,j};
    VPA524(:,j)=eig(Anewm524{1,j});
    moduleVP2(j)=sqrt(real(VPA524(1,j))^2+imag(VPA524(1,j))^2);
end
plot(dt524,moduleVP2)
hold on;
plot(dt524,real(VPA524(1,:)))
hold on;
plot(dt524,imag(VPA524(1,:)))
legend('module','partie reelle','partie imaginaire')

syms dt5240 w0
Bnewm5240=[1+beta2*dt5240*dt5240*w0*w0,0;gama2*dt5240*w0*w0,1];
Cnewm5240=[1-(0.5-beta2)*dt5240*dt5240*w0*w0,dt5240; -(1-
gama2)*dt5240*w0*w0,1];
```

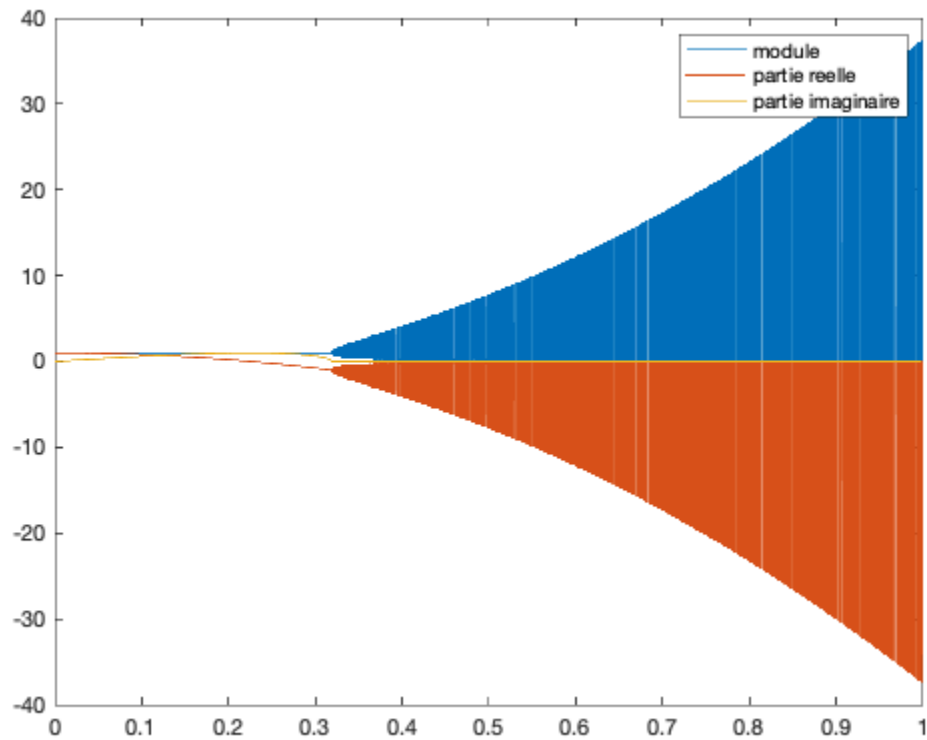


```
Anewm5240=inv(Bnewm5240)*Cnewm5240;  
VPA5240=eig(Anewm5240)
```

```
% le pas de temps de critique est 1/pi  
% donc alpha=1
```

```
VPA5240 =
```

```
1 - (dt5240*w0*((dt5240*w0 - 2)*(dt5240*w0 + 2))^(1/2))/2 -  
(dt5240^2*w0^2)/2  
(dt5240*w0*((dt5240*w0 - 2)*(dt5240*w0 + 2))^(1/2))/2 -  
(dt5240^2*w0^2)/2 + 1
```



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