Oscillateur conservatif linéaire à un degré de liberté

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0

```
%Retrouver l’équation du mouvement du pendule simple avec les
 équations de Lagrange
A = imread('IMG_0324.jpg');
imshow(A);
```

On sait que l'équation de Lagrange

$L = E_c - E_p$. $\quad \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}_i}\right] - \frac{\partial L}{\partial x_i} = Q_i$ avec $\frac{\partial E_p}{\partial \dot{x}_i} = 0$

Et pour pendule simple, on a $\begin{cases} E_c = \frac{I}{2}\dot{\theta}^2 \\ E_p = -mgd\cos\theta + cte. \\ \delta W = 0. \end{cases}$

Donc $L = E_c - E_p = \frac{I}{2}\dot{\theta}^2 + mgd\cos\theta + cte$

$\delta W = \sum_{i=1}^{N} Q_i \delta q_i = 0 \Rightarrow Q_i = 0$

Selon l'équation de Lagrange, $\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}}\right] - \frac{\partial L}{\partial \theta} = Q$.

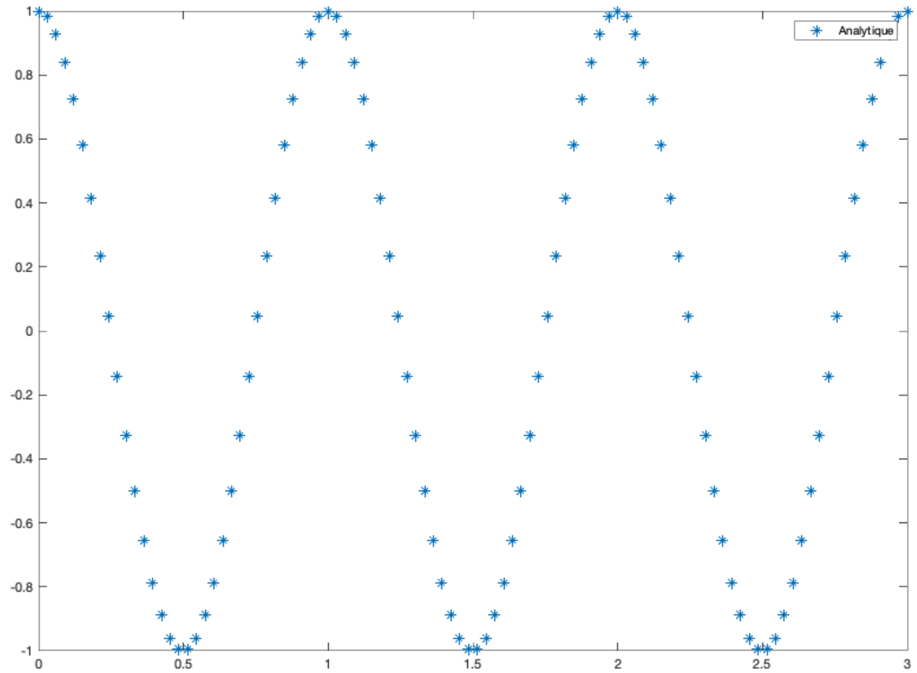
On a l'équation du mouvement du pendule simple

$\Rightarrow I\ddot{\theta} + mgd\sin\theta = 0$

1.1

```
clear all
clf;
W0=2*pi;
syms q t ;
b='D2q=-(2*pi)^2*q';
q = simplify(dsolve(b,'q(0)=1','Dq(0)=0'));
% on trouve q =cos(2*pi*t)
T0 = 3;
t = linspace(0,T0,100);
q= cos(2*pi*t); %on a deja trouver
plot(t,q,'*')
legend('Analytique');
```

```
##: Support of character vectors and strings will be removed in a
 future
release. Use sym objects to define differential equations instead.
```



1.2

```
%dq = simplify(diff(q,t));
%E =1/2*(dq^2+(2*pi)^2*q^2);
%E = simplify(E);
E = 2*pi^2;
% E est une constante
```

2.1

```
B = imread('IMG_0325.jpg');
imshow(B);
```

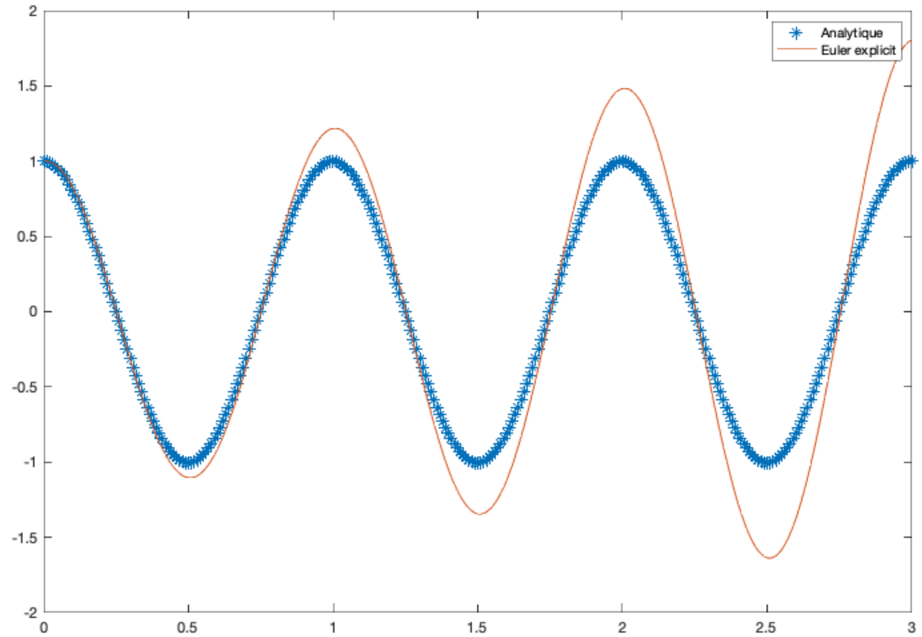
On a
$$\begin{cases} \begin{vmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{vmatrix} = \begin{vmatrix} q_j \\ \dot{q}_j \end{vmatrix} + \Delta t \begin{vmatrix} \dot{q}_j \\ \ddot{q}_j \end{vmatrix} \\ \ddot{q}_j = -\omega_0^2 q_j \end{cases}$$

$$\begin{vmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{vmatrix} = \begin{vmatrix} q_j + \Delta t \, \dot{q}_j \\ \dot{q}_j + \Delta t \, \ddot{q}_j \end{vmatrix} = \begin{vmatrix} q_j + \Delta t \, \dot{q}_j \\ \dot{q}_j - \omega_0^2 \Delta t \, q_j \end{vmatrix}$$

Donc
$$\begin{vmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{vmatrix} = \begin{bmatrix} 1 & \Delta t \\ -\omega_0^2 \Delta t & 1 \end{bmatrix} \begin{vmatrix} q_j \\ \dot{q}_j \end{vmatrix}$$

2.2

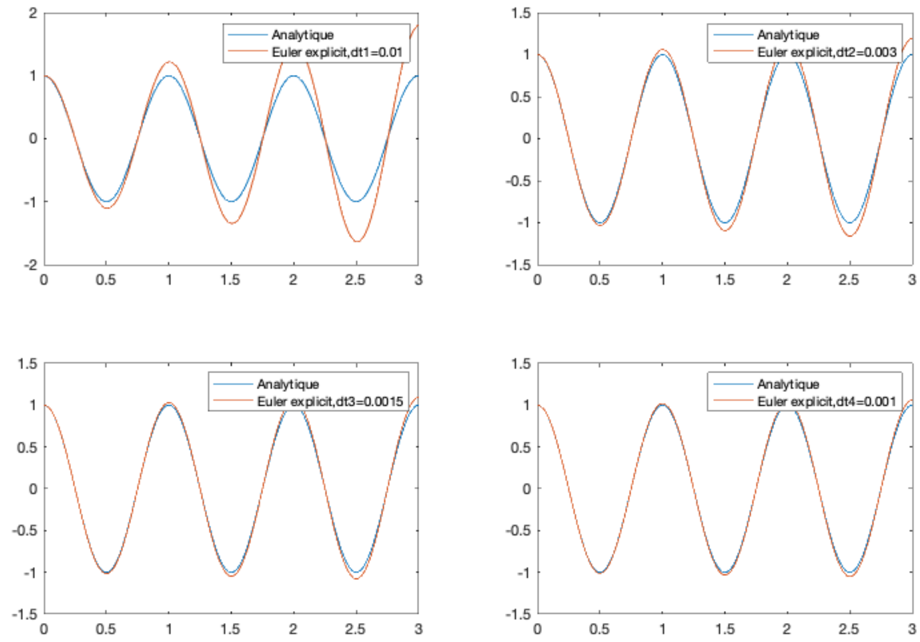
```
clf;
dt=0.01;
t=0:dt:T0;
q0=1;
dq0=0;
A=[1,dt; -W0^2*dt,1];
U(:,1)=[q0;dq0];
for j=1:(length(t)-1)
    U(:,j+1)= A*U(:,j);
end
q = cos(W0*t);
plot(t,q,'*',t,U(1,:));
legend('Analytique','Euler explicit');
```



2.3

```
clf;
dt1=0.01;
dt2=0.003;
dt3=0.0015;
dt4=0.001;
t1=0:dt1:T0;
t2=0:dt2:T0;
t3=0:dt3:T0;
t4=0:dt4:T0;
q0=1;
dq0=0;
A1=[1,dt1; -W0^2*dt1,1];
U1(:,1)=[q0;dq0];
for j=1:(length(t1)-1)
    U1(:,j+1)= A1*U1(:,j);
end
q1 = cos(W0*t1);
A2=[1,dt2; -W0^2*dt2,1];
U2(:,1)=[q0;dq0];
for j=1:(length(t2)-1)
    U2(:,j+1)= A2*U2(:,j);
end
q2 = cos(W0*t2);
A3=[1,dt3; -W0^2*dt3,1];
U3(:,1)=[q0;dq0];
for j=1:(length(t3)-1)
    U3(:,j+1)= A3*U3(:,j);
```

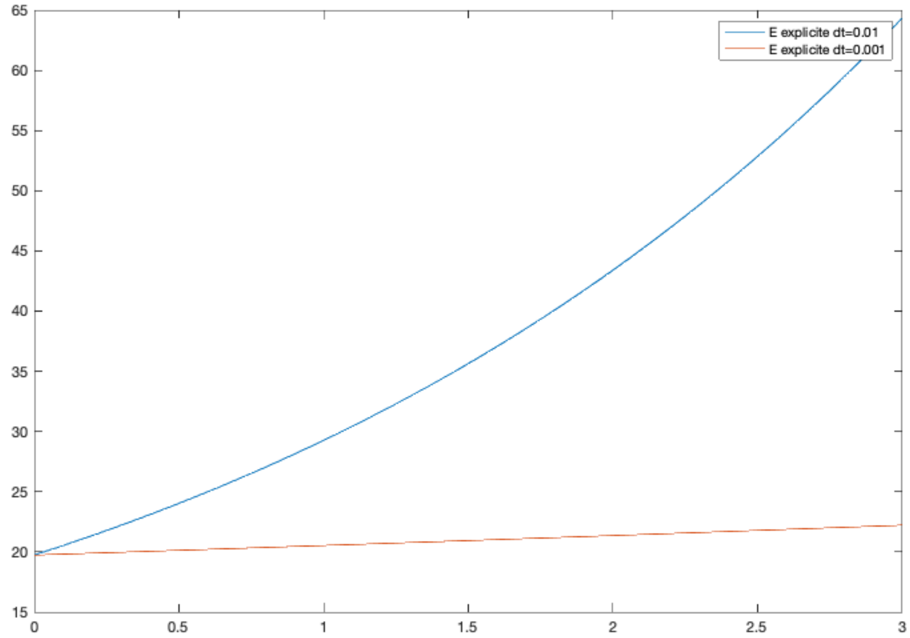
```
end
q3 = cos(W0*t3);
A4=[1,dt4; -W0^2*dt4,1];
U4(:,1)=[q0;dq0];
for j=1:(length(t4)-1)
    U4(:,j+1)= A4*U4(:,j);
end
q4 = cos(W0*t4);
subplot(2,2,1)
plot(t1,q1,t1,U1(1,:))
legend('Analytique','Euler explicit,dt1=0.01')
subplot(2,2,2)
plot(t2,q2,t2,U2(1,:))
legend('Analytique','Euler explicit,dt2=0.003')
subplot(2,2,3)
plot(t3,q3,t3,U3(1,:))
legend('Analytique','Euler explicit,dt3=0.0015')
subplot(2,2,4)
plot(t4,q4,t4,U4(1,:))
legend('Analytique','Euler explicit,dt4=0.001')
%On peut voir que plus le pas de temps ?t est petit, plus la
 divergence est lent
```



2.4

```
clf
for j=1:length(t)
    E1(j)=1/2*(U(2,j)*U(2,j)+4*pi*pi*U(1,j)*U(1,j));
end
```

```
for j=1:length(t4)
    E4(j)=1/2*(U4(2,j)*U4(2,j)+4*pi*pi*U4(1,j)*U4(1,j));
end
plot(t,E1,t4,E4)
legend('E explicite dt=0.01','E explicite dt=0.001')
%On peut voir que E explicite est plus grande que celle calculée à
 partir de la solution exacte. Et elle n'est pas une constante
%E augment moins vite si dt est plus petit et devient plus près avec E
 exacte.
```



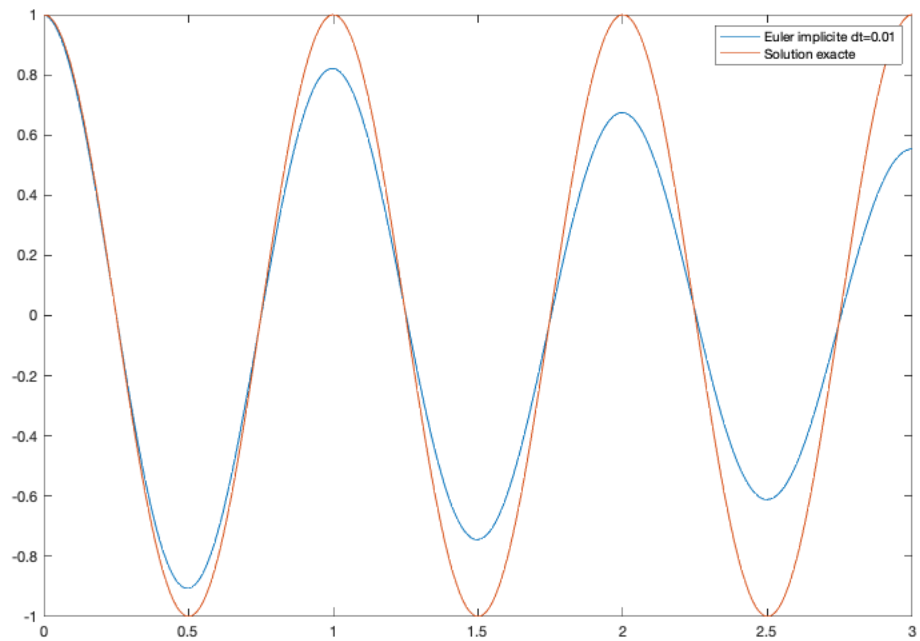
2.5

```
val1=eig(A1)
val2=eig(A2)
val3=eig(A3)
val4=eig(A4)
% On voit que le schema d'Euler explicite est toujours instable
% les valeurs absolues des valeurs propres sont plus grandes si dt est
 plus grandes


val1 =

   1.0000 + 0.0628i
   1.0000 - 0.0628i


val2 =
```

```
   1.0000 + 0.0188i
   1.0000 - 0.0188i


val3 =

   1.0000 + 0.0094i
   1.0000 - 0.0094i


val4 =

   1.0000 + 0.0063i
   1.0000 - 0.0063i
```

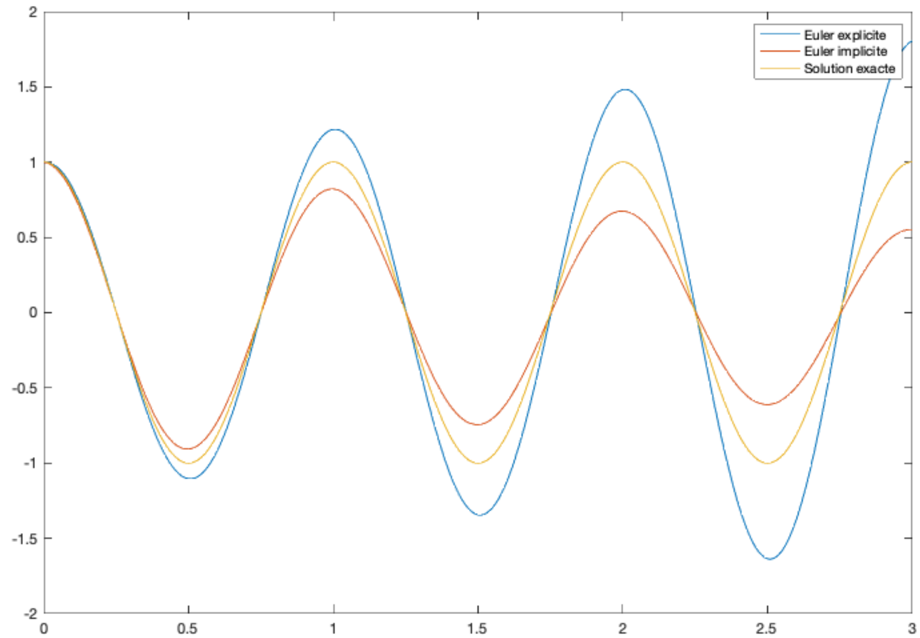
3.1

```
dt=0.01;
t=0:dt:T0;
B=[1,-dt; W0^2*dt,1];
B=inv(B);
Uim(:,1)=[q0;dq0];
for j=1:(length(t)-1)
    Uim(:,j+1)= B*Uim(:,j);
end
plot(t,Uim(1,:),t,q)
hold on;
legend('Euler implicite dt=0.01','Solution exacte')
```



3.2

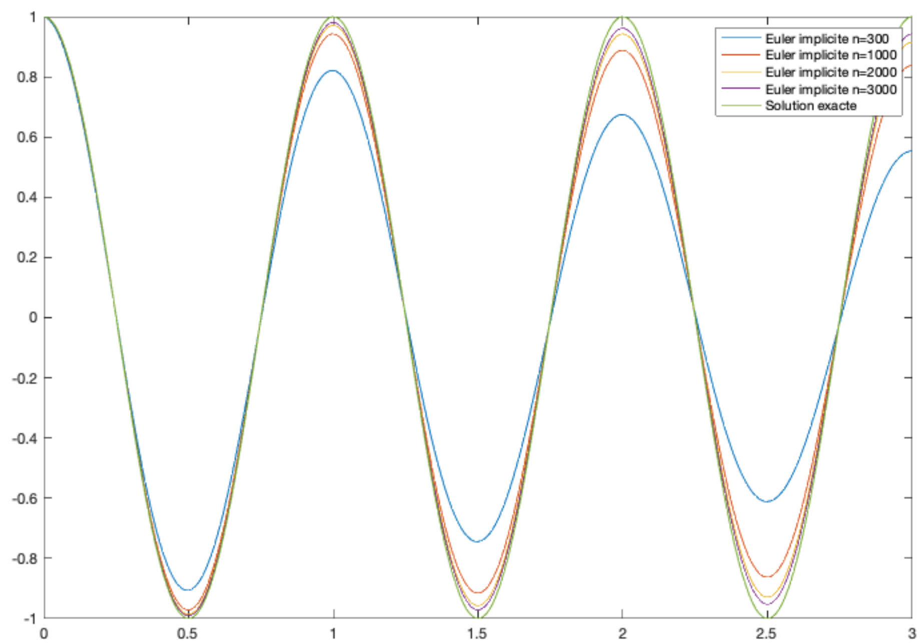
```
clf;
plot(t,U(1,:),t,Uim(1,:),t,q)
legend('Euler explicite','Euler implicite','Solution exacte')
```



3.3

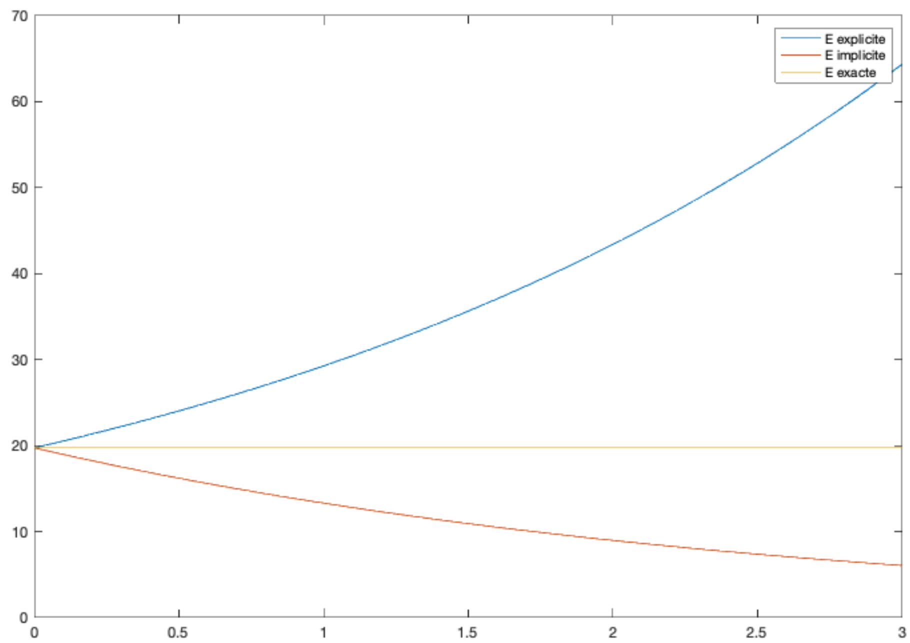
```
clf;
plot(t,Uim(1,:))
hold on;

n1=1000;
dtim1=T0/n1;
tim1=0:dtim1:T0;
Uim1(:,1)=[q0;dq0];
B1=[1,-dtim1; W0^2*dtim1,1];
B1=inv(B1);
for j=1:(length(tim1)-1)
    Uim1(:,j+1)= B1*Uim1(:,j);
end
plot(tim1,Uim1(1,:))

hold on;
n2=2000;
dtim2=T0/n2;
tim2=0:dtim2:T0;
Uim2(:,1)=[q0;dq0];
```

```
B2=[1,-dtim2; W0^2*dtim2,1];
B2=inv(B2);
for j=1:(length(tim2)-1)
    Uim2(:,j+1)= B2*Uim2(:,j);
end
plot(tim2,Uim2(1,:))
hold on;

n3=3000;
dtim3=T0/n3;
tim3=0:dtim3:T0;
Uim3(:,1)=[q0;dq0];
B3=[1,-dtim3; W0^2*dtim3,1];
B3=inv(B3);
for j=1:(length(tim3)-1)
    Uim3(:,j+1)= B3*Uim3(:,j);
end
plot(tim3,Uim3(1,:))
hold on;
plot(t,q)
legend('Euler implicite n=300','Euler implicite n=1000','Euler
 implicite n=2000','Euler implicite n=3000','Solution exacte')
%En testant différents pas de temps, on peut voir que un amortissement
 num ?erique
%On pourra remarquer cependant que plus le pas de temps ?t est petit,
 plus l’att ?enuation des oscillations est faible.
```



3.4

```
clf;
for j=1:length(t)
    Eimp(j)=1/2*(Uim(2,j)*Uim(2,j)+4*pi*pi*Uim(1,j)*Uim(1,j));
end
plot(t,E1)
hold on;
plot(t,Eimp)
hold on;
Eexa(1:length(t))=E;
plot(t,Eexa)
legend('E explicite','E implicite','E exacte')
```



3.5

```
valim=eig(B)
valim1=eig(B1)
valim2=eig(B2)
valim3=eig(B3)
%Quand dt est plus de 0.0015 On voit que le schema d'Euler implicite
 est toujours stable
%mais quand dt est moins de 0.001,le schema d'Euler implicite est
 toujours instable
```

valim =

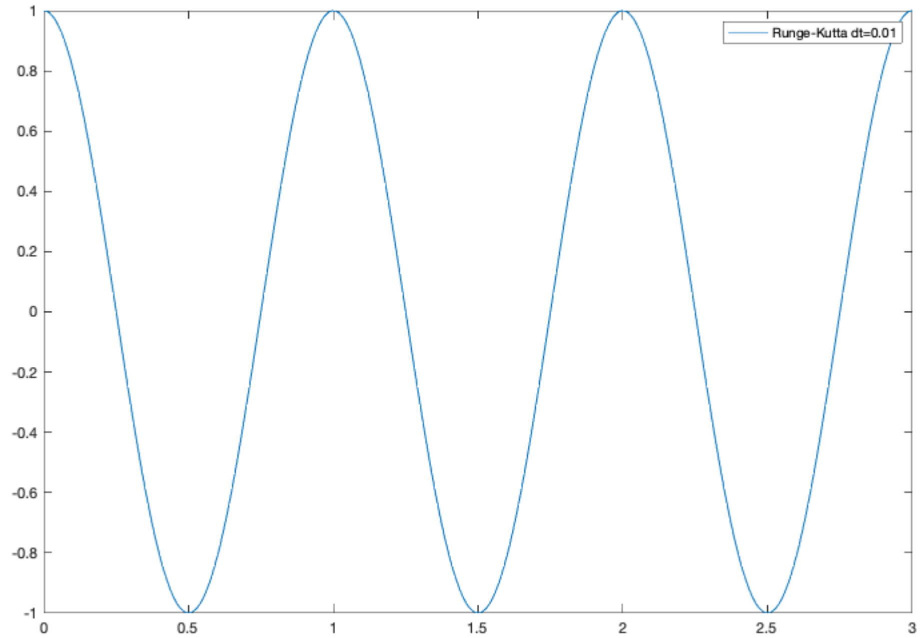
```
   0.9961 + 0.0626i
   0.9961 - 0.0626i


valim1 =

   0.9996 + 0.0188i
   0.9996 - 0.0188i


valim2 =

   0.9999 + 0.0094i
   0.9999 - 0.0094i


valim3 =

   1.0000 + 0.0063i
   1.0000 - 0.0063i
```

4.1

```
%x=q dx=dq
%X=[x;dx]
%M = [0 , 1; -W0^2, 0]
%dX = M * X;
%Afin d'obtenir une formulation adaptée aux schémas du premier ordre.
```

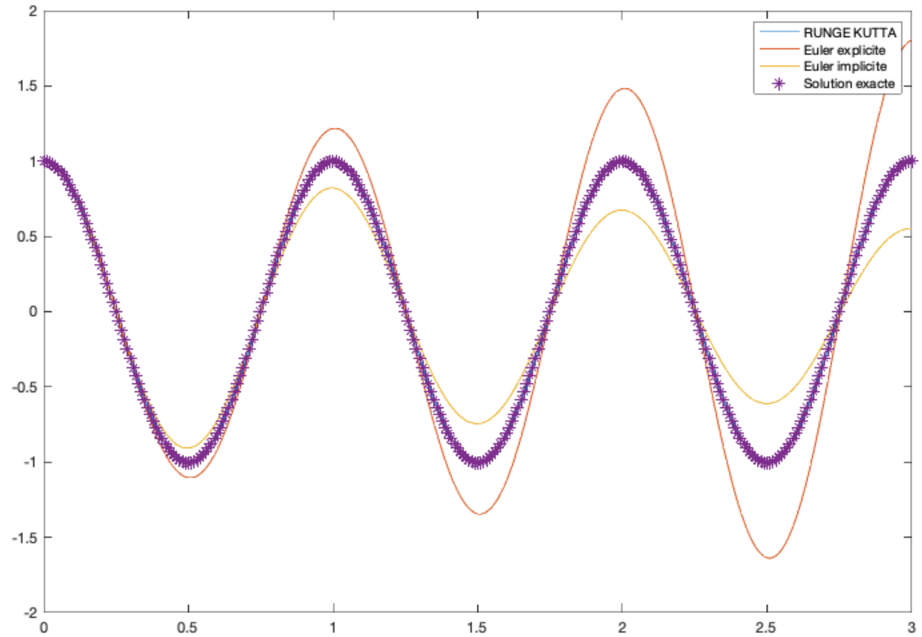
4.2

```
clf;
dt = 0.01;
t=0:dt:T0;
C=[0,1; -W0^2,0];
Urk(:,1)=[q0;dq0];
for j=1:(length(t)-1)
    k1=C*Urk(:,j);
    k2=C*(Urk(:,j)+1/2*k1*dt);
    k3=C*(Urk(:,j)+1/2*k2*dt);
    k4=C*(Urk(:,j)+k3*dt);
    Urk(:,j+1)= Urk(:,j)+1/6*dt*(k1+2*k2+2*k3+k4);
end
 plot(t,Urk(1,:));
 legend('Runge-Kutta dt=0.01')
```



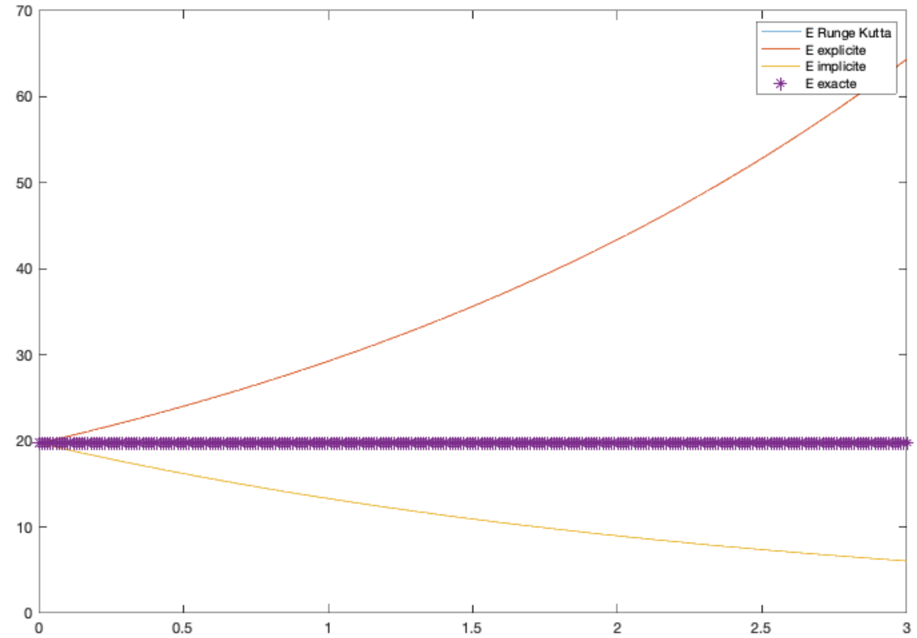
4.3

```
clf;
plot(t,Urk(1,:))
hold on;
plot(t,U(1,:))
hold on;
plot(t,Uim(1,:))
hold on;
plot(t,q,'*')
legend('RUNGE KUTTA','Euler explicite','Euler implicite','Solution
 exacte')
 %on peut voir que Runge Kutta est plus pres du solution exacte
```



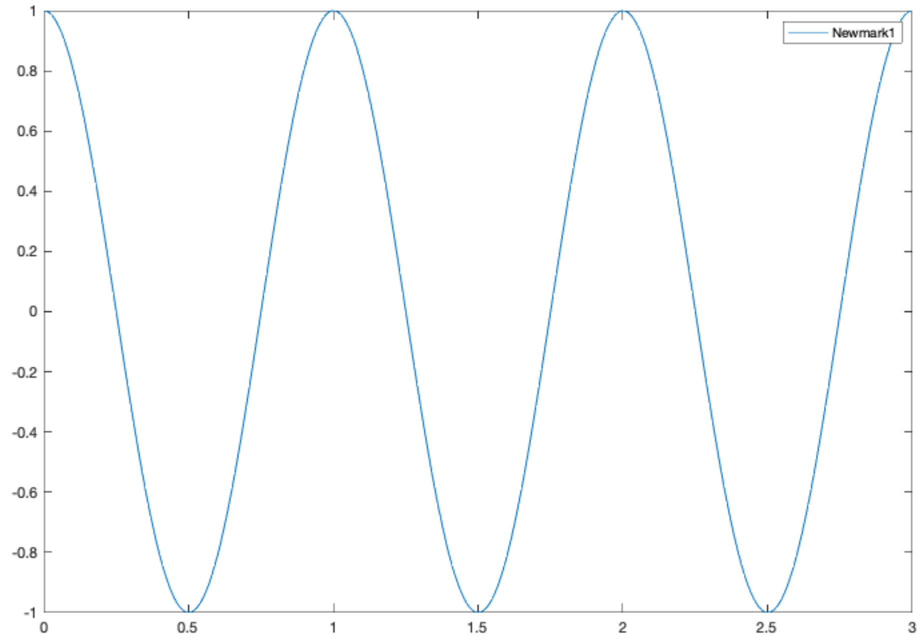
4.4

```
clf;
for j=1:length(t)
    Erk(j)=1/2*(Urk(2,j)*Urk(2,j)+4*pi*pi*Urk(1,j)*Urk(1,j));
end
plot(t,Erk)
hold on;
plot(t,E1)
hold on;
plot(t,Eimp)
hold on;
plot(t,Eexa,'*')
legend('E Runge Kutta','E explicite','E implicite','E exacte')
%on peut voir E de Runge Kutta est plus pres du solution exacte
```



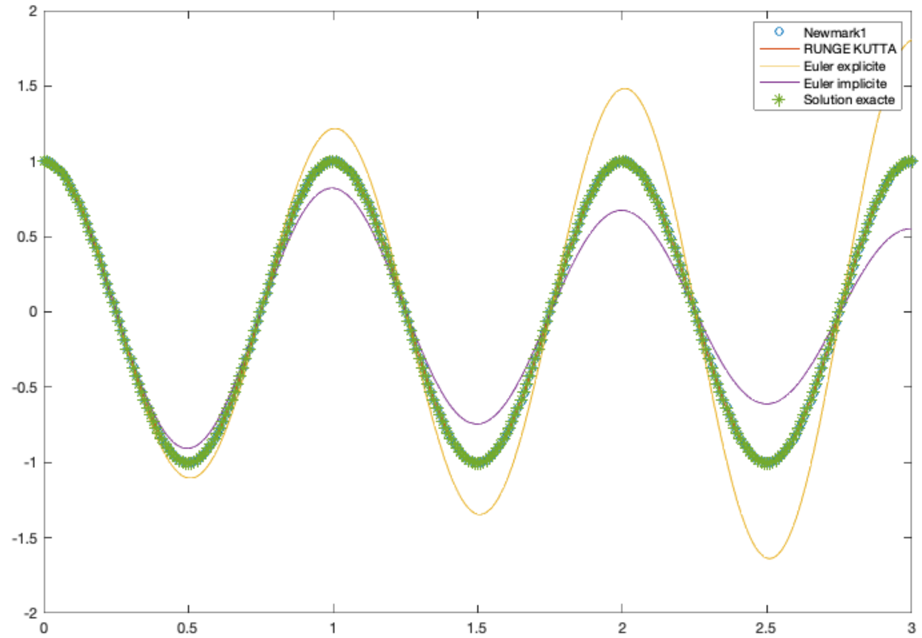
5.1.1

```
clf;
w0 = 2*pi;
n=300;dt=T0/n;
t=0:dt:T0;
gama1=0.5;beta1=0.25;
Bnewm1=[1+beta1*dt*dt*w0*w0,0;gama1*dt*w0*w0,1];
Cnewm1=[1-(0.5-beta1)*dt*dt*w0*w0,dt; -(1-gama1)*dt*w0*w0,1];
Anewm1=inv(Bnewm1)*Cnewm1; % La matrice d'amplification
Unewm1(:,1)=[q0;dq0];
for j=1:(length(t)-1)
    Unewm1(:,j+1)=Anewm1*Unewm1(:,j);
end
plot(t,Unewm1(1,:))
legend('Newmark1')
```



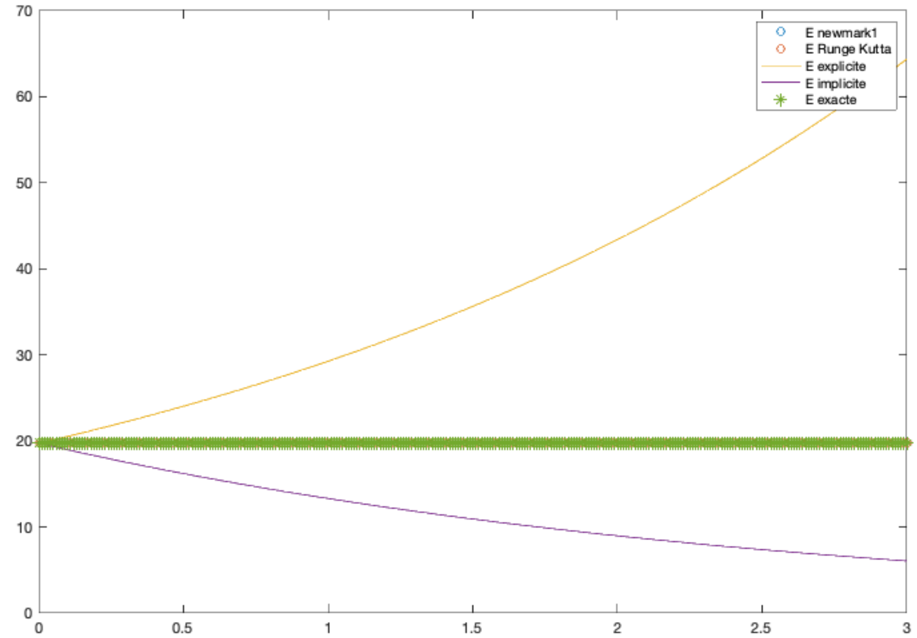
5.1.2

```
clf;
plot(t,Unewm1(1,:),'o')
hold on;
plot(t,Urk(1,:))
hold on;
plot(t,U(1,:))
hold on;
plot(t,Uim(1,:))
hold on;
plot(t,q,'*')
legend('Newmark1','RUNGE KUTTA','Euler explicite','Euler
 implicite','Solution exacte')
```



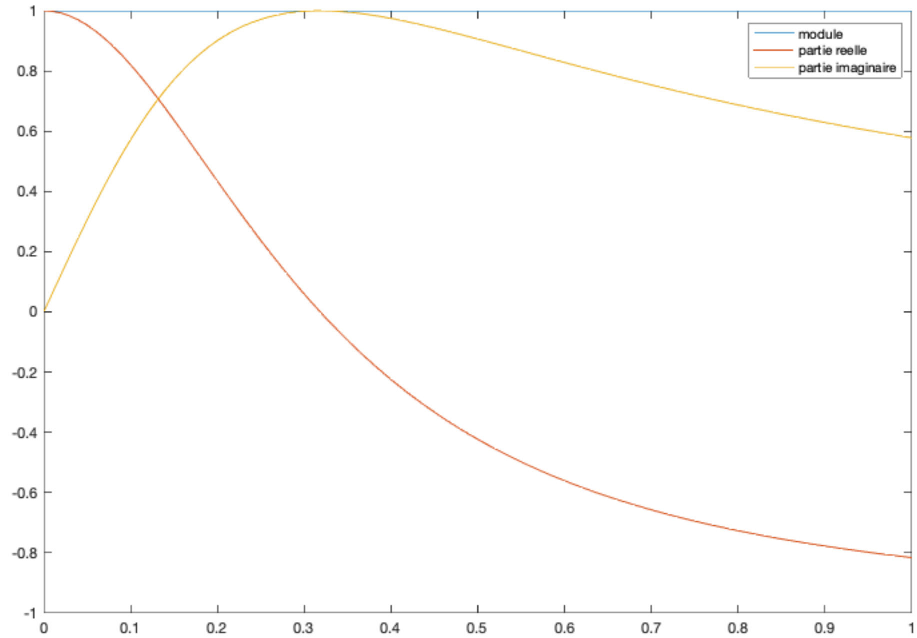
5.1.3

```
for j=1:length(t)
 Enewm1(j)=1/2*(Unewm1(2,j)*Unewm1(2,j)+4*pi*pi*Unewm1(1,j)*Unewm1(1,j));
end
clf;
plot(t,Enewm1,'o')
hold on;
plot(t,Erk,'o')
hold on;
plot(t,E1)
hold on;
plot(t,Eimp)
hold on;
plot(t,Eexa,'*')
legend('E newmark1','E Runge Kutta','E explicite','E implicite','E
 exacte')
%le schema de NEWMARK gama=0.5 beta=0.25 et le schema de RUNGE KUTTA
%ces deux converge vers la solution exacte
```



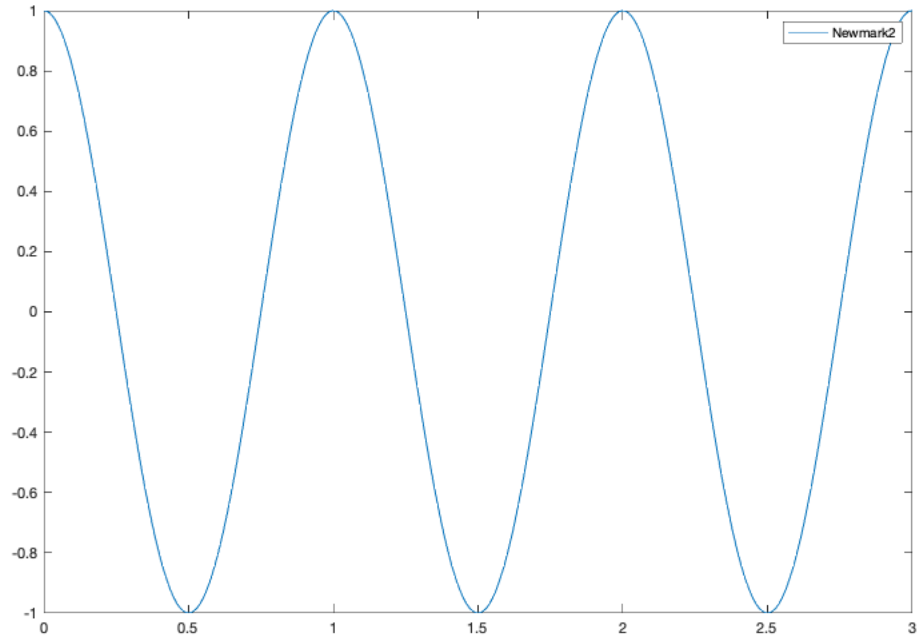
5.1.4

```
clf;
dt514=0:0.01:1;
for j=1:length(dt514)

 Bnewm514{1,j}=[1+beta1*dt514(j)*dt514(j)*w0*w0,0;gama1*dt514(j)*w0*w0,1];
    Cnewm514{1,j}=[1-(0.5-beta1)*dt514(j)*dt514(j)*w0*w0,dt514(j); -
(1-gama1)*dt514(j)*w0*w0,1];
    Anewm514{1,j}=inv(Bnewm514{1,j})*Cnewm514{1,j};
    VPA514(:,j)=eig(Anewm514{1,j});
    moduleVP(j)=sqrt(real(VPA514(1,j))^2+imag(VPA514(1,j))^2);
end
plot(dt514,moduleVP)
hold on;
plot(dt514,real(VPA514(1,:)))
hold on;
plot(dt514,imag(VPA514(1,:)))
legend('module','partie reelle','partie imaginaire')
%plus le pas de temps dt est petit, plus la partie reelle est petit
%la module ne change pas avec dt
```



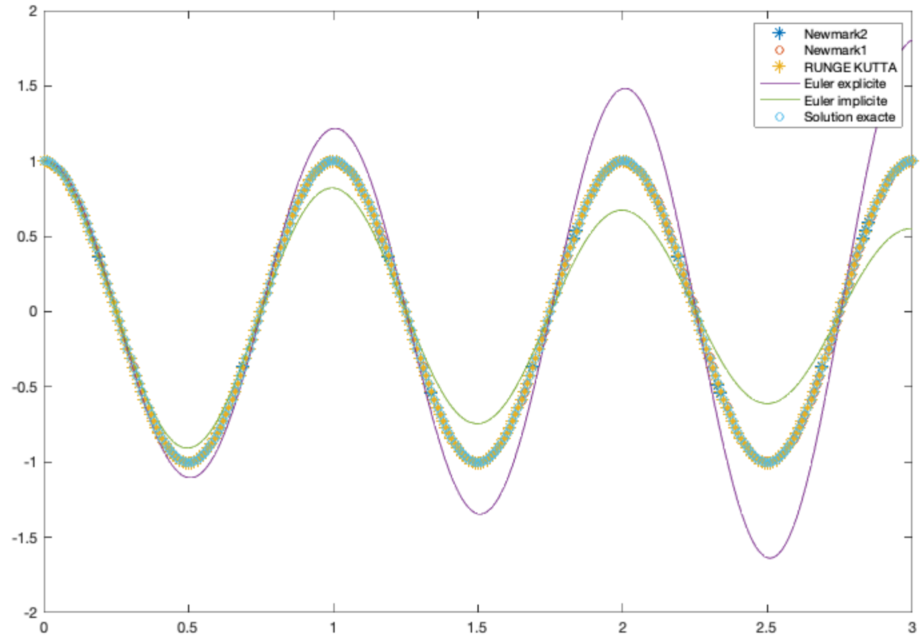
5.2.1

```
clf;
n=300;dt=T0/n;
t=0:dt:T0;
gama2=0.5;beta2=0;
Bnewm2=[1+beta2*dt*dt*w0*w0,0;gama2*dt*w0*w0,1];
Cnewm2=[1-(0.5-beta2)*dt*dt*w0*w0,dt; -(1-gama2)*dt*w0*w0,1];
% La matrice d'amplification
Anewm2=inv(Bnewm2)*Cnewm2;
Unewm2(:,1)=[q0;dq0];
for j=1:(length(t)-1)
    Unewm2(:,j+1)=Anewm2*Unewm2(:,j);
end
plot(t,Unewm2(1,:))
legend('Newmark2')
```



5.2.2

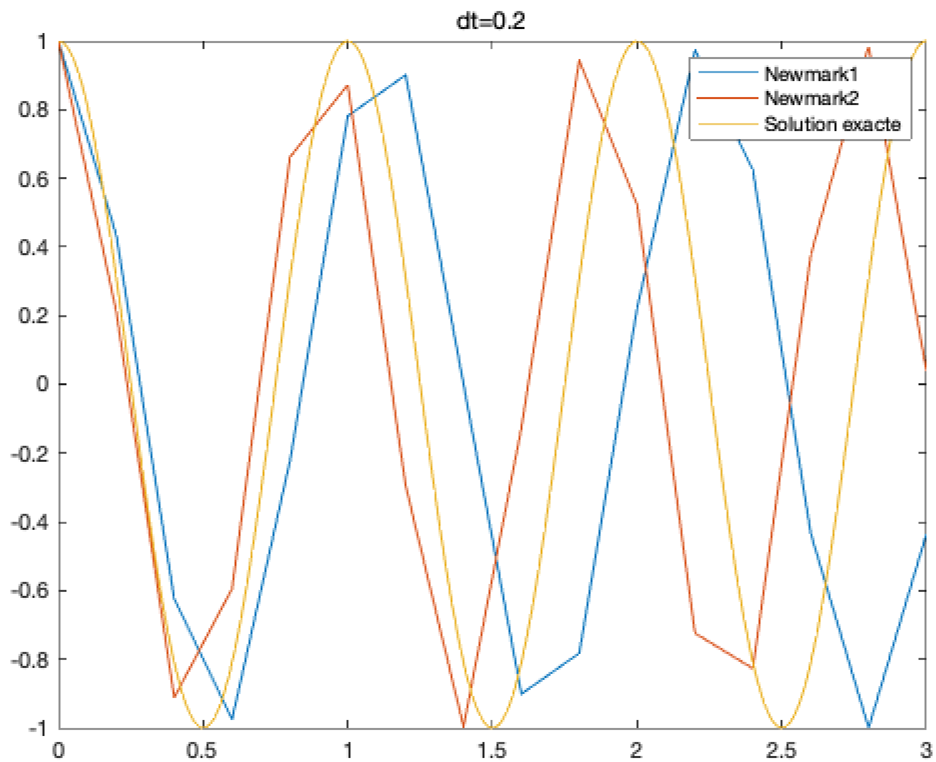
```
clf;
plot(t,Unewm2(1,:),'*')
hold on;
plot(t,Unewm1(1,:),'o')
hold on;
plot(t,Urk(1,:),'*')
hold on;
plot(t,U(1,:))
hold on;
plot(t,Uim(1,:))
hold on;
plot(t,q,'o')
legend('Newmark2','Newmark1','RUNGE KUTTA','Euler explicite','Euler
 implicite','Solution exacte')
% On peut voir que les solutions a partie des schemas de NEWMARK et
 RUNGE-KUTTA beaucoup plus pres avec la solution exacte
```

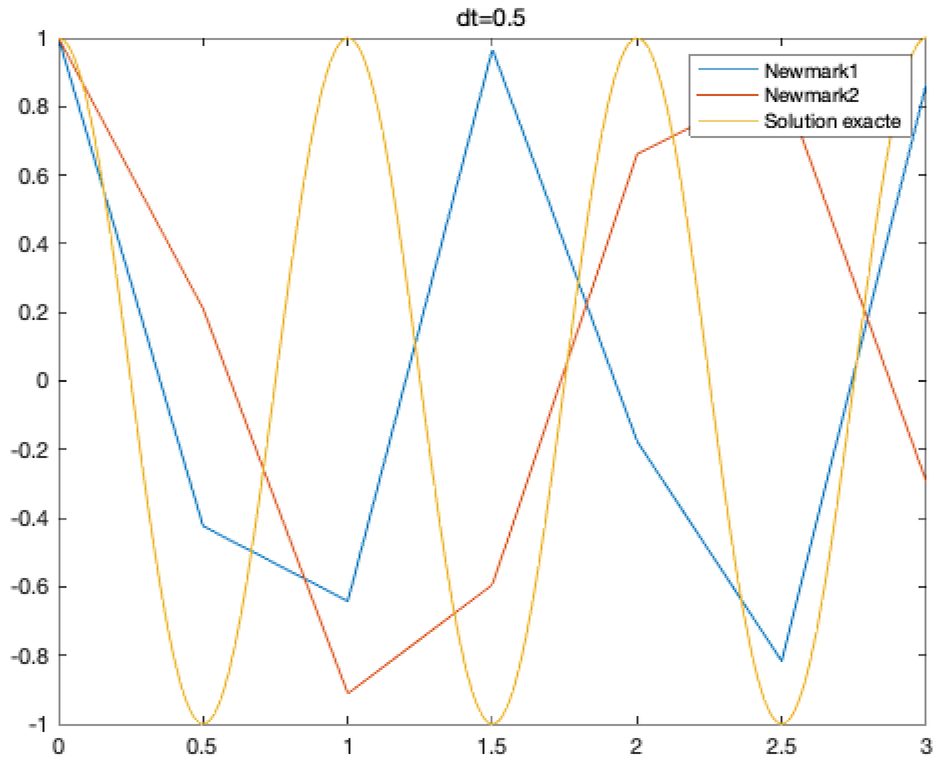


5.2.3

```
%dt=0.2
clf;
dt1=0.2;
t1=0:dt1:T0;
gama1=0.5;beta1=0.25;
Bnewm1=[1+beta1*dt1*dt1*w0*w0,0;gama1*dt1*w0*w0,1];
Cnewm1=[1-(0.5-beta1)*dt1*dt1*w0*w0,dt1; -(1-gama1)*dt1*w0*w0,1];
% La matrice d'amplification
Anewm1=inv(Bnewm1)*Cnewm1;
Unewm15231(:,1)=[q0;dq0];
for j=1:length(t1)
                Unewm15231(:,j+1)=Anewm1*Unewm15231(:,j);
end
Unewm15231(:,length(t1)+1)=[];
gama2=0.5;beta2=0;
Bnewm2=[1+beta2*dt1*dt1*w0*w0,0;gama2*dt1*w0*w0,1];
Cnewm2=[1-(0.5-beta2)*dt1*dt1*w0*w0,dt1; -(1-gama2)*dt1*w0*w0,1];
% La matrice d'amplification
Anewm2=inv(Bnewm2)*Cnewm2;
Unewm25231(:,1)=[q0;dq0];
for j=1:length(t1)
                Unewm25231(:,j+1)=Anewm2*Unewm25231(:,j);
end
Unewm25231(:,length(t1)+1)=[];
figure(1)
plot(t1,Unewm15231(1,:))
hold on;
```

```
plot(t1,Unewm25231(1,:))
hold on;
plot(t,q)
legend('Newmark1','Newmark2','Solution exacte')
title('dt=0.2')

%dt=0.5
dt2=0.5;
t2=0:dt2:T0;
gama1=0.5;beta1=0.25;
Bnewm1=[1+beta1*dt2*dt2*w0*w0,0;gama1*dt2*w0*w0,1];
Cnewm1=[1-(0.5-beta1)*dt2*dt2*w0*w0,dt2; -(1-gama1)*dt2*w0*w0,1];
% La matrice d'amplification
Anewm1=inv(Bnewm1)*Cnewm1;
Unewm15232(:,1)=[q0;dq0];
for j=1:length(t2)
        Unewm15232(:,j+1)=Anewm1*Unewm15232(:,j);
end
Unewm15232(:,length(t2)+1)=[];
gama2=0.5;beta2=0;
Bnewm25=[1+beta2*dt2*dt2*w0*w0,0;gama2*dt*w0*w0,1];
Cnewm2=[1-(0.5-beta2)*dt2*dt2*w0*w0,dt2; -(1-gama2)*dt2*w0*w0,1];
% La matrice d'amplification
Unewm25232(:,1)=[q0;dq0];
for j=1:length(t2)
        Unewm25232(:,j+1)=Anewm2*Unewm25232(:,j);
end
    Unewm25232(:,length(t2)+1)=[];
figure(2)
plot(t2,Unewm15232(1,:))
hold on;
plot(t2,Unewm25232(1,:))
hold on;
plot(t,q)
legend('Newmark1','Newmark2','Solution exacte')
title('dt=0.5')
%On peut voir que si le pas de temps est tres grand, les solutions ne
 converge pas bien avec la solution exacte.
```


dt=0.2
Newmark1
Newmark2
Solution exacte
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
3

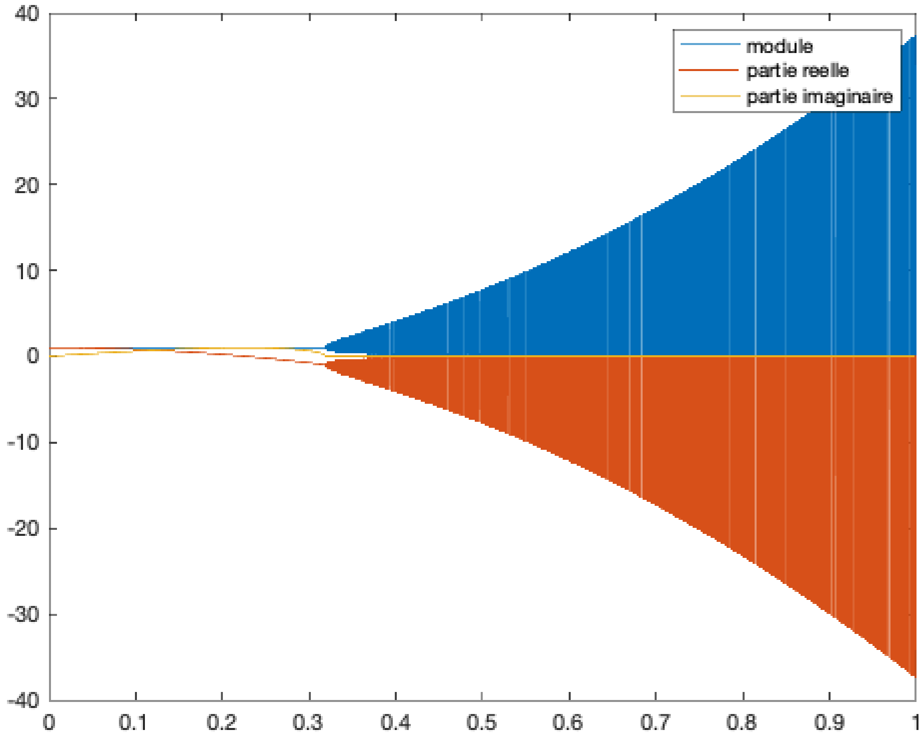


5.2.4

```
clf
close all
dt524=0:0.0001:1;
for j=1:length(dt524)

 Bnewm524{1,j}=[1+beta2*dt524(j)*dt524(j)*w0*w0,0;gama2*dt524(j)*w0*w0,1];
    Cnewm524{1,j}=[1-(0.5-beta2)*dt524(j)*dt524(j)*w0*w0,dt524(j); -
(1-gama2)*dt524(j)*w0*w0,1];
    Anewm524{1,j}=inv(Bnewm524{1,j})*Cnewm524{1,j};
    VPA524(:,j)=eig(Anewm524{1,j});
    moduleVP2(j)=sqrt(real(VPA524(1,j))^2+imag(VPA524(1,j))^2);
end
plot(dt524,moduleVP2)
hold on;
plot(dt524,real(VPA524(1,:)))
hold on;
plot(dt524,imag(VPA524(1,:)))
legend('module','partie reelle','partie imaginaire')

 syms dt5240 w0
 Bnewm5240=[1+beta2*dt5240*dt5240*w0*w0,0;gama2*dt5240*w0*w0,1];
 Cnewm5240=[1-(0.5-beta2)*dt5240*dt5240*w0*w0,dt5240; -(1-
 gama2)*dt5240*w0*w0,1];
```

```
 Anewm5240=inv(Bnewm5240)*Cnewm5240;
 VPA5240=eig(Anewm5240)

% le pas de temps de critique est 1/pi
% donc alpha=1
```

```
VPA5240 =

 1 - (dt5240*w0*((dt5240*w0 - 2)*(dt5240*w0 + 2))^(1/2))/2 -
 (dt5240^2*w0^2)/2
 (dt5240*w0*((dt5240*w0 - 2)*(dt5240*w0 + 2))^(1/2))/2 -
 (dt5240^2*w0^2)/2 + 1
```



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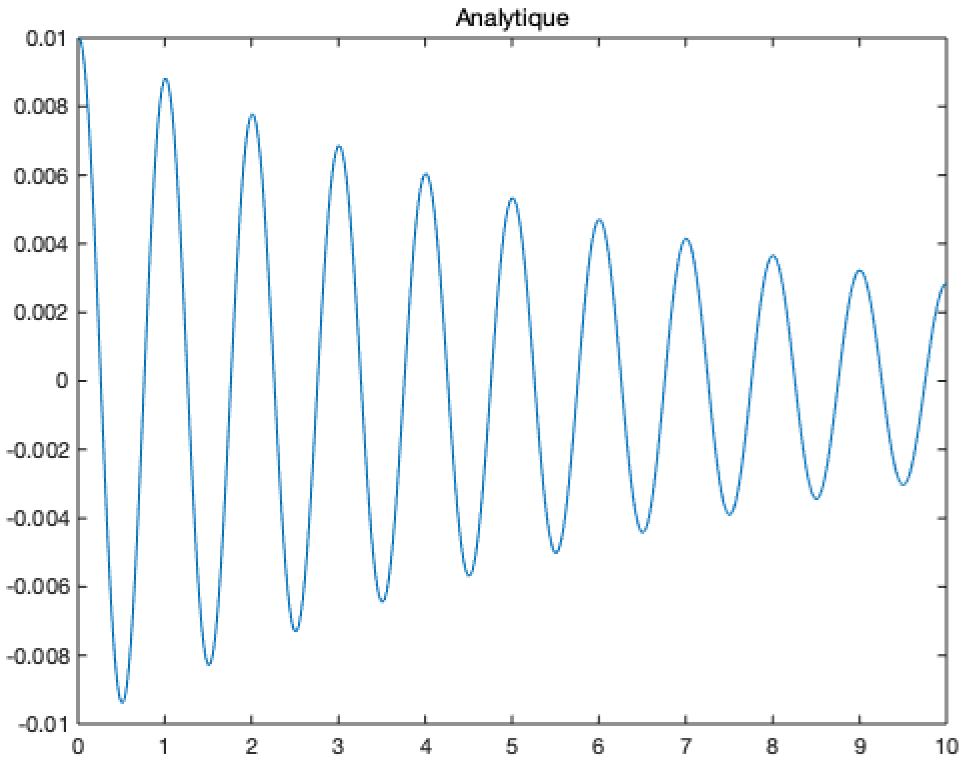
Etude d'un oscillateur linéaire marti à un degré de liberté

Table of Contents

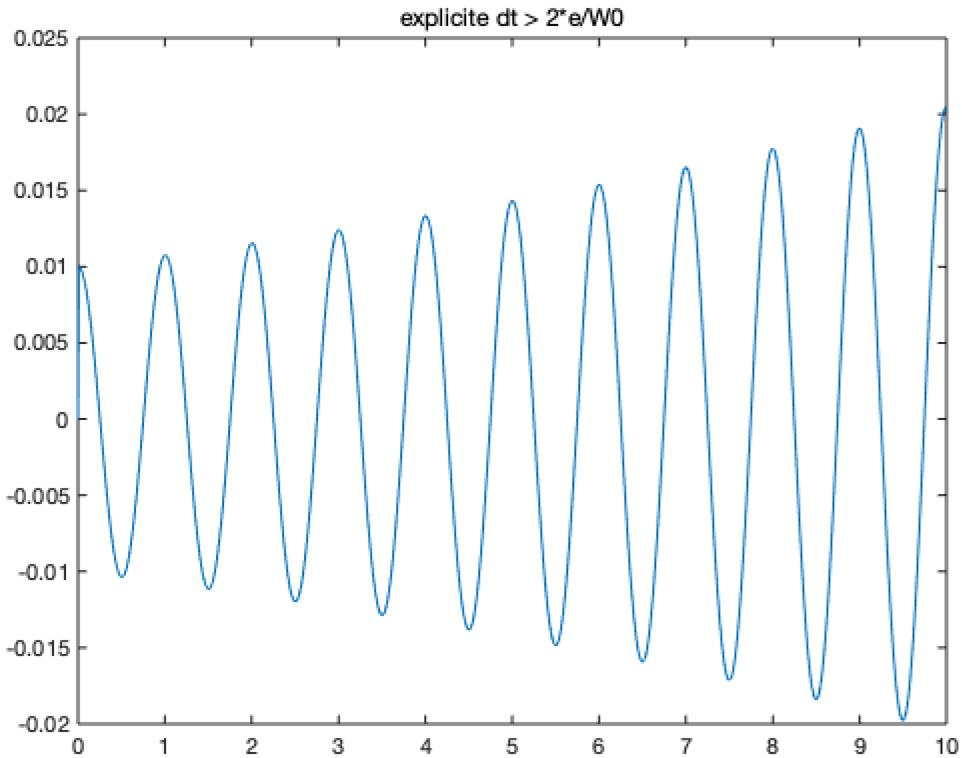
1.1

```
T0 = 1;
W0 = 2*pi/T0;
e = 0.02;
m = 1;
b = 2*e*W0*m;
X0 = 0.01;
dX0 = 0;
omiga = W0*(1-e^2)^0.5; x = [];
x(1) = X0;
n = 1;
for t = 0:0.01:10*T0
    n = n + 1;
    x(n) = exp(-e*W0*t)*(X0*cos(omiga*t) + (e*W0*X0 + dX0)/
 omiga*sin(omiga*t));
end
t = linspace(0,10*T0,n);
plot(t,x);
title('Analytique');
```



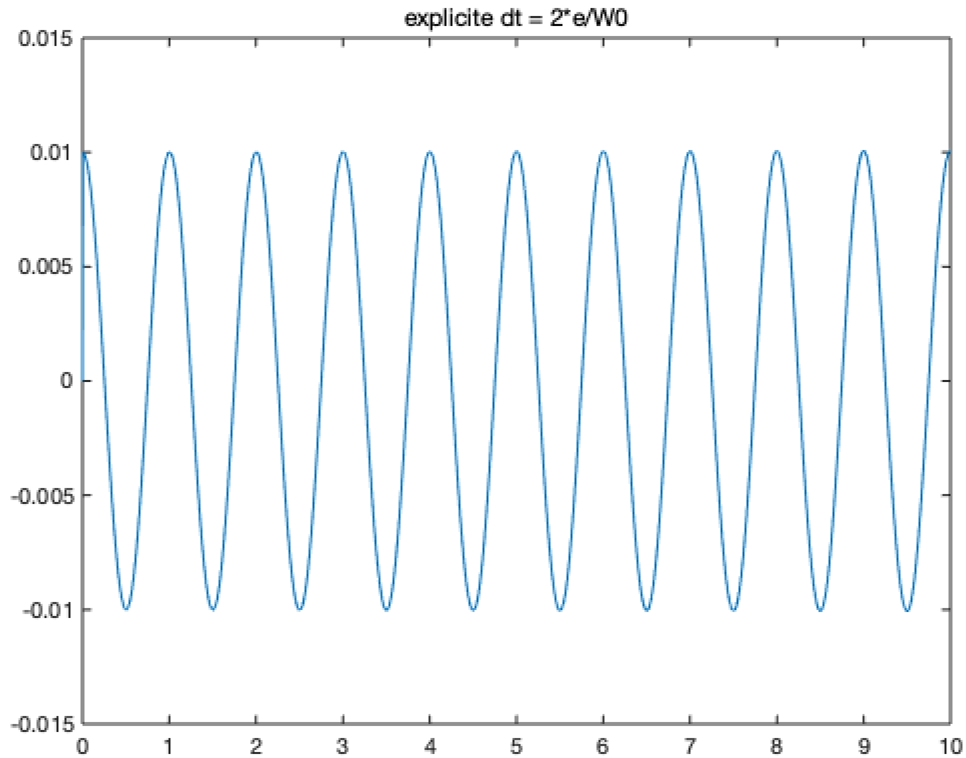
1.1.a

```
%2*e/W0=0.0064
clf;
dta=0.01;
A = [1,dta;-dta*W0^2,1-2*dta*e*W0];
X = [X0;dX0];
n=1;
x1a=[];
dx1a=[];
for t = 0:dta:10*T0
    n = n + 1;
    X = A*X;
    x1a(n) = X(1,1);
    dx1a(n) = X(2,1);
end
ta =linspace(0,10*T0,n);
plot(ta,x1a);
title('explicite dt > 2*e/W0');
%on peut voir que x diverge
```



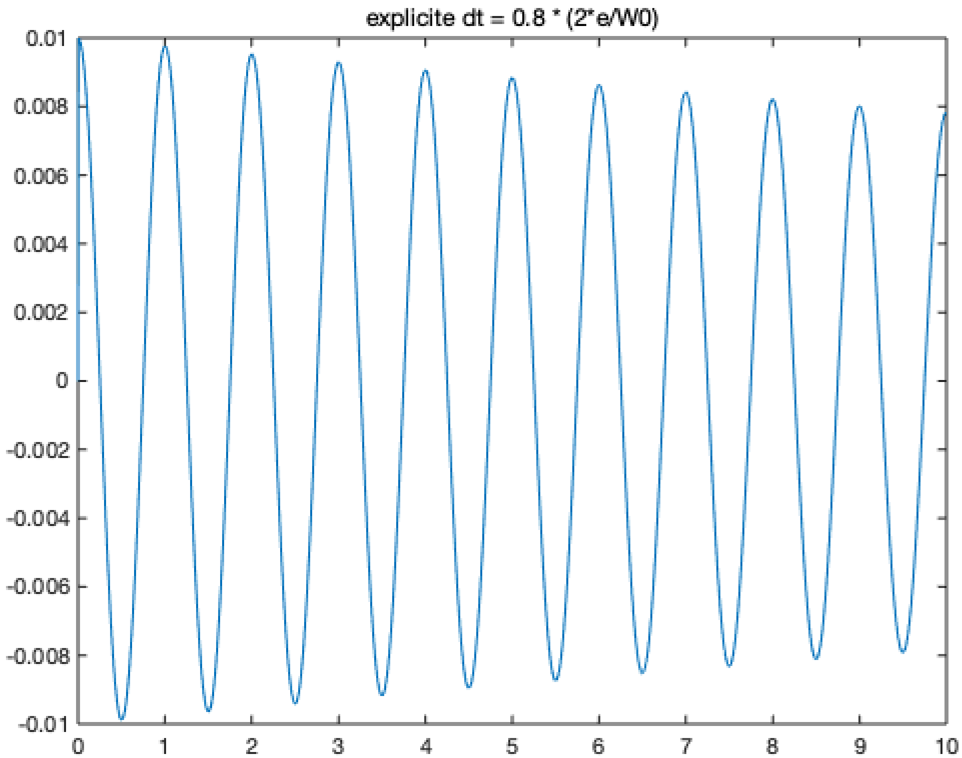
1.1.b

```
clf;
dtb=0.0064;
A = [1,dtb;-dtb*W0^2,1-2*dtb*e*W0];
X = [X0;dX0];
n=1;
x1b=[];
dx1b=[];
for t = 0:dtb:10*T0
    n = n + 1;
    X = A*X;
    x1b(n) = X(1,1);
    dx1b(n) = X(2,1);
end
tb =linspace(0,10*T0,n);
plot(tb,x1b);
title('explicite dt = 2*e/W0');
%on peut voir que x est sinusoidale,
%ni converge ni diverge.
```



1.1.c

```
clf;
dtc=0.0064*0.8;
A = [1,dtc;-dtc*W0^2,1-2*dtc*e*W0];
X = [X0;dX0];
n=1;
x1c=[];
dx1c=[];
for t = 0:dtc:10*T0
    n = n + 1;
    X = A*X;
    x1c(n) = X(1,1);
    dx1c(n) = X(2,1);
end
tc =linspace(0,10*T0,n);
plot(tc,x1c);
title('explicite dt = 0.8 * (2*e/W0)');
%on peut voir que x converge
```



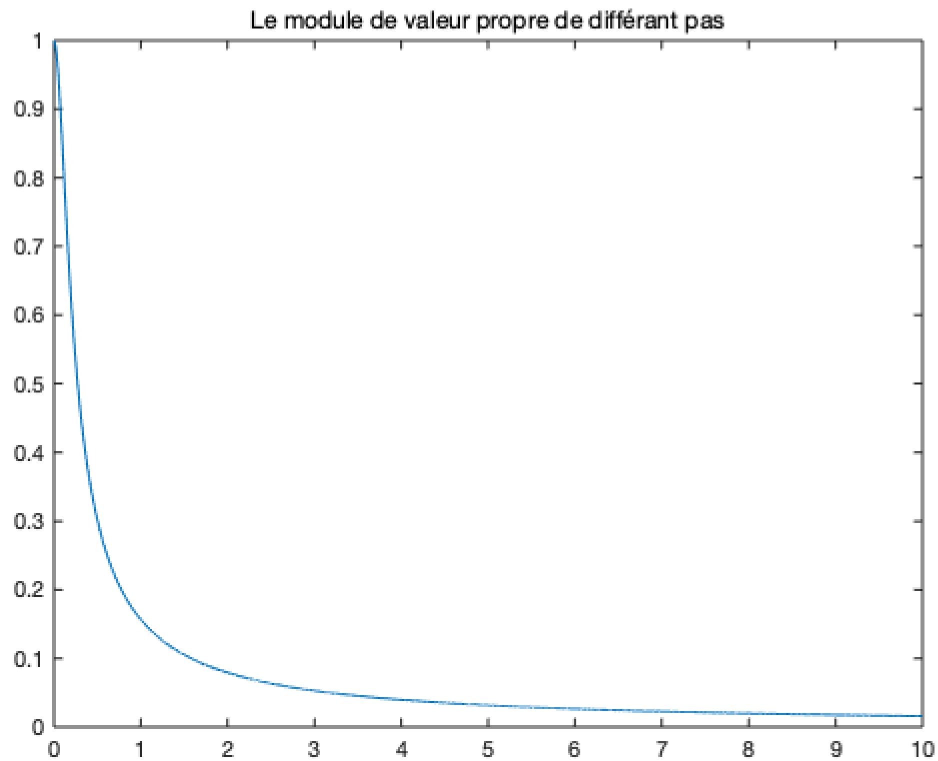
1.1.d

```
%la valeur de ddx+2*e*W0*dx+W0*x est un critere qui permet de etudier
 la
%precision de la solution, si cette valeur est pres de 0, la solution
 est
%precis
%la valeur de dt est un critere qui permet de etudier la precision de
 la solution
%et le rapport de dt/(2*e/W0) doit etre plus petit que 1 pour la
 solution est precis
```

1.2

```
clf;
dt=0:0.001:10*T0;
for j=1:length(dt)
    A_im{1,j} = [1+2*dt(j)*e*W0,dt(j);-dt(j)*W0^2,1]/(1 + 2*dt(j)*e*W0
 + dt(j)^2*W0^2);
    VPA(1,j)=max(abs(eig(A_im{1,j})));
end
plot(dt,VPA)
title('Le module de valeur propre de différant pas');
%les valeurs absolus des valeurs propres sont toujours moins de 1
%schéma inconditionnellement stable
```

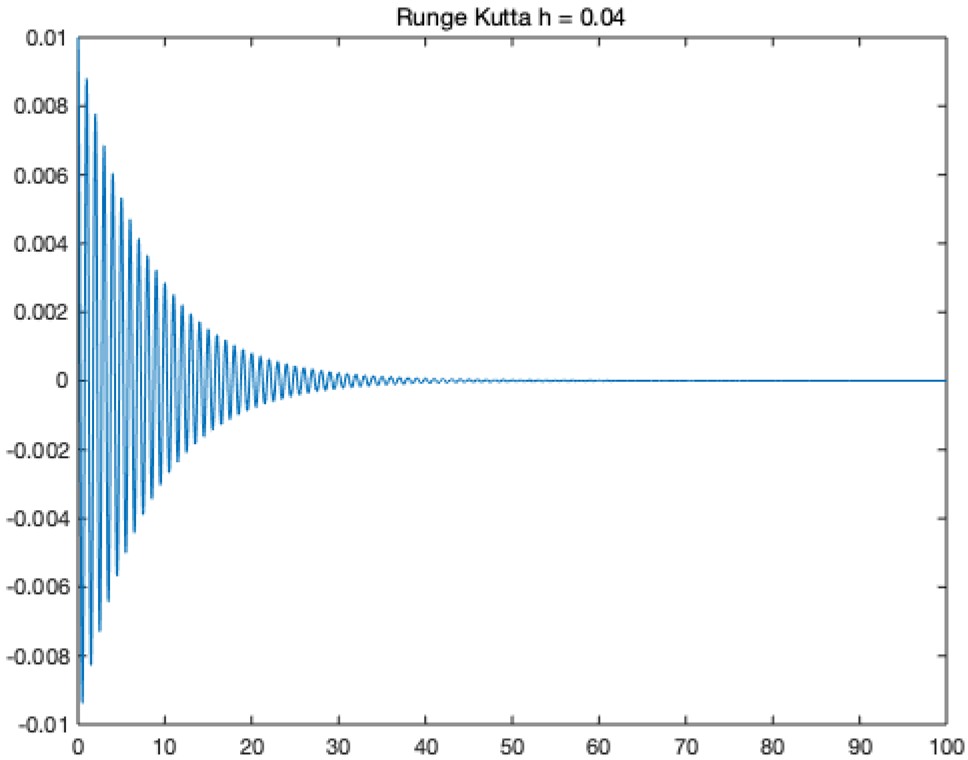
```
%donc pour implicite,il n'y a pas de temps critique
```



1.3.a h=0.04

```
clf;
h = 0.04;
dt = h*2*2^0.5/W0;
M = [0,1;-W0^2,-2*e*W0];
X = [X0;dX0];
x_rg = [];
dx_rg = [];
x_rg(1) = X0;
dx_rg(1) = dX0;
n = 1;
for t = 0:dt:100*T0
    n = n + 1;
    k1 = M * X;
    k2 = M * (X + k1 * dt/2);
    k3 = M * (X + k2 * dt/2);
    k4 = M * (X + k3 * dt);
    K = (k1 + 2*k2 + 2*k3 + k4)/6; X = X + K *dt;
    x_rg(n) = X(1,1);
    dx_rg(n) = X(2,1);
end
t = linspace(0,100*T0,n);
plot(t,x_rg);
```

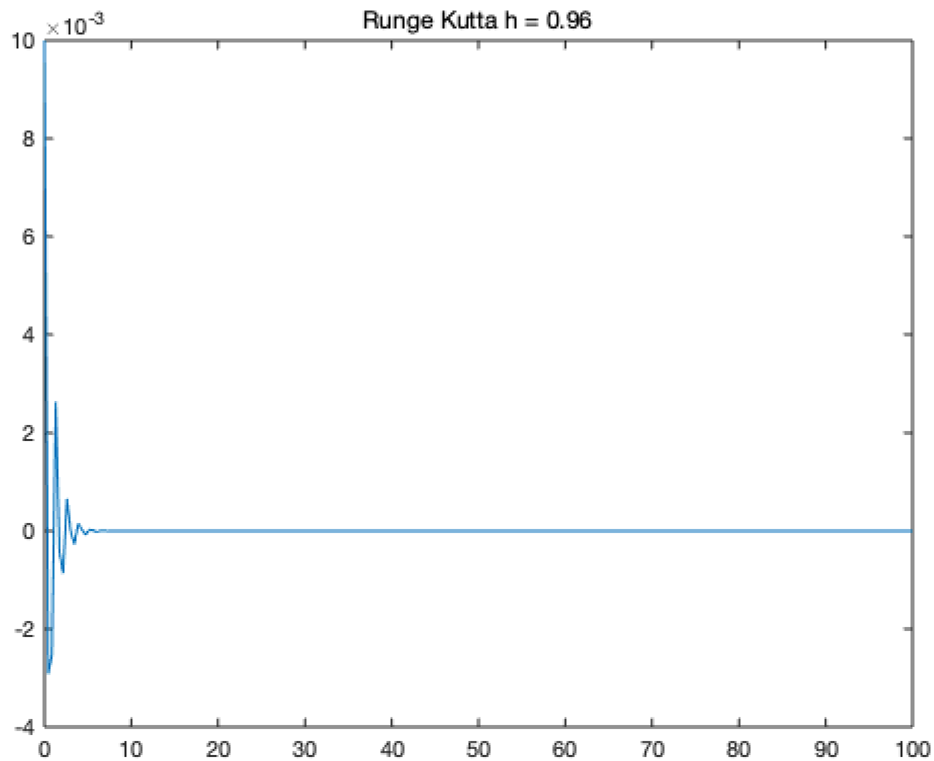
```
title('Runge Kutta h = 0.04')
```



1.3.a h=0.96

```
clf;
h = 0.96;
dt = h*2*2^0.5/W0;
M = [0,1;-W0^2,-2*e*W0];
X = [X0;dX0];
x_rg = [];
dx_rg = [];
x_rg(1) = X0;
dx_rg(1) = dX0;
n = 1;
for t = 0:dt:100*T0
    n = n + 1;
    k1 = M * X;
    k2 = M * (X + k1 * dt/2);
    k3 = M * (X + k2 * dt/2);
    k4 = M * (X + k3 * dt);
    K = (k1 + 2*k2 + 2*k3 + k4)/6; X = X + K *dt;
    x_rg(n) = X(1,1);
    dx_rg(n) = X(2,1);
end
t = linspace(0,100*T0,n);
plot(t,x_rg);
```

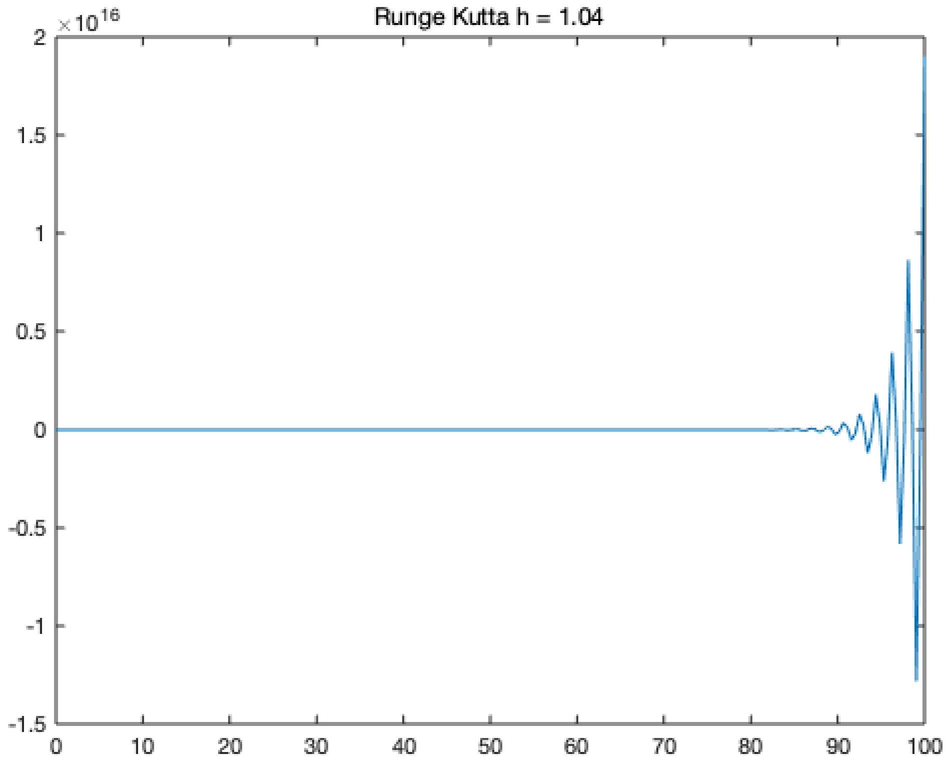
```
title('Runge Kutta h = 0.96')
```



1.3.a h=1.04

```
clf;
h = 1.04;
dt = h*2*2^0.5/W0;
M = [0,1;-W0^2,-2*e*W0];
X = [X0;dX0];
x_rg = [];
dx_rg = [];
x_rg(1) = X0;
dx_rg(1) = dX0;
n = 1;
for t = 0:dt:100*T0
    n = n + 1;
    k1 = M * X;
    k2 = M * (X + k1 * dt/2);
    k3 = M * (X + k2 * dt/2);
    k4 = M * (X + k3 * dt);
    K = (k1 + 2*k2 + 2*k3 + k4)/6; X = X + K *dt;
    x_rg(n) = X(1,1);
    dx_rg(n) = X(2,1);
end
t = linspace(0,100*T0,n);
plot(t,x_rg);
```

```
title('Runge Kutta h = 1.04')
```



1.3.a

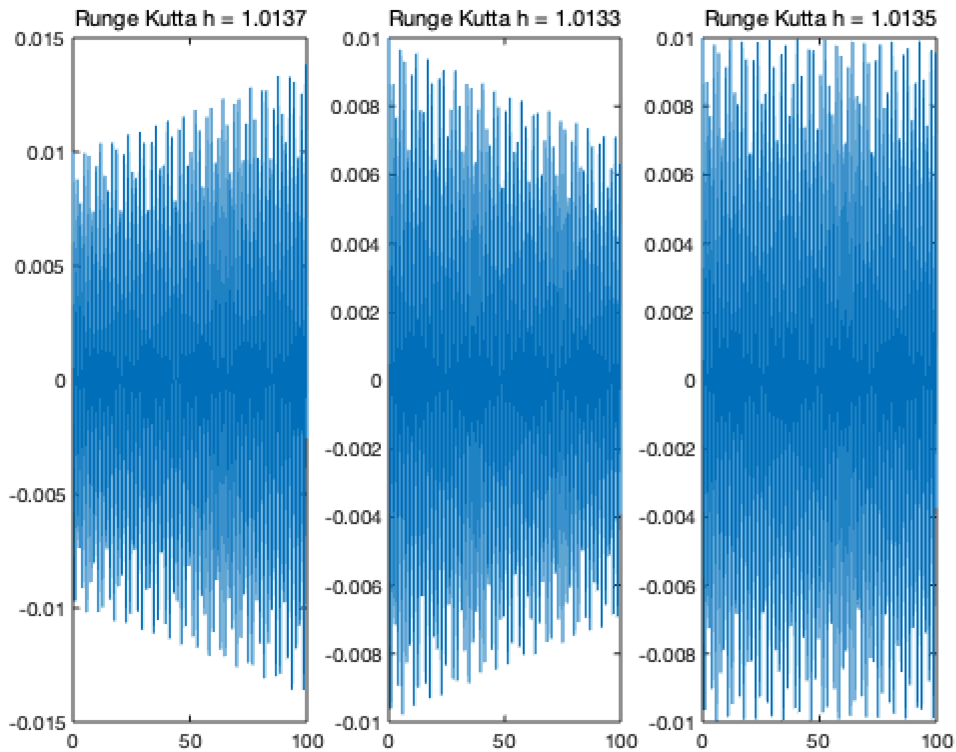
```
%la stabilite de x depend de h
% h depasse un valeur critique, dt depasse le pas de temps critique,x
 diverge.
```

1.3.b

```
%hmax = 1.0137 diverge un peu
clf;
h = 1.0137;
dt = h*2*2^0.5/W0;
M = [0,1;-W0^2,-2*e*W0];
X = [X0;dX0];
x_rg = [];
dx_rg = [];
x_rg(1) = X0;
dx_rg(1) = dX0;
n = 1;
for t = 0:dt:100*T0
    n = n + 1;
    k1 = M * X;
    k2 = M * (X + k1 * dt/2);
```

```
    k3 = M * (X + k2 * dt/2);
    k4 = M * (X + k3 * dt);
    K = (k1 + 2*k2 + 2*k3 + k4)/6; X = X + K *dt;
    x_rg(n) = X(1,1);
    dx_rg(n) = X(2,1);
end
t = linspace(0,100*T0,n);
subplot(1,3,1)
plot(t,x_rg);
title('Runge Kutta h = 1.0137')
%hmin = 1.0133 converge un peu
h = 1.0133;
dt = h*2*2^0.5/W0;
M = [0,1;-W0^2,-2*e*W0];
X = [X0;dX0];
x_rg = [];
dx_rg = [];
x_rg(1) = X0;
dx_rg(1) = dX0;
n = 1;
for t = 0:dt:100*T0
    n = n + 1;
    k1 = M * X;
    k2 = M * (X + k1 * dt/2);
    k3 = M * (X + k2 * dt/2);
    k4 = M * (X + k3 * dt);
    K = (k1 + 2*k2 + 2*k3 + k4)/6; X = X + K *dt;
    x_rg(n) = X(1,1);
    dx_rg(n) = X(2,1);
end
t = linspace(0,100*T0,n);
subplot(1,3,2)
plot(t,x_rg);
title('Runge Kutta h = 1.0133')
%hc = 1.0135;
h = 1.0135;
dt = h*2*2^0.5/W0;
M = [0,1;-W0^2,-2*e*W0];
X = [X0;dX0];
x_rg = [];
dx_rg = [];
x_rg(1) = X0;
dx_rg(1) = dX0;
n = 1;
for t = 0:dt:100*T0
    n = n + 1;
    k1 = M * X;
    k2 = M * (X + k1 * dt/2);
    k3 = M * (X + k2 * dt/2);
    k4 = M * (X + k3 * dt);
    K = (k1 + 2*k2 + 2*k3 + k4)/6; X = X + K *dt;
    x_rg(n) = X(1,1);
    dx_rg(n) = X(2,1);
end
```

```
t = linspace(0,100*T0,n);
subplot(1,3,3)
plot(t,x_rg);
title('Runge Kutta h = 1.0135')
%une valeur approximative du pas de temps critique
tc = hc * 2*2^0.5/W0; % tc = 0.4562
```



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Etude d’un double pendule avec l’hypothese des petits mouvements

Table of Contents

1.1

```
syms m;
syms a;
syms g;
syms F0;
syms w;
syms beta;
syms gamma;
syms dt;
syms n;
I = [1, 0; 0, 1];
% D'apres l'euqaiton(1),On sait que m * a * a * M1 * d2q + m * g * a *
 M2 * q = F0 * sin(w * t) * M3
M1 = [2, 1; 1, 1];
M2 = [2, 0; 0, 1];
M3 = [a; a / sqrt(2)];
% q = [theta1; theta2]
% dq = [dtheta1; dtheta2]
% d2q = [d2theta1; d2theta2]
M4 = - inv(M1) * g / a * M2;
M5 = inv(M1) * F0 / m / a / a * M3;
%Donc maitenant on a  d2q = M4 * q + M5 * sin(w * t)
% En utilisant les relation (2) , on a
% M6 * qn1 = M7 * qn + M8 * dqn + M9
M6 = I - dt * dt * beta * M4;
M7 = I + dt * dt * (0.5 - beta) * M4;
M8 = I * dt;
```

```
M9 = dt * dt * (0.5 - beta) * M5 * sin(w * n * dt) + dt * dt * beta *
 M5 * sin(w * (n + 1) * dt);
% En utilisant les relation (3), on a
% M10 * qn1 + M11 * dqn1 = M12 * qn + M13 * dqn + M14
M10 = - dt * gamma * M4;
M11 = I;
M12 = dt * (1 - gamma) * M4;
M13 = I;
M14 = dt * (1 - gamma) * M5 * sin(w * n * dt) + dt * gamma * M5 *
 sin(w * (n + 1) * dt);
% Soit U = [q; dq], alors on peut trouver M15 * Un1 = M16 * Un + M17
 avec
M15 = [M6, 0 * I; M10, M11];
M16 = [M7, M8; M12, M13];
M17 = [M9; M14];
% Alors, on a Un1 = A * Un + B avec
A = inv(M15) * M16
B = inv(M15) * M17
%On peut trouver la matrice d'amplification
m = 2;
a = 0.5;
g = 9.81;
F0 = 20;
w = 2 * pi;
beta = 0;
gamma = 0.5;
%On peut aussi trouver la matrice d'amplification numeriquement
An=eval(A)
Bn=eval(B)
```

```
A =

[                                                    (((2*g*(beta -
 1/2)*dt^2)/a + 1)*(a^2 + 2*beta*g*a*dt^2))/(a^2 + 4*a*beta*dt^2*g
 + 2*beta^2*dt^4*g^2) - (2*beta*dt^4*g^2*(beta - 1/2))/(a^2 +
 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
              (a*beta*dt^2*g*((2*g*(beta - 1/2)*dt^2)/a + 1))/
(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2) - (dt^2*g*(a^2 +
 2*beta*g*a*dt^2)*(beta - 1/2))/(a*(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)),                    (dt*(a^2 + 2*beta*g*a*dt^2))/
(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
       (a*beta*dt^3*g)/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)]
[                                       (2*a*beta*dt^2*g*((2*g*(beta
 - 1/2)*dt^2)/a + 1))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)
 - (2*dt^2*g*(a^2 + 2*beta*g*a*dt^2)*(beta - 1/2))/(a*(a^2 +
 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)),
                        (((2*g*(beta - 1/2)*dt^2)/a + 1)*(a^2 +
 2*beta*g*a*dt^2))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)
 - (2*beta*dt^4*g^2*(beta - 1/2))/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2),                                (2*a*beta*dt^3*g)/
(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
      (dt*(a^2 + 2*beta*g*a*dt^2))/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)]
```

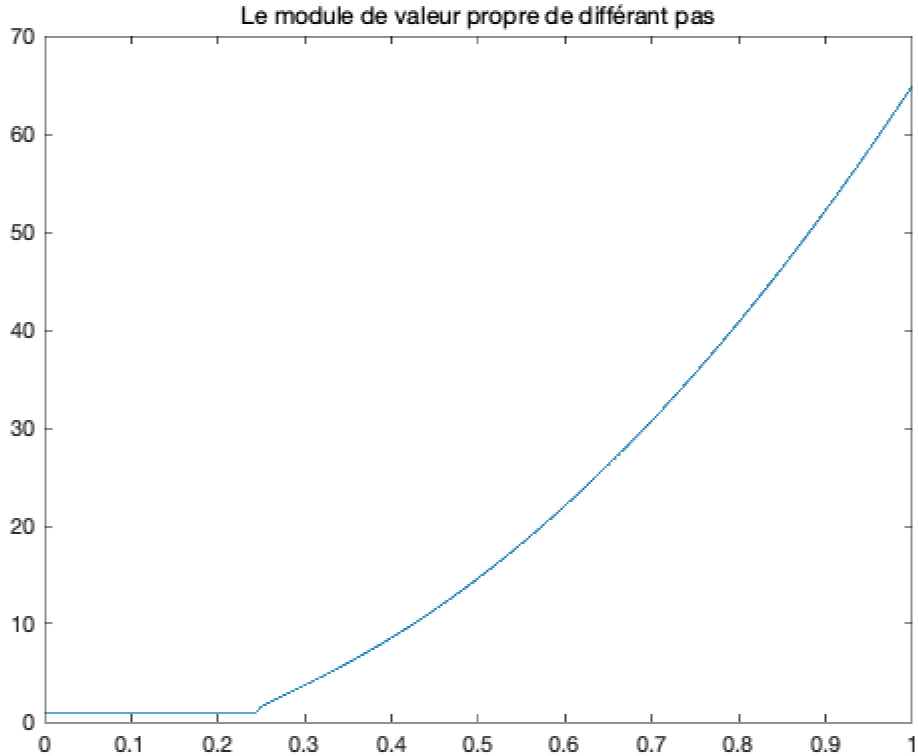
```
[           (2*dt*g*(gamma - 1))/a - (2*(beta*gamma*dt^3*g^2
 + a*gamma*dt*g)*((2*g*(beta - 1/2)*dt^2)/a + 1))/(a^2 +
 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2) - (2*dt^3*g^2*gamma*(beta
 - 1/2))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
 (2*dt^2*g*(beta*gamma*dt^3*g^2 + a*gamma*dt*g)*(beta - 1/2))/
(a*(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)) - (dt*g*(gamma
 - 1))/a + (a*dt*g*gamma*((2*g*(beta - 1/2)*dt^2)/a + 1))/(a^2 +
 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2), 1 - (2*dt*(beta*gamma*dt^3*g^2
 + a*gamma*dt*g))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
                           (a*dt^2*g*gamma)/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)]
[ (4*dt^2*g*(beta*gamma*dt^3*g^2 + a*gamma*dt*g)*(beta -
 1/2))/(a*(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)) -
 (2*dt*g*(gamma - 1))/a + (2*a*dt*g*gamma*((2*g*(beta -
 1/2)*dt^2)/a + 1))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
      (2*dt*g*(gamma - 1))/a - (2*(beta*gamma*dt^3*g^2 +
 a*gamma*dt*g)*((2*g*(beta - 1/2)*dt^2)/a + 1))/(a^2 + 4*a*beta*dt^2*g
 + 2*beta^2*dt^4*g^2) - (2*dt^3*g^2*gamma*(beta - 1/2))/(a^2 +
 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
 (2*a*dt^2*g*gamma)/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2), 1 -
 (2*dt*(beta*gamma*dt^3*g^2 + a*gamma*dt*g))/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)]


B =



     ((a^2 + 2*beta*g*a*dt^2)*(beta*dt^2*sin(dt*w*(n + 1))*(F0/(a*m)
 - (2^(1/2)*F0)/(2*a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/
(2*a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2) -
 (a*beta*dt^2*g*(beta*dt^2*sin(dt*w*(n + 1))*(F0/(a*m) - (2^(1/2)*F0)/
(a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/(a*m))*(beta -
 1/2)))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)


         (2*a*beta*dt^2*g*(beta*dt^2*sin(dt*w*(n + 1))*(F0/(a*m) -
 (2^(1/2)*F0)/(2*a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/
(2*a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)
 - ((a^2 + 2*beta*g*a*dt^2)*(beta*dt^2*sin(dt*w*(n + 1))*(F0/(a*m)
 - (2^(1/2)*F0)/(a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/
(a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)
 dt*gamma*sin(dt*w*(n + 1))*(F0/(a*m) - (2^(1/2)*F0)/(2*a*m)) -
 (2*(beta*gamma*dt^3*g^2 + a*gamma*dt*g)*(beta*dt^2*sin(dt*w*(n
 + 1))*(F0/(a*m) - (2^(1/2)*F0)/(2*a*m)) - dt^2*sin(dt*n*w)*(F0/
(a*m) - (2^(1/2)*F0)/(2*a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g
 + 2*beta^2*dt^4*g^2) - dt*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/
(2*a*m))*(gamma - 1) - (a*dt*g*gamma*(beta*dt^2*sin(dt*w*(n +
 1))*(F0/(a*m) - (2^(1/2)*F0)/(a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m)
 - (2^(1/2)*F0)/(a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)
   (2*(beta*gamma*dt^3*g^2 + a*gamma*dt*g)*(beta*dt^2*sin(dt*w*(n
 + 1))*(F0/(a*m) - (2^(1/2)*F0)/(a*m)) - dt^2*sin(dt*n*w)*(F0/
(a*m) - (2^(1/2)*F0)/(a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g
```

```
 + 2*beta^2*dt^4*g^2) - dt*gamma*sin(dt*w*(n + 1))*(F0/(a*m) -
 (2^(1/2)*F0)/(a*m)) + dt*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/
(a*m))*(gamma - 1) + (2*a*dt*g*gamma*(beta*dt^2*sin(dt*w*(n +
 1))*(F0/(a*m) - (2^(1/2)*F0)/(2*a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m)
 - (2^(1/2)*F0)/(2*a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)


An =

[
  1 - (981*dt^2)/50,
                         (981*dt^2)/100,                dt,
     0]
[
      (981*dt^2)/50,
                      1 - (981*dt^2)/50,                 0,
    dt]
[ (981*dt*((981*dt^2)/50 - 1))/50 - (981*dt)/50 +
 (6772013501556091*dt^3)/35184372088832,
 (981*dt)/100 - (981*dt*((981*dt^2)/50 - 1))/100 - (962361*dt^3)/5000,
 1 - (981*dt^2)/50,    (981*dt^2)/100]
[                     (981*dt)/50 - (981*dt*((981*dt^2)/50 - 1))/50
 - (962361*dt^3)/2500, (981*dt*((981*dt^2)/50 - 1))/50 - (981*dt)/50
 + (6772013501556091*dt^3)/35184372088832,     (981*dt^2)/50, 1 -
 (981*dt^2)/50]


Bn =



 (51526319965141*dt^2*sin(2*pi*dt*n))/17592186044416


 (36434610256939*dt^2*sin(2*pi*dt*n))/8796093022208
 (51526319965141*dt*sin(2*pi*dt*(n + 1)))/17592186044416 -
 (7402483611873081*dt^3*sin(2*pi*dt*n))/439804651110400 +
 (51526319965141*dt*sin(2*pi*dt*n))/17592186044416
  (36434610256939*dt*sin(2*pi*dt*(n + 1)))/8796093022208 -
 (20937385438310997*dt^3*sin(2*pi*dt*n))/879609302220800 +
 (36434610256939*dt*sin(2*pi*dt*n))/8796093022208
```

1.2

```
e1=[];
xt=0:0.001:1;
for j = 1:length(xt)
    dt= xt(j);
    e1 = [e1, max(abs(eig(eval(A))))];
end
plot(xt,e1);
```

```
title('Le module de valeur propre de différant pas');
% On trouve que quand le pas est inférieure à 0.244, tous les modules
 de valeur propre est presque égale à 1,
% Mais quand le pas est supérieure à 0.244, les modules de valeur
 propre supérieure à 1.
%Donc le pas de temps critique est 0.244
```



1.3

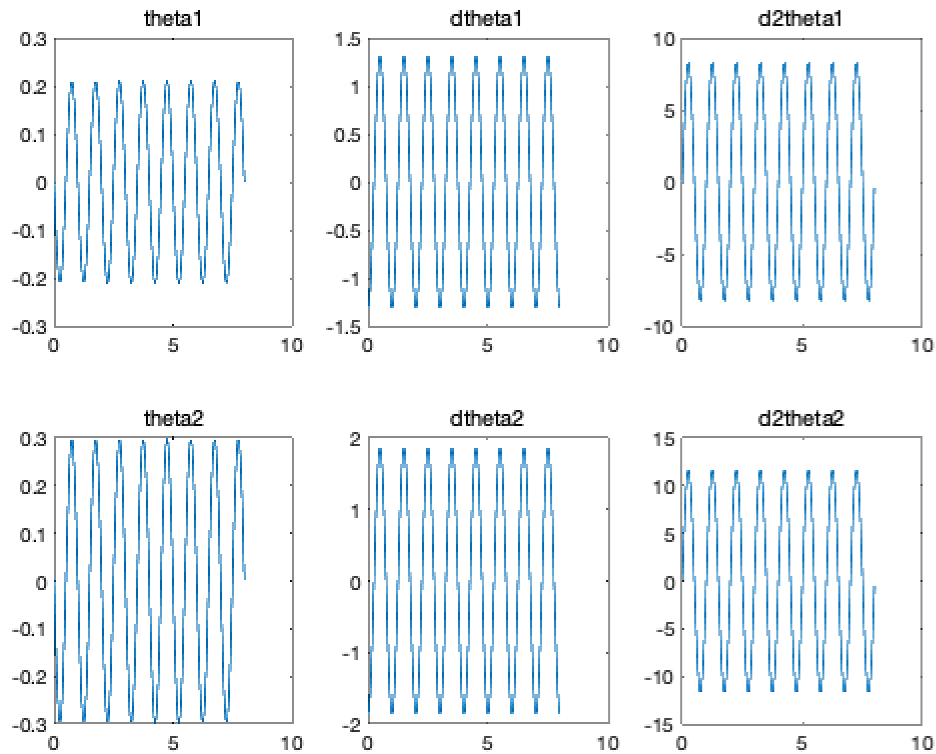
```
theta1_0 = 0;
theta2_0 = 0;
dtheta1_0 = - 1.31519275;
dtheta2_0 = - 1.85996342;
q0 = [theta1_0; theta2_0];
dq0 = [dtheta1_0; dtheta2_0];
d2q0 = eval(M4) * q0;
```

1.4

```
%Un = [qn; dqn]
%Un1 = [qn1;dqn1]
%les trois relations
% Un1 = A * Un + B
% d2qn = M4 * qn + M5 * sin(w * tn)
% d2qn1 = M4 * qn1 + M5 * sin(w * tn1)
```

1.5

```
T0 = 8;
dt = 0.01;
U = [q0; dq0];
q = [q0];
dq = [dq0];
d2q = [d2q0];
for n = 0 : (T0 / dt - 1)
    U = eval(A) * U + eval(B);
    q = [q, U(1:2)];
    dq = [dq, U(3:4)];
    d2q = [d2q, eval(M4 * U(1:2) + M5 * sin(w * n * dt))];
end

t = 0 :dt:T0;
subplot(2, 3, 1);
plot(t, q(1, :));
title('theta1');
subplot(2, 3, 2);
plot(t, dq(1, :));
title('dtheta1');
subplot(2, 3, 3);
plot(t, d2q(1, :));
title('d2theta1');
subplot(2, 3, 4);
plot(t, q(2, :));
title('theta2');
subplot(2, 3, 5);
plot(t, dq(2, :));
title('dtheta2');
subplot(2, 3, 6);
plot(t, d2q(2, :));
title('d2theta2');
```



1.6

```
clf;
T0 = 8;
dt = 0.02;
U = [q0; dq0];
q = [q0];
dq = [dq0];
d2q = [d2q0];
for n = 0 : (T0 / dt - 1)
    U = eval(A) * U + eval(B);
    q = [q, U(1:2)];
    dq = [dq, U(3:4)];
    d2q = [d2q, eval(M4 * U(1:2) + M5 * sin(w * n * dt))];
end
q(:, 1 : 3)% ce sont les valeurs de q à 0s , dt , 2dt.
q(:, 0.5 / dt + 1) %c'est le valeur de q à 0.5s.
% Ce sont
% 0   -0.0263   -0.0522   -0.299e-3
% 0   -0.0372   -0.0738   -0.423e-3
dq(:, 1 : 3) % ce sont les valeurs de dq à 0s , dt , 2dt.
dq(:, 0.5 / dt + 1) %c'est le valeur de dq à 0.5s.
% Ce sont
% -1.32   -1.30   -1.27   1.31
% -1.86   -1.85   -1.80   1.86
```

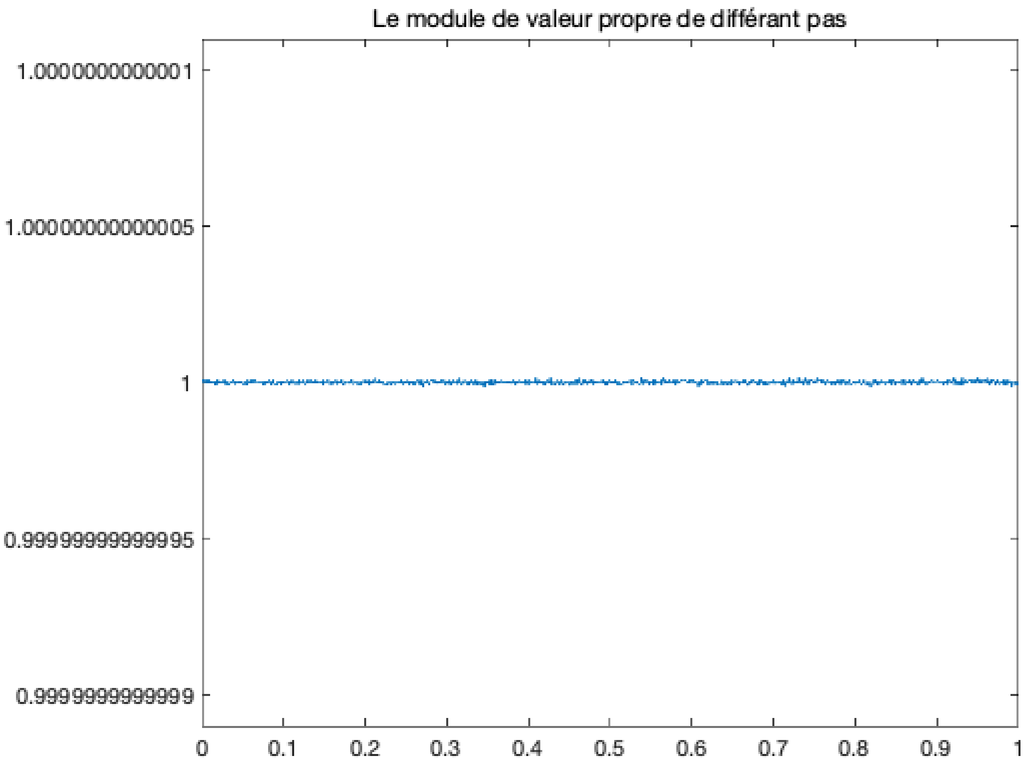
```
d2q(:, 1 : 3) % ce sont les valeurs de d2q à 0s , dt , 2dt.
d2q(:, 0.5 / dt + 1) %c'est le valeur de d2q à 0.5s.
% Ce sont
% 0    0.302    1.33   0.737
% 0    0.428    1.89   1.04
%Ces valeurs seront donnees avec trois chiffres significatifs.


ans =

         0   -0.0263   -0.0522
         0   -0.0372   -0.0738


ans =

   1.0e-03 *

   -0.2988
   -0.4226


ans =

   -1.3152   -1.3048   -1.2739
   -1.8600   -1.8453   -1.8016


ans =

    1.3143
    1.8587


ans =

         0    0.3023    1.3340
         0    0.4275    1.8866


ans =

    0.7376
    1.0432
```

2.1

```
beta=0.25;
gamma=0.5;
eval(A)
eval(B)


ans =

    0.9922    0.0039    0.0199    0.0000
    0.0078    0.9922    0.0001    0.0199
   -0.7802    0.3893    0.9922    0.0039
    0.7787   -0.7802    0.0078    0.9922


ans =

   -0.0001
   -0.0001
   -0.0073
   -0.0104
```

2.2

```
clf;
e=[];
xt=0:0.001:1;
for j = 1:length(xt)
    dt= xt(j);
    e = [e, max(abs(eig(eval(A))))];
end
plot(xt,e);
title('Le module de valeur propre de différant pas');
% d'apres le schema
%on peut trouver que le module de valeur propre est toujours presque
 de 1
```



2.3

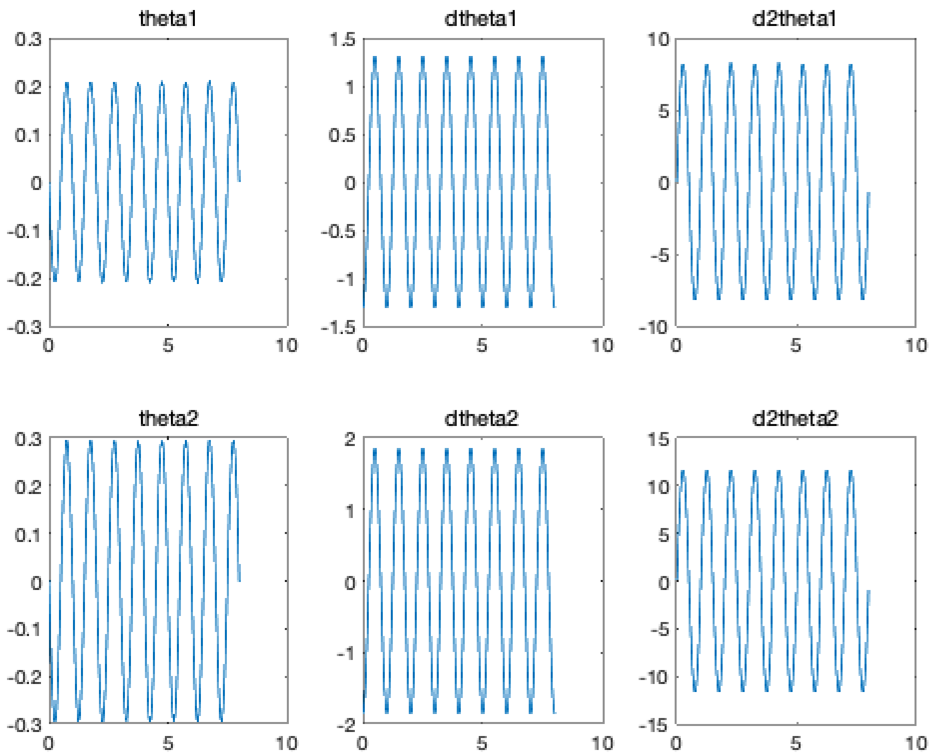
```
theta1_0 = 0;
theta2_0 = 0;
dtheta1_0 = - 1.31519275;
dtheta2_0 = - 1.85996342;
q0 = [theta1_0; theta2_0];
dq0 = [dtheta1_0; dtheta2_0];
d2q0 = eval(M4) * q0;
```

2.4

```
%Un = [qn; dqn]
%Un1 = [qn1;dqn1]
%les trois relations
% Un1 = A * Un + B
% d2qn = M4 * qn + M5 * sin(w * tn)
% d2qn1 = M4 * qn1 + M5 * sin(w * tn1)
```

2.5

```
clf;
T0 = 8;
dt = 0.02;
U = [q0; dq0];
q = [q0];
dq = [dq0];
d2q = [d2q0];
for n = 0 : (T0 / dt - 1)
    U = eval(A) * U + eval(B);
    q = [q, U(1:2)];
    dq = [dq, U(3:4)];
    d2q = [d2q, eval(M4 * U(1:2) + M5 * sin(w * n * dt))];
end

t = 0 :dt:T0;
subplot(2, 3, 1);
plot(t, q(1, :));
title('theta1');
subplot(2, 3, 2);
plot(t, dq(1, :));
title('dtheta1');
subplot(2, 3, 3);
plot(t, d2q(1, :));
title('d2theta1');
subplot(2, 3, 4);
plot(t, q(2, :));
title('theta2');
subplot(2, 3, 5);
plot(t, dq(2, :));
title('dtheta2');
subplot(2, 3, 6);
plot(t, d2q(2, :));
title('d2theta2');
```



2.6

```
q(:, 1 : 3)% ce sont les valeurs de q à 0s , dt , 2dt.
q(:, 0.5 / dt + 1)%c'est le valeur de q à 0.5s.
% Ce sont
% 0   -0.0262   -0.0520   -0.000900
% 0   -0.0371   -0.0735   -0.0013
dq(:, 1 : 3)% ce sont les valeurs de dq à 0s , dt , 2dt.
dq(:, 0.5 / dt + 1)%c'est le valeur de dq à 0.5s.
% Ce sont
% -1.32   -1.30   -1.27   1.31
% -1.86   -1.85   -1.80   1.86
%Ces valeurs seront donnees avec trois chiffres significatifs.
```

```
ans =

         0   -0.0262   -0.0520
         0   -0.0371   -0.0735


ans =

   -0.0009
   -0.0013
```

```
ans =

   -1.3152   -1.3048   -1.2739
   -1.8600   -1.8453   -1.8016


ans =

    1.3124
    1.8561
```

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Oscillateur non linéaire à un degré de liberté(1)

Table of Contents

1.1

```
q0 = 2;
dq0 = 0;
w0 = 2*pi;
alpha = 0.1;
ddq0 = - w0^2*q0*(1+alpha*q0^2);
T0=6;
gama1=0.5;beta1=0;
%on sait les relations
%q1(inc+1) = q1(inc) + dt1 * dq1(inc)+ dt1*dt1*0.5*ddq1(inc)
%ddq1(inc+1)=- w0^2*q1(inc+1)*(1+alpha*q1(inc+1)^2)
%dq1(inc+1) = dq1(inc) +0.5*dt1 * (ddq1(inc) + ddq1(inc+1))
```

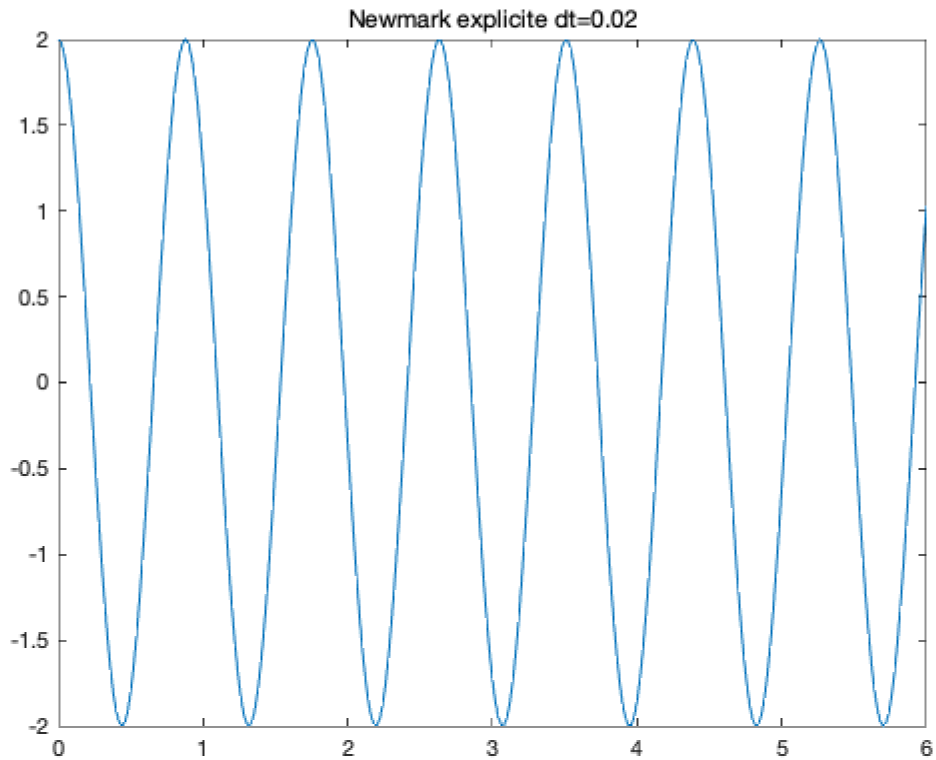
1.2

```
dt1 =0.02;
t1 =(0:dt1:T0)';
np1=size(t1,1);
q1=zeros(np1,1);
dq1=zeros(np1,1);
ddq1=zeros(np1,1);
energ1=zeros(np1,1);

q1(1)=q0;
dq1(1)=dq0;
ddq1(1)=ddq0;

for inc =1:(np1-1)
```

```
    q1(inc+1) = q1(inc) + dt1 * dq1(inc)+ dt1*dt1*0.5*ddq1(inc);
    ddq1(inc+1)=- w0^2*q1(inc+1)*(1+alpha*q1(inc+1)^2);
    dq1(inc+1) = dq1(inc) +0.5*dt1 * (ddq1(inc) + ddq1(inc+1));
end
plot(t1,q1)
title('Newmark explicite dt=0.02')
```



1.3

```
clf;
q1(1)%t=0
q1(2)%t=dt
q1(3)%t=2*dt
q1(301)%t=T0
```

```
ans =

     2


ans =

    1.9779
```

```
ans =

    1.9123


ans =

    1.0329
```

2.1

```
gama2=0.5;beta2=0.25;
%on cherche à minimiser la valeur absolue de: ddq+w0^2*q*(1+alpha*q^2)
%on voudrais cette valeur egale 0
```

2.2

```
A = imread('IMG_0326.jpg');
imshow(A);
```

On sait que : $$\Delta \ddot{q}_{n+1} = - \frac{f(\ddot{q}^*_{n+1}, \dot{q}^*_{n+1}, q^*_{n+1})}{\frac{\partial f}{\partial \ddot{q}^*_{n+1}} + \frac{\partial f}{\partial q^*_{n+1}} \beta \Delta t^2}$$

et $$\Delta q_{n+1} = \beta \Delta t^2 \Delta \ddot{q}_{n+1}$$

$$f = \ddot{q} + w_0^2 q (1 + a q^2) \qquad w_0^2 q + w_0^2 a q^3$$

$$\frac{\partial f}{\partial \ddot{q}^*_{n+1}} = 1 \qquad \frac{\partial f}{\partial q^*_{n+1}} = w_0^2 + 3 w_0^2 a q^{*2}_{n+1}$$

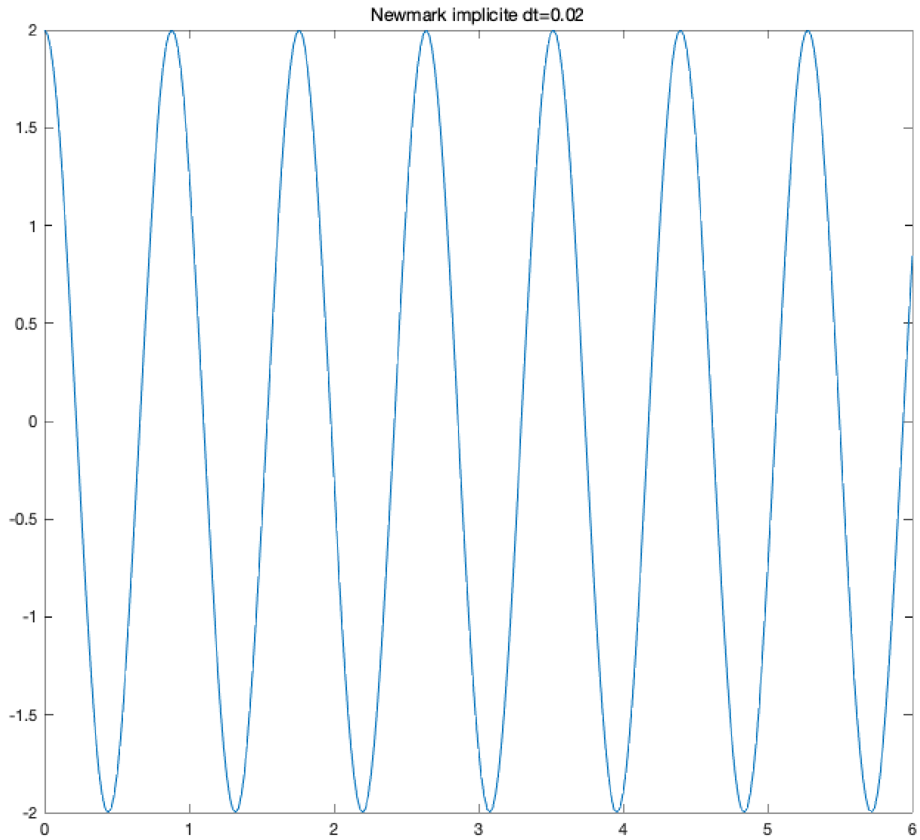
Donc l'expression analytique de la correction de $\ddot{q}^*_{n+1}$ est :

$$\Delta \ddot{q}_{n+1} = \frac{-\left(\ddot{q}^*_{n+1} + w_0^2 q^*_{n+1} (1 + a q^{*2}_{n+1})\right)}{1 + \beta \Delta t^2 (w_0^2 + 3 w_0^2 a q^{*2}_{n+1})}$$

2.3

```
q2=zeros(np1,1);
dq2=zeros(np1,1);
ddq2=zeros(np1,1);
energ2=zeros(np1,1);
q2(1)=q0;
dq2(1)=dq0;
ddq2(1)=ddq0;
e=0.01; %supposons le erreur est 0.01 pour verifier abs(ddq
+w0*w0*q*(1+alpha*q*q))<e
for inc =1:(np1-1)
    q2(inc+1) = q2(inc) + dt1 * dq2(inc)+ dt1*dt1*(0.5-
beta2)*ddq2(inc);
    dq2(inc+1) = dq2(inc) +dt1 *(1-gama2)*ddq2(inc);
    ddq2(inc+1)=0;
    while  abs(ddq2(inc+1)+w0*w0*q2(inc+1)*(1+alpha*q2(inc+1)*q2(inc
+1)))> e
        cddq2 = (-(ddq2(inc+1)+w0*w0*q2(inc+1)*(1+alpha*q2(inc
+1)*q2(inc+1))))/(1+beta2*dt1*dt1*(w0*w0+3*w0*w0*alpha*q2(inc
+1)*q2(inc+1)));
        cdq2=gama2*dt1* cddq2;
```

```
            cq2=beta2*dt1*dt1* cddq2;
            q2(inc+1)=q2(inc+1)+cq2;
            dq2(inc+1)=dq2(inc+1)+cdq2;
            ddq2(inc+1)=ddq2(inc+1)+cddq2;
        end
    end
    plot(t1,q2)
    title('Newmark implicite dt=0.02')
```



2.4

```
q2(1)%t=0
q2(2)%t=dt1
q2(3)%t=2*dt1
q2(301)%t=T0
```

ans =

2

```
ans =

    1.9781


ans =

    1.9131


ans =

    0.8478
```

3.1

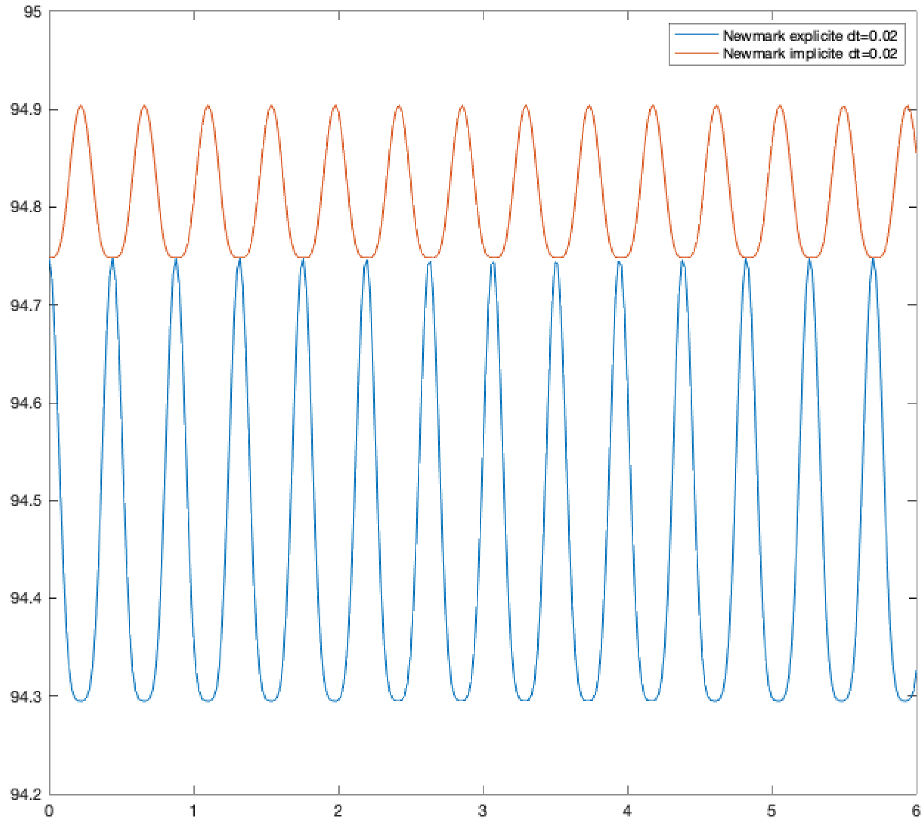
```
%il y a deux partie : l'energie cinetique et l'energie potentiel
%pour l'energie cinetique, c'est 0.5*dq^2
%pour l'energie potentiel,on fait un integrale,
%c'est 0.5*w0*w0*q*q+0.25*alpha*w0*w0*q^4
```

3.2

```
for inc =1:np1
    energ1(inc)= 0.5*dq1(inc)^2 +
 0.5*w0*w0*q1(inc)*q1(inc)+0.25*alpha*w0*w0*q1(inc)^4;
    energ2(inc)= 0.5*dq2(inc)^2 +
 0.5*w0*w0*q2(inc)*q2(inc)+0.25*alpha*w0*w0*q2(inc)^4;
end
```

3.3

```
clf;
plot(t1,energ1,t1,energ2);
legend('Newmark explicite dt=0.02','Newmark implicite dt=0.02')
%l'energie implicite est toujours plus grande de l'energie explicite
%mais, quelque fois, ils ont la meme l'energie#quand il retourne a q=2
%l'amplitude de vibration de l'energie implicite est plus petit que
 l'energie explicite
```



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