Etude d'un double pendule avec l'hypothese des petits mouvements

Table of Contents

1.1

```
syms m;
syms a;
syms g;
syms F0;
syms w;
syms beta;
syms gamma;
syms dt;
syms n;
I = [1, 0; 0, 1];
% D'apres l'euqaiton(1),On sait que m * a * a * M1 * d2q + m * g * a *
 M2 * q = F0 * sin(w * t) * M3
M1 = [2, 1; 1, 1];
M2 = [2, 0; 0, 1];
M3 = [a; a / sqrt(2)];
% q = [theta1; theta2]
% dq = [dtheta1; dtheta2]
% d2q = [d2theta1; d2theta2]
M4 = - inv(M1) * g / a * M2;
M5 = inv(M1) * F0 / m / a / a * M3;
%Donc maitenant on a  d2q = M4 * q + M5 * sin(w * t)
% En utilisant les relation (2) , on a
% M6 * qn1 = M7 * qn + M8 * dqn + M9
M6 = I - dt * dt * beta * M4;
M7 = I + dt * dt * (0.5 - beta) * M4;
M8 = I * dt;
```

```
M9 = dt * dt * (0.5 - beta) * M5 * sin(w * n * dt) + dt * dt * beta *
 M5 * sin(w * (n + 1) * dt);
% En utilisant les relation (3), on a
% M10 * qn1 + M11 * dqn1 = M12 * qn + M13 * dqn + M14
M10 = - dt * gamma * M4;
M11 = I;
M12 = dt * (1 - gamma) * M4;
M13 = I;
M14 = dt * (1 - gamma) * M5 * sin(w * n * dt) + dt * gamma * M5 *
 sin(w * (n + 1) * dt);
% Soit U = [q; dq], alors on peut trouver M15 * Un1 = M16 * Un + M17
 avec
M15 = [M6, 0 * I; M10, M11];
M16 = [M7, M8; M12, M13];
M17 = [M9; M14];
% Alors, on a Un1 = A * Un + B avec
A = inv(M15) * M16
B = inv(M15) * M17
%On peut trouver la matrice d'amplification
m = 2;
a = 0.5;
g = 9.81;
F0 = 20;
w = 2 * pi;
beta = 0;
gamma = 0.5;
%On peut aussi trouver la matrice d'amplification numeriquement
An=eval(A)
Bn=eval(B)


A =

[                                                    (((2*g*(beta -
 1/2)*dt^2)/a + 1)*(a^2 + 2*beta*g*a*dt^2))/(a^2 + 4*a*beta*dt^2*g
 + 2*beta^2*dt^4*g^2) - (2*beta*dt^4*g^2*(beta - 1/2))/(a^2 +
 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
              (a*beta*dt^2*g*((2*g*(beta - 1/2)*dt^2)/a + 1))/
(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2) - (dt^2*g*(a^2 +
 2*beta*g*a*dt^2)*(beta - 1/2))/(a*(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)),                    (dt*(a^2 + 2*beta*g*a*dt^2))/
(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
       (a*beta*dt^3*g)/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)]
[                                       (2*a*beta*dt^2*g*((2*g*(beta
 - 1/2)*dt^2)/a + 1))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)
 - (2*dt^2*g*(a^2 + 2*beta*g*a*dt^2)*(beta - 1/2))/(a*(a^2 +
 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)),
                        (((2*g*(beta - 1/2)*dt^2)/a + 1)*(a^2 +
 2*beta*g*a*dt^2))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)
 - (2*beta*dt^4*g^2*(beta - 1/2))/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2),                              (2*a*beta*dt^3*g)/
(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
      (dt*(a^2 + 2*beta*g*a*dt^2))/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)]
```

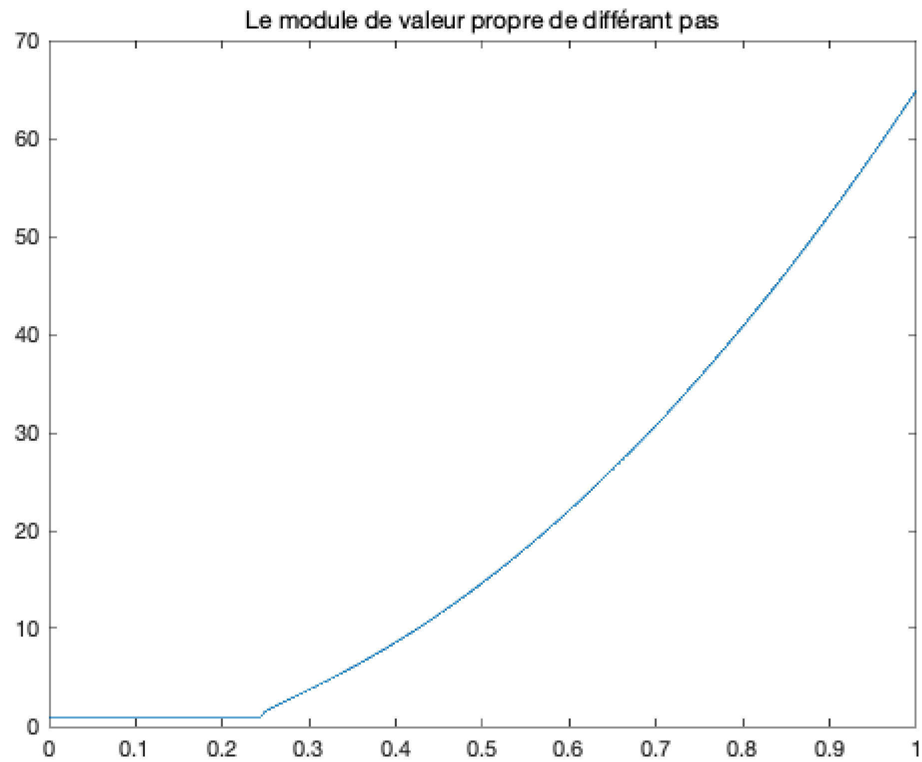
```
[          (2*dt*g*(gamma - 1))/a - (2*(beta*gamma*dt^3*g^2
 + a*gamma*dt*g)*((2*g*(beta - 1/2)*dt^2)/a + 1))/(a^2 +
 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2) - (2*dt^3*g^2*gamma*(beta
 - 1/2))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
 (2*dt^2*g*(beta*gamma*dt^3*g^2 + a*gamma*dt*g)*(beta - 1/2))/
(a*(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)) - (dt*g*(gamma
 - 1))/a + (a*dt*g*gamma*((2*g*(beta - 1/2)*dt^2)/a + 1))/(a^2 +
 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2), 1 - (2*dt*(beta*gamma*dt^3*g^2
 + a*gamma*dt*g))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
                           (a*dt^2*g*gamma)/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)]
[ (4*dt^2*g*(beta*gamma*dt^3*g^2 + a*gamma*dt*g)*(beta -
 1/2))/(a*(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)) -
 (2*dt*g*(gamma - 1))/a + (2*a*dt*g*gamma*((2*g*(beta -
 1/2)*dt^2)/a + 1))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
      (2*dt*g*(gamma - 1))/a - (2*(beta*gamma*dt^3*g^2 +
 a*gamma*dt*g)*((2*g*(beta - 1/2)*dt^2)/a + 1))/(a^2 + 4*a*beta*dt^2*g
 + 2*beta^2*dt^4*g^2) - (2*dt^3*g^2*gamma*(beta - 1/2))/(a^2 +
 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2),
 (2*a*dt^2*g*gamma)/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2), 1 -
 (2*dt*(beta*gamma*dt^3*g^2 + a*gamma*dt*g))/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)]


B =


     ((a^2 + 2*beta*g*a*dt^2)*(beta*dt^2*sin(dt*w*(n + 1))*(F0/(a*m)
 - (2^(1/2)*F0)/(2*a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/
(2*a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2) -
 (a*beta*dt^2*g*(beta*dt^2*sin(dt*w*(n + 1))*(F0/(a*m) - (2^(1/2)*F0)/
(a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/(a*m))*(beta -
 1/2)))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)


         (2*a*beta*dt^2*g*(beta*dt^2*sin(dt*w*(n + 1))*(F0/(a*m) -
 (2^(1/2)*F0)/(2*a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/
(2*a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)
 - ((a^2 + 2*beta*g*a*dt^2)*(beta*dt^2*sin(dt*w*(n + 1))*(F0/(a*m)
 - (2^(1/2)*F0)/(a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/
(a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g + 2*beta^2*dt^4*g^2)
 dt*gamma*sin(dt*w*(n + 1))*(F0/(a*m) - (2^(1/2)*F0)/(2*a*m)) -
 (2*(beta*gamma*dt^3*g^2 + a*gamma*dt*g)*(beta*dt^2*sin(dt*w*(n
 + 1))*(F0/(a*m) - (2^(1/2)*F0)/(2*a*m)) - dt^2*sin(dt*n*w)*(F0/
(a*m) - (2^(1/2)*F0)/(2*a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g
 + 2*beta^2*dt^4*g^2) - dt*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/
(2*a*m))*(gamma - 1) - (a*dt*g*gamma*(beta*dt^2*sin(dt*w*(n +
 1))*(F0/(a*m) - (2^(1/2)*F0)/(a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m)
 - (2^(1/2)*F0)/(a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)
   (2*(beta*gamma*dt^3*g^2 + a*gamma*dt*g)*(beta*dt^2*sin(dt*w*(n
 + 1))*(F0/(a*m) - (2^(1/2)*F0)/(a*m)) - dt^2*sin(dt*n*w)*(F0/
(a*m) - (2^(1/2)*F0)/(a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g
```

```
 + 2*beta^2*dt^4*g^2) - dt*gamma*sin(dt*w*(n + 1))*(F0/(a*m) -
 (2^(1/2)*F0)/(a*m)) + dt*sin(dt*n*w)*(F0/(a*m) - (2^(1/2)*F0)/
(a*m))*(gamma - 1) + (2*a*dt*g*gamma*(beta*dt^2*sin(dt*w*(n +
 1))*(F0/(a*m) - (2^(1/2)*F0)/(2*a*m)) - dt^2*sin(dt*n*w)*(F0/(a*m)
 - (2^(1/2)*F0)/(2*a*m))*(beta - 1/2)))/(a^2 + 4*a*beta*dt^2*g +
 2*beta^2*dt^4*g^2)


An =

[
  1 - (981*dt^2)/50,
                          (981*dt^2)/100,                 dt,
     0]
[
      (981*dt^2)/50,
                      1 - (981*dt^2)/50,                   0,
    dt]
[ (981*dt*((981*dt^2)/50 - 1))/50 - (981*dt)/50 +
 (6772013501556091*dt^3)/35184372088832,
 (981*dt)/100 - (981*dt*((981*dt^2)/50 - 1))/100 - (962361*dt^3)/5000,
 1 - (981*dt^2)/50,    (981*dt^2)/100]
[                      (981*dt)/50 - (981*dt*((981*dt^2)/50 - 1))/50
 - (962361*dt^3)/2500, (981*dt*((981*dt^2)/50 - 1))/50 - (981*dt)/50
 + (6772013501556091*dt^3)/35184372088832,     (981*dt^2)/50, 1 -
 (981*dt^2)/50]


Bn =



 (51526319965141*dt^2*sin(2*pi*dt*n))/17592186044416


 (36434610256939*dt^2*sin(2*pi*dt*n))/8796093022208
 (51526319965141*dt*sin(2*pi*dt*(n + 1)))/17592186044416 -
 (7402483611873081*dt^3*sin(2*pi*dt*n))/439804651110400 +
 (51526319965141*dt*sin(2*pi*dt*n))/17592186044416
  (36434610256939*dt*sin(2*pi*dt*(n + 1)))/8796093022208 -
 (20937385438310997*dt^3*sin(2*pi*dt*n))/879609302220800 +
 (36434610256939*dt*sin(2*pi*dt*n))/8796093022208
```

1.2

```
e1=[];
xt=0:0.001:1;
for j = 1:length(xt)
    dt= xt(j);
    e1 = [e1, max(abs(eig(eval(A))))];
end
plot(xt,e1);
```

```
title('Le module de valeur propre de différant pas');
% On trouve que quand le pas est inférieure à 0.244, tous les modules
 de valeur propre est presque égale à 1,
% Mais quand le pas est supérieure à 0.244, les modules de valeur
 propre supérieure à 1.
%Donc le pas de temps critique est 0.244
```



1.3

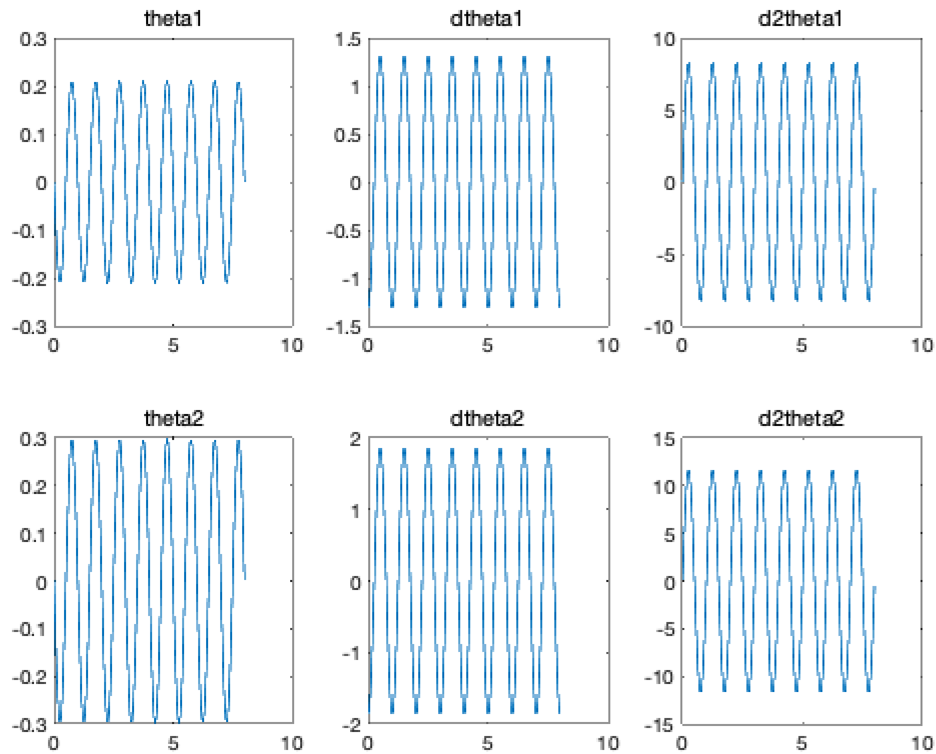
```
theta1_0 = 0;
theta2_0 = 0;
dtheta1_0 = - 1.31519275;
dtheta2_0 = - 1.85996342;
q0 = [theta1_0; theta2_0];
dq0 = [dtheta1_0; dtheta2_0];
d2q0 = eval(M4) * q0;
```

1.4

```
%Un = [qn; dqn]
%Un1 = [qn1;dqn1]
%les trois relations
% Un1 = A * Un + B
% d2qn = M4 * qn + M5 * sin(w * tn)
% d2qn1 = M4 * qn1 + M5 * sin(w * tn1)
```

1.5

```
T0 = 8;
dt = 0.01;
U = [q0; dq0];
q = [q0];
dq = [dq0];
d2q = [d2q0];
for n = 0 : (T0 / dt - 1)
    U = eval(A) * U + eval(B);
    q = [q, U(1:2)];
    dq = [dq, U(3:4)];
    d2q = [d2q, eval(M4 * U(1:2) + M5 * sin(w * n * dt))];
end

t = 0 :dt:T0;
subplot(2, 3, 1);
plot(t, q(1, :));
title('theta1');
subplot(2, 3, 2);
plot(t, dq(1, :));
title('dtheta1');
subplot(2, 3, 3);
plot(t, d2q(1, :));
title('d2theta1');
subplot(2, 3, 4);
plot(t, q(2, :));
title('theta2');
subplot(2, 3, 5);
plot(t, dq(2, :));
title('dtheta2');
subplot(2, 3, 6);
plot(t, d2q(2, :));
title('d2theta2');
```



1.6

```
clf;
T0 = 8;
dt = 0.02;
U = [q0; dq0];
q = [q0];
dq = [dq0];
d2q = [d2q0];
for n = 0 : (T0 / dt - 1)
    U = eval(A) * U + eval(B);
    q = [q, U(1:2)];
    dq = [dq, U(3:4)];
    d2q = [d2q, eval(M4 * U(1:2) + M5 * sin(w * n * dt))];
end
q(:, 1 : 3)% ce sont les valeurs de q à 0s , dt , 2dt.
q(:, 0.5 / dt + 1) %c'est le valeur de q à 0.5s.
% Ce sont
% 0   -0.0263   -0.0522   -0.299e-3
% 0   -0.0372   -0.0738   -0.423e-3
dq(:, 1 : 3) % ce sont les valeurs de dq à 0s , dt , 2dt.
dq(:, 0.5 / dt + 1) %c'est le valeur de dq à 0.5s.
% Ce sont
% -1.32   -1.30   -1.27   1.31
% -1.86   -1.85   -1.80   1.86
```

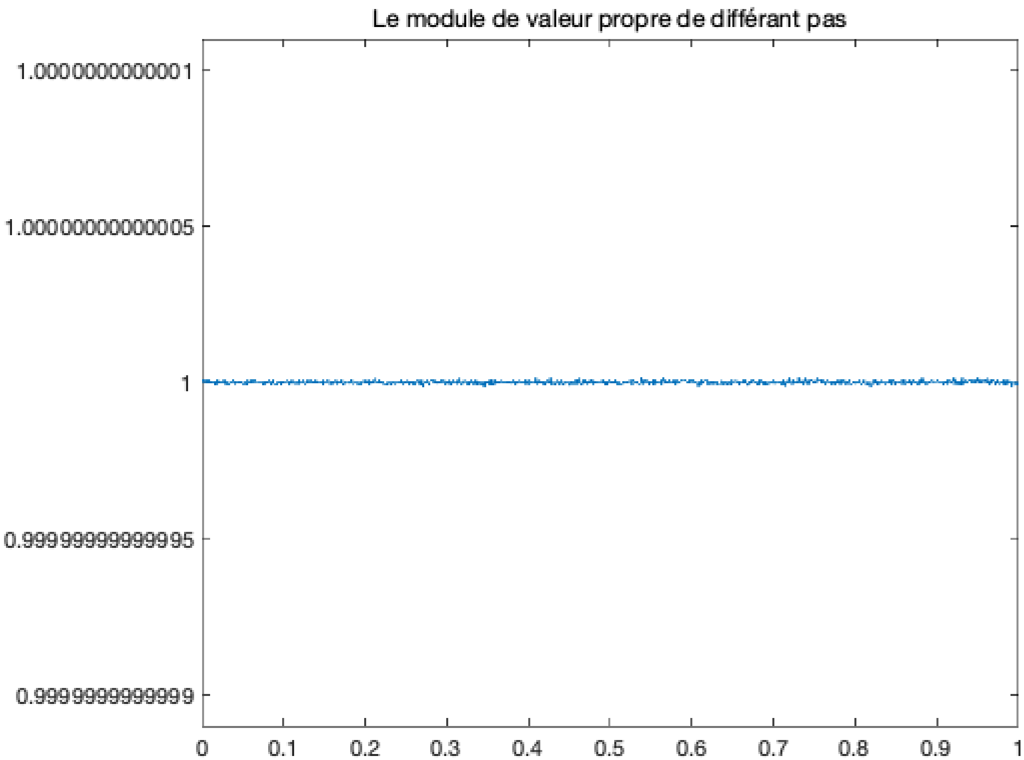
```
d2q(:, 1 : 3) % ce sont les valeurs de d2q à 0s , dt , 2dt.
d2q(:, 0.5 / dt + 1) %c'est le valeur de d2q à 0.5s.
% Ce sont
% 0    0.302    1.33   0.737
% 0    0.428    1.89   1.04
%Ces valeurs seront donnees avec trois chiffres significatifs.


ans =

         0   -0.0263   -0.0522
         0   -0.0372   -0.0738


ans =

   1.0e-03 *

   -0.2988
   -0.4226


ans =

   -1.3152   -1.3048   -1.2739
   -1.8600   -1.8453   -1.8016


ans =

    1.3143
    1.8587


ans =

         0    0.3023    1.3340
         0    0.4275    1.8866


ans =

    0.7376
    1.0432
```

2.1

```
beta=0.25;
gamma=0.5;
eval(A)
eval(B)


ans =

    0.9922    0.0039    0.0199    0.0000
    0.0078    0.9922    0.0001    0.0199
   -0.7802    0.3893    0.9922    0.0039
    0.7787   -0.7802    0.0078    0.9922


ans =

   -0.0001
   -0.0001
   -0.0073
   -0.0104
```

2.2

```
clf;
e=[];
xt=0:0.001:1;
for j = 1:length(xt)
    dt= xt(j);
    e = [e, max(abs(eig(eval(A))))];
end
plot(xt,e);
title('Le module de valeur propre de différant pas');
% d'apres le schema
%on peut trouver que le module de valeur propre est toujours presque
 de 1
```



2.3

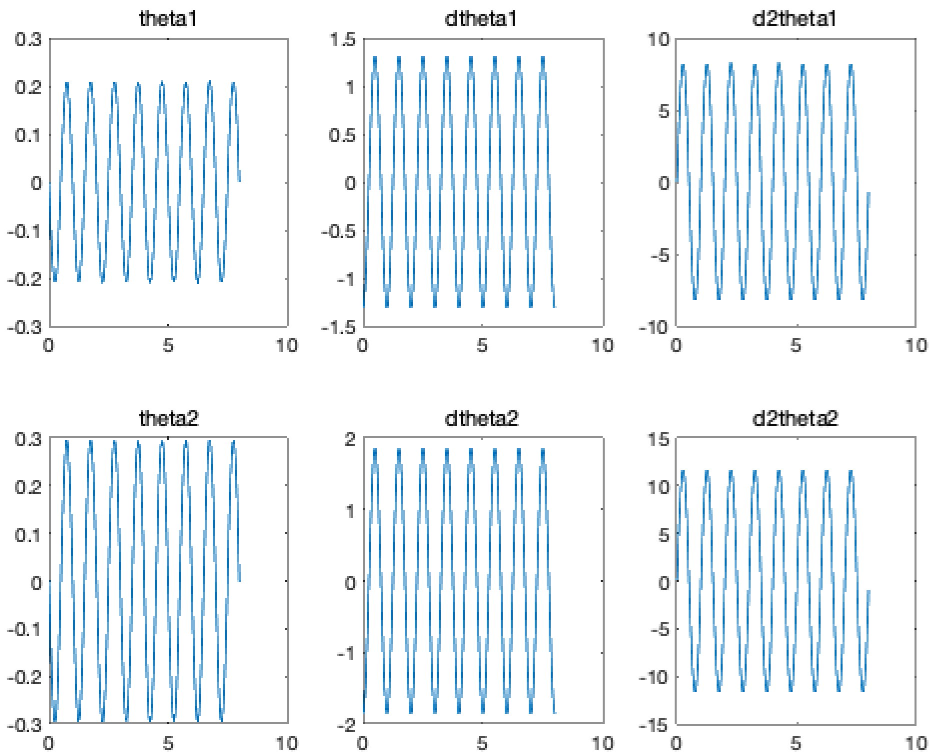
```
theta1_0 = 0;
theta2_0 = 0;
dtheta1_0 = - 1.31519275;
dtheta2_0 = - 1.85996342;
q0 = [theta1_0; theta2_0];
dq0 = [dtheta1_0; dtheta2_0];
d2q0 = eval(M4) * q0;
```

2.4

```
%Un = [qn; dqn]
%Un1 = [qn1;dqn1]
%les trois relations
% Un1 = A * Un + B
% d2qn = M4 * qn + M5 * sin(w * tn)
% d2qn1 = M4 * qn1 + M5 * sin(w * tn1)
```

2.5

```
clf;
T0 = 8;
dt = 0.02;
U = [q0; dq0];
q = [q0];
dq = [dq0];
d2q = [d2q0];
for n = 0 : (T0 / dt - 1)
    U = eval(A) * U + eval(B);
    q = [q, U(1:2)];
    dq = [dq, U(3:4)];
    d2q = [d2q, eval(M4 * U(1:2) + M5 * sin(w * n * dt))];
end

t = 0 :dt:T0;
subplot(2, 3, 1);
plot(t, q(1, :));
title('theta1');
subplot(2, 3, 2);
plot(t, dq(1, :));
title('dtheta1');
subplot(2, 3, 3);
plot(t, d2q(1, :));
title('d2theta1');
subplot(2, 3, 4);
plot(t, q(2, :));
title('theta2');
subplot(2, 3, 5);
plot(t, dq(2, :));
title('dtheta2');
subplot(2, 3, 6);
plot(t, d2q(2, :));
title('d2theta2');
```



2.6

```
q(:, 1 : 3)% ce sont les valeurs de q à 0s , dt , 2dt.
q(:, 0.5 / dt + 1)%c'est le valeur de q à 0.5s.
% Ce sont
% 0   -0.0262   -0.0520   -0.000900
% 0   -0.0371   -0.0735   -0.0013
dq(:, 1 : 3)% ce sont les valeurs de dq à 0s , dt , 2dt.
dq(:, 0.5 / dt + 1)%c'est le valeur de dq à 0.5s.
% Ce sont
% -1.32   -1.30   -1.27   1.31
% -1.86   -1.85   -1.80   1.86
%Ces valeurs seront donnees avec trois chiffres significatifs.


ans =

         0   -0.0262   -0.0520
         0   -0.0371   -0.0735


ans =

   -0.0009
   -0.0013
```

```
ans =

   -1.3152   -1.3048   -1.2739
   -1.8600   -1.8453   -1.8016


ans =

    1.3124
    1.8561
```

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