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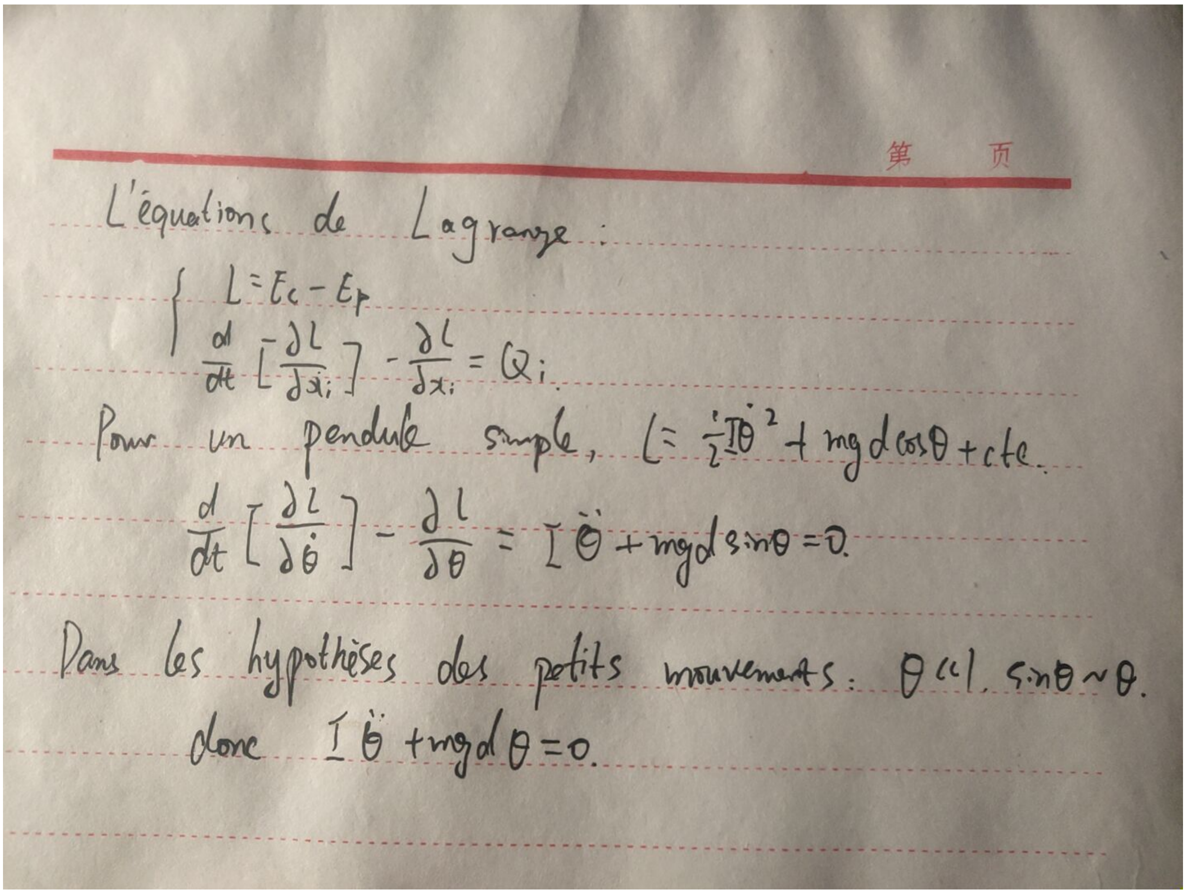
Les 4 devoirs sont tous dans ce fichier.

Retrouver l'équation du mouvement du pendule simple avec les équations de Lagrange



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L'équations de Lagrange :

$$\begin{cases} L = E_c - E_p \\ \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}_i}\right] - \frac{\partial L}{\partial x_i} = Q_i \end{cases}$$

Pour un pendule simple, $L = \frac{1}{2} I \dot{\theta}^2 + mgd\cos\theta + cte.$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}}\right] - \frac{\partial L}{\partial \theta} = I\ddot{\theta} + mgd\sin\theta = 0.$$

Dans les hypothèses des petits mouvements : $\theta \ll 1$, $\sin\theta \sim \theta$.

donc $I\ddot{\theta} + mgd\,\theta = 0.$

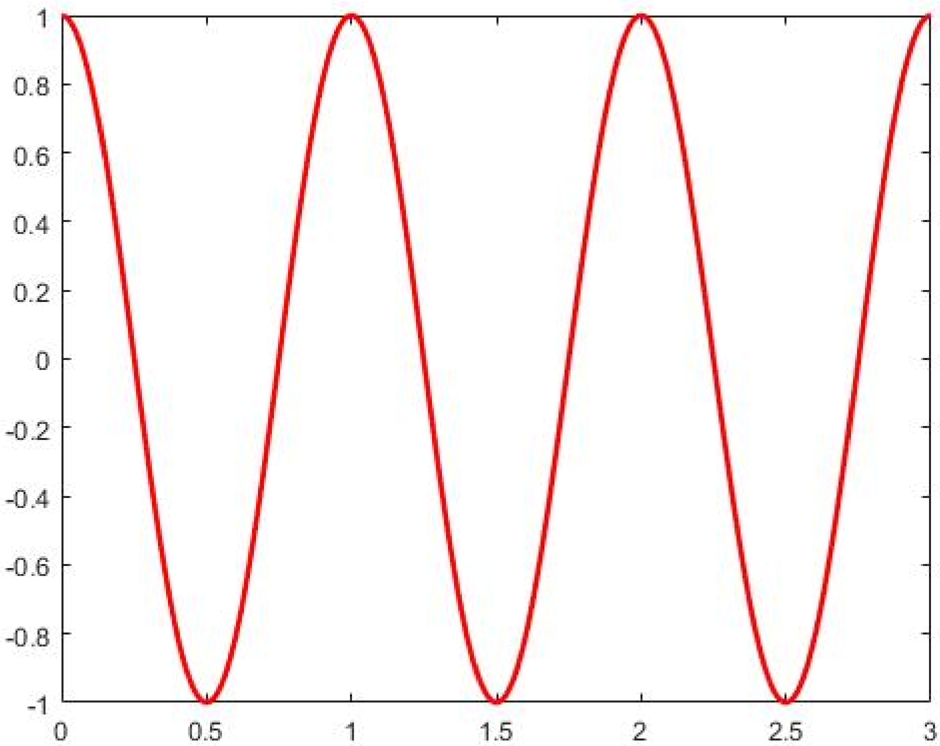
Oscillateur conservatif à un degré de liberté

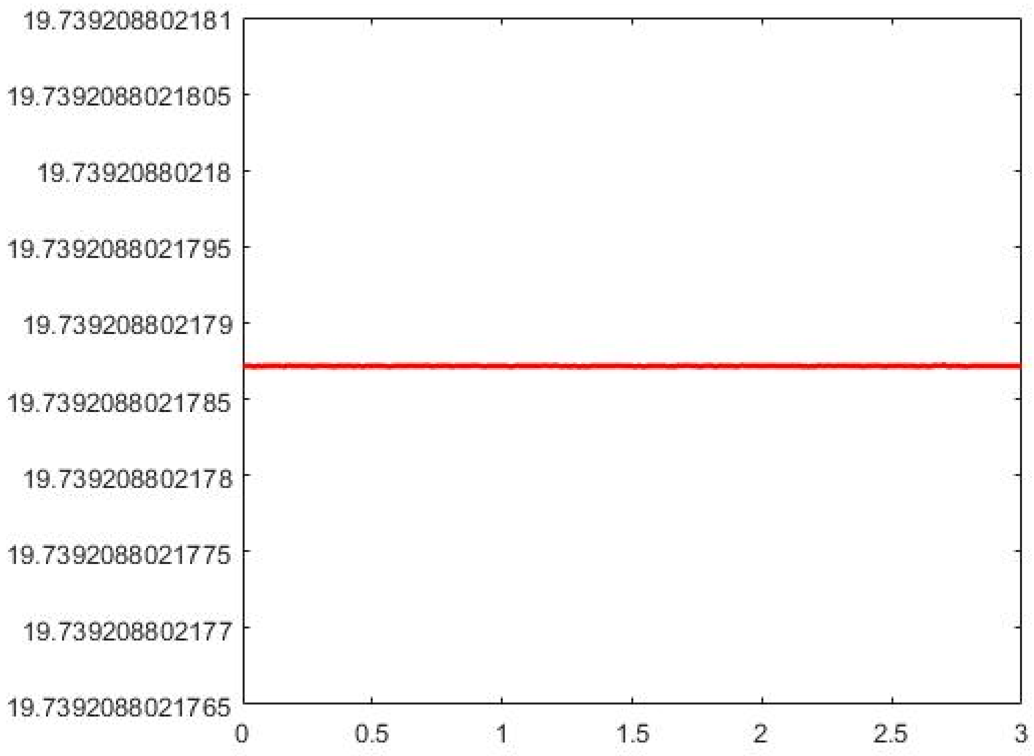
Solution analytique de l'équation

```
clear all; close all; clc
T0 = 3;
w0 = 2*pi;
w0c = w0 * w0;
q0 = 1; dq0 = 0.0; dte = 0.01;
te = (0:dte:T0)';
```

```
npe = size(te,1);
qe = zeros(npe,1);
dqe = zeros(npe,1);
energe = zeros(npe,1);
tic;
qe = q0*cos(w0*te) + dq0/w0*sin(w0*te);
dqe = -w0*q0*sin(w0*te)+dq0*cos(w0*te);
ddqe = -w0c*qe;
energe = 0.5*(dqe.*dqe + w0c.*(qe.^2)); toc;
plot(te,qe,'-r','Linewidth',2)
% plot(te,energe,'-r','Linewidth',2)
```

La solution et la quantité:



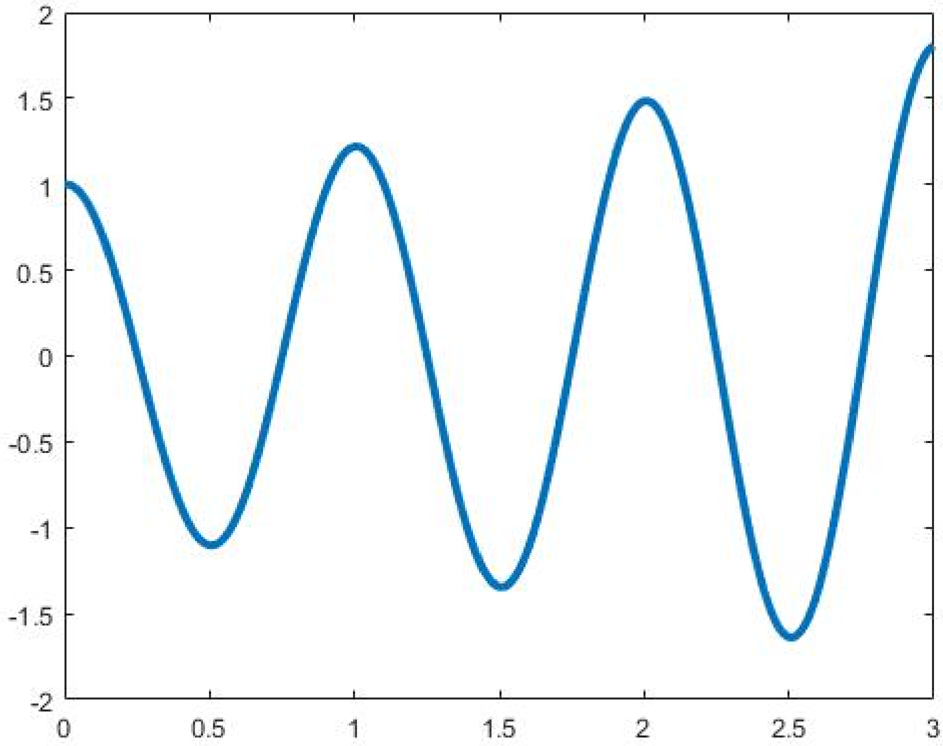


La quantité est constante.

Résolution de l'équation (1) avec un schéma d'EULER explicite

```
clear all; close all; clc
t0=3;
w0=2*pi;
w0c = w0*w0;
q0=1;
dq0=0.0;
dt1=0.01;
t1=(0:dt1:t0)';
np1=size(t1,1);
q1=zeros(np1,1);
dq1=zeros(np1,1);
ddq1=zeros(np1,1);
energ1=zeros(np1,1);
q1(1)=q0;
dq1(1)=dq0;
ddq1(1)=-w0c*q1(1);

for inc=2:np1
    q1(inc)=q1(inc-1)+dt1*dq1(inc-1);
    dq1(inc)=dq1(inc-1)+dt1*ddq1(inc-1);
    ddq1(inc)=-w0c*q1(inc);
end;
ener1=0.5*(dq1.*dq1+w0c*(q1.^2));
plot(t1,q1,'Linewidth',3)
```

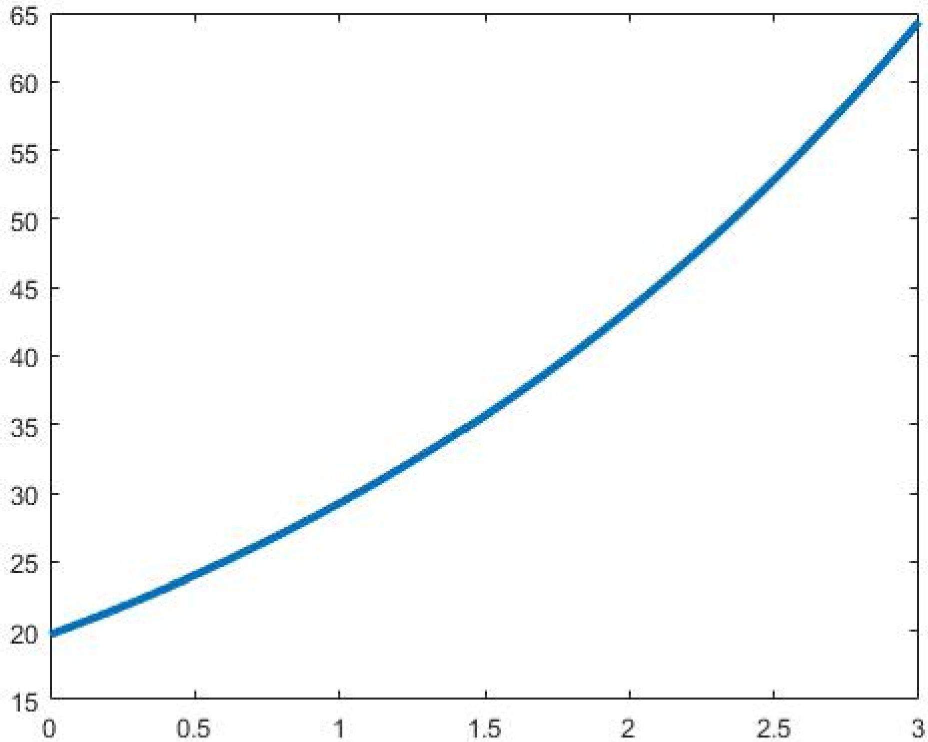
Resultat:



La solution est divergente. Et si l'on prend dt1 plus petite, la divergence est plus lente.

la quantité:

```
plot(t1,ener1,'Linewidth',3)
```



i l'on prend dt1 plus petite, la quantité augmente plus lentement.

Stabilité:

```
clear all; close all; clc;
dt1=sym('dt1','real');
%dt1=0.01;
w0=sym('w0','real');
%w0=2*pi;
A=[1,dt1;-1*w0*w0*dt1,1]
[z,d]=eig(A)
re=real(d);
im=imag(d)
mo=abs(d)
zm=inv(z)
C=z*(d*zm)
C=simplify(C)
```

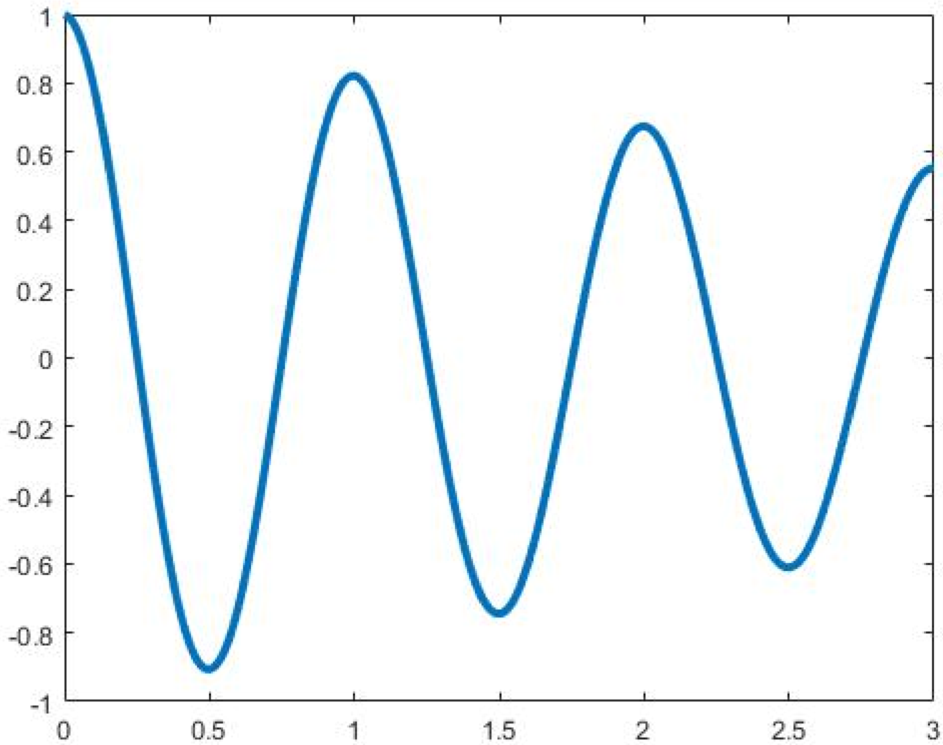
Quand dt1=0.01 et w0=2*pi, mo =[1.0020   0; 0   1.0020]

Donc e caractère inconditionnellement est instable.

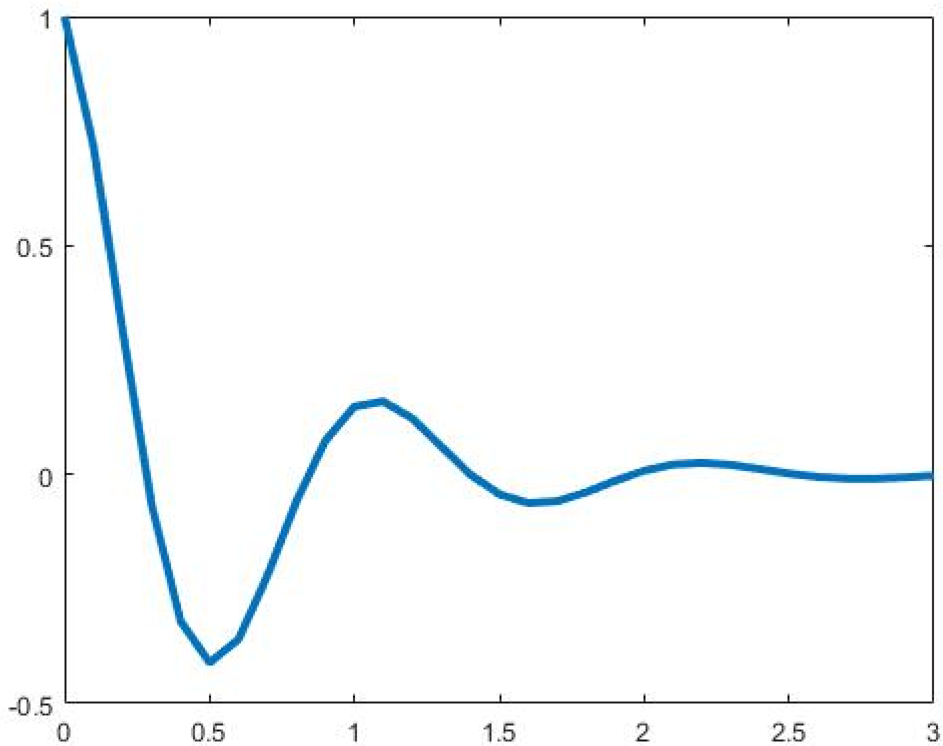
Résolution de l'équation (1) avec un schéma d'EULER implicite

```
clear all; close all; clc;
t0=3;
w0=2*pi;
w0c = w0*w0;
q0=1;
dq0=0.0;
dt1=0.01;
t1=(0:dt1:t0)';
np1=size(t1,1);
q1=zeros(np1,1);
dq1=zeros(np1,1);
ddq1=zeros(np1,1);
energ1=zeros(np1,1);
q1(1)=q0;
dq1(1)=dq0;
ddq1(1)=-w0c*q1(1);

for inc=2:np1
    q1(inc) = (q1(inc-1)+dt1*dq1(inc-1))/(1+dt1^2*w0c);
    ddq1(inc) = -w0c*q1(inc);
    dq1(inc) = dq1(inc-1)+dt1*ddq1(inc);
end;
ener1=0.5*(dq1.*dq1+w0c*(q1.^2));
plot(t1,q1,'Linewidth',3)
```

Résultat:



Si l'on prend dt1=0.1 (plus grand), moins l'atténuation des oscillations est faible.

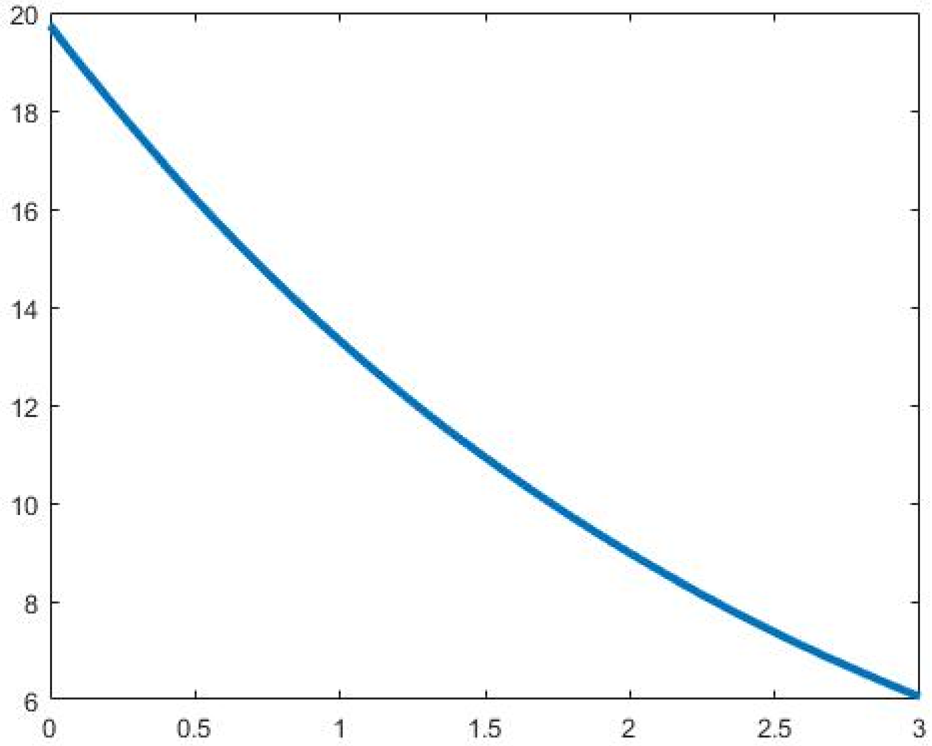


la quantité associée au schéma d'EULER implicite:

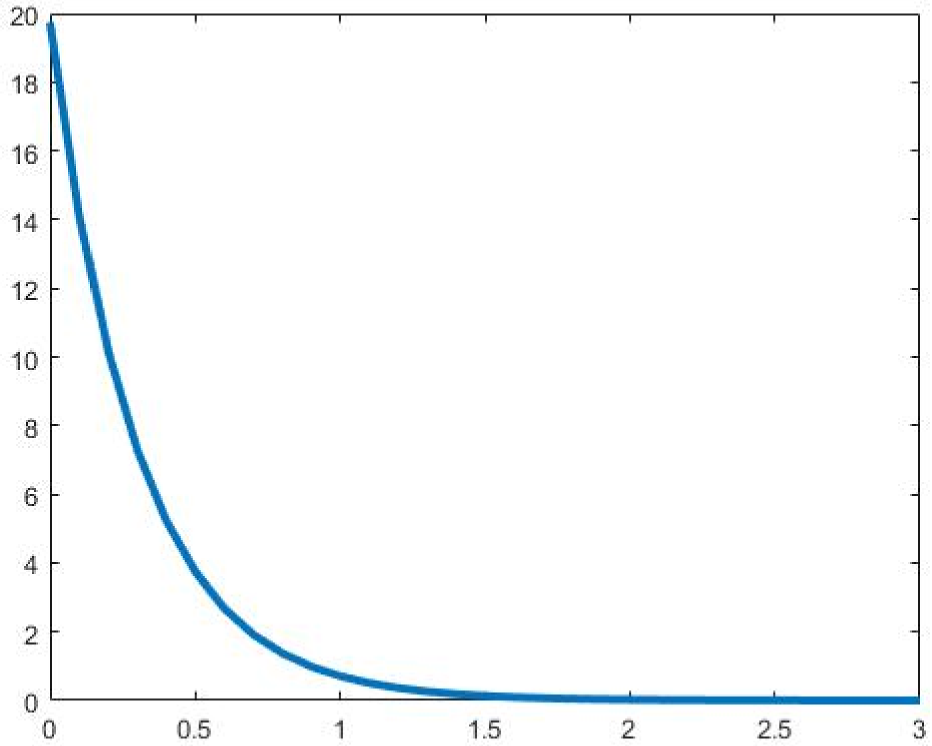
```
plot(t1,ener1,'Linewidth',3)
```

la quantité est de plus en plus faible.

Si l'on prend dt1=0.01:



Si l'on prend dt1=0.1, l’atténuation est plus grande.



Les valeurs propres et stabilite:

```
clear all; close all; clc; format short e;
%dt1 = sym('dt1','real'); w0 = sym('w0','real');
w0 = 2*pi;
dt1 = 0.01;
%be = sym('be','real'); ga = sym('ga','real');
be = 0;
```

```
ga = 0.5;
A = [1-dt1*dt1*(0.5-be)*w0*w0,dt1;-(1-ga)*dt1*w0*w0,1];
B = [1+be*dt1*dt1*w0*w0,0;ga*dt1*w0*w0,1];
%vecteur et valeur propres
[z,d] = eig((inv(B))*A);
re = real(d);
im = imag(d);
mo = abs(d);
% mo = simplify(mo);
% C est matrice initiale! apres changement de base!
zm = inv(z);
C = z*(d*zm);
% C = simplify(C);
d
mo
```

On a:

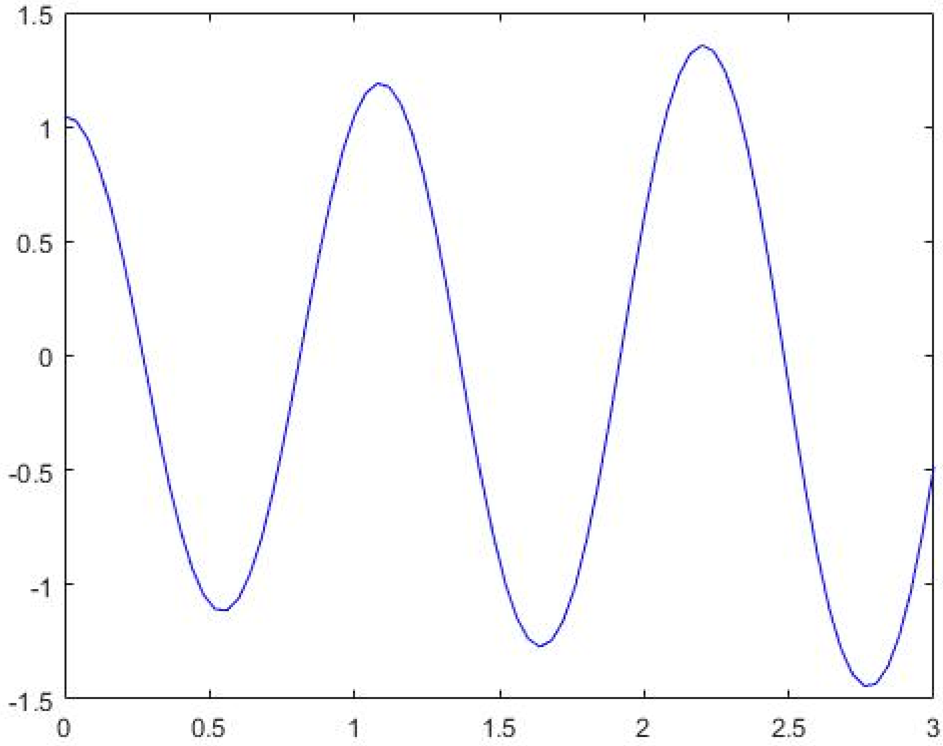
mo=[1 0; 0 1], le caractère inconditionnellement est stable.

Résolution de l'équation (1) avec un schéma de RUNGE KUTTA

```
clc;clear;
% range kutta
%q''+w0c*sinq=0
w0=2*pi;
w0c=w0^2;

dt6=0.04;
t6=(0:dt6:3)';
np6=size(t6,1)
q0=pi/3;
dq0=0;
q6=zeros(np6,1);
dq6=zeros(np6,1);
dq=zeros(np6,1);
e6=zeros(np6,1);
ener1=zeros(np6,1);
q6(1)=q0;
dq6(1)=dq0;
qj=[q0;dq0];
for inc=2:np6
    tc=t6(inc-1);
    xc=qj;
    k1=cal_f2(xc,tc,w0c);
    xc=qj+k1*dt6/2;
    k2=cal_f2(xc,tc+dt6/2,w0c);
    xc=qj+k2*dt6/2;
    k3=cal_f2(xc,tc+dt6/2,w0c);
    xc=qj+k3*dt6/2;
    k4=cal_f2(xc,tc+dt6,w0c);
    dq=(k1+2*k2+2*k3+k4)/6;
    qj=qj+dq*dt6;
    q6(inc)=qj(1);
    dq6(inc)=qj(2);
```

```
end
ener1=0.5*(dq6.*dq6+w0c*(q6.^2));
plot(t6,q6,'b');
plot(t6,ener1,'b');
```

La solution et la quantité:



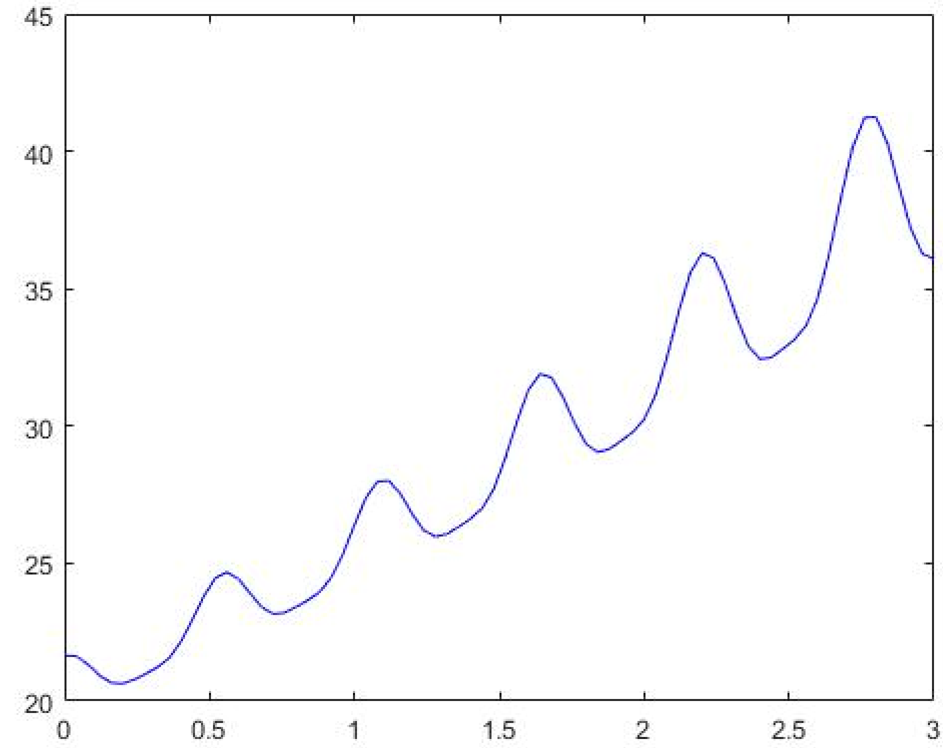


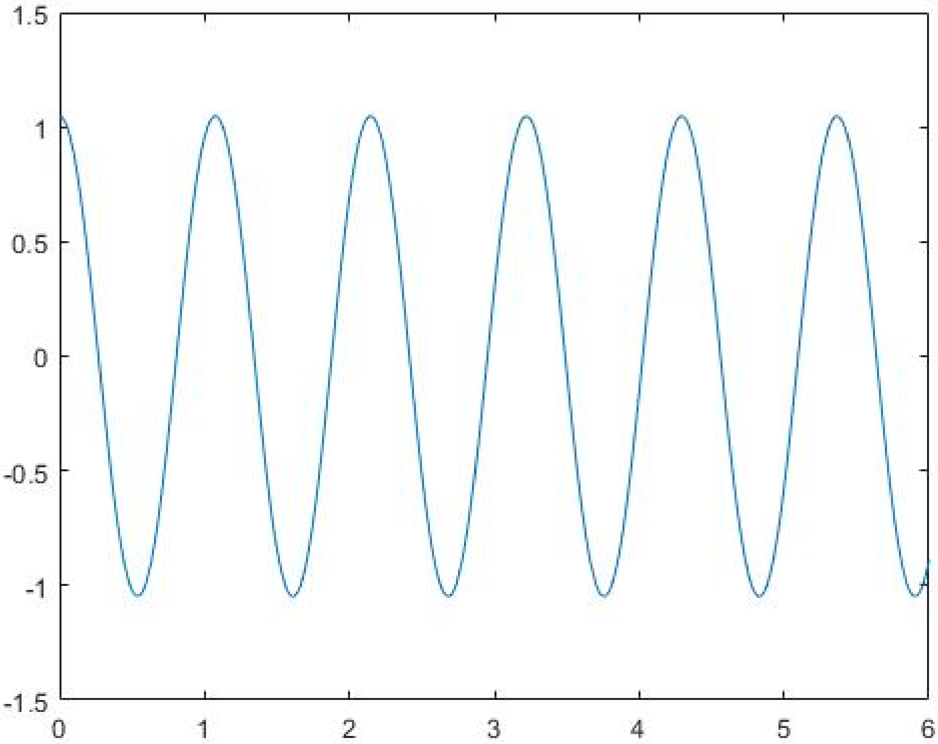
schéma de NEWMARK

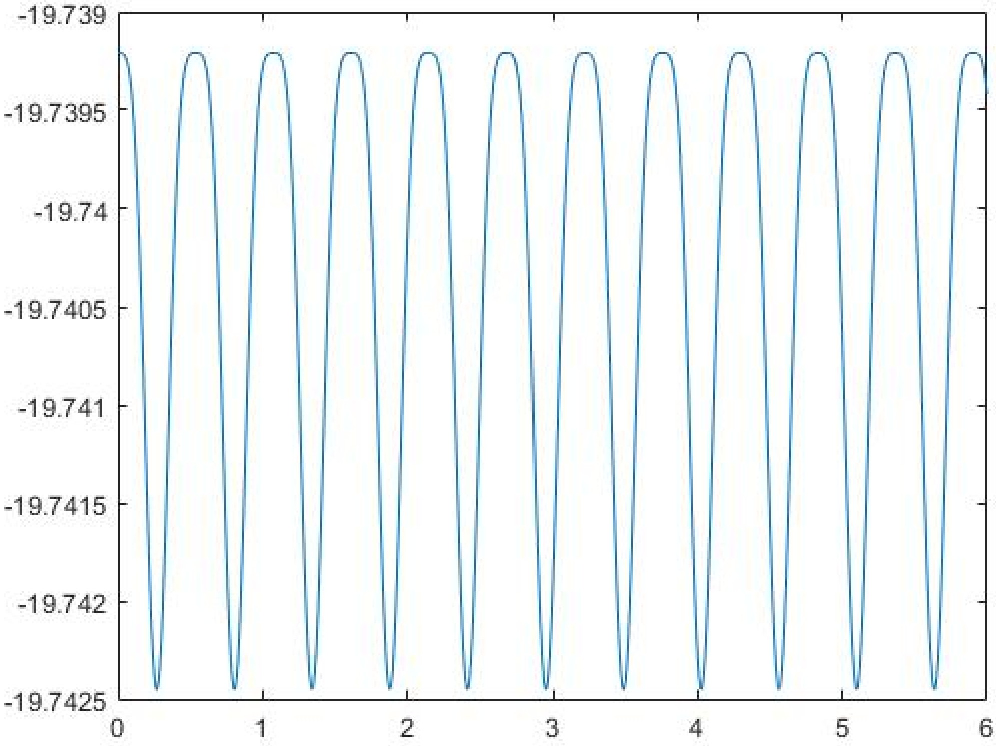
Quand gamma=0.5, beta=0.25:

```
clear all;
close all;
clc;

%  ddqc = -w0c*sin(q1(inc));

dt3 = 0.01;
T0 = 6;
t3 = (0:dt3:T0)';
np3 = size(t3,1);
q3 = zeros(np3,1);
dq3 = zeros(np3,1);
ddq3 = zeros(np3,1);
energ3 = zeros(np3,1);
residu = zeros(np3,1);
niter = zeros(np3,1);
threshold = 1e-8;
nitermax = 20;

q0 = pi/3;
dq0 = 0;
w0 = 2*pi;
w0c = w0*w0;

gam = 0.5;
gam1 = (1-gam)*dt3;
gam2 = gam*dt3;
bet = 0.25;
bet1 = (0.5-bet)*dt3*dt3;
bet2 = bet * dt3 * dt3;
q3(1) = q0;
dq3(1) = dq0;
ddq3(1) = -w0c * sin(q0);

for inc = 2:np3
    ddqc = 0;
    dqc = dq3(inc - 1)+gam1*ddq3(inc -1);
    qc = q3(inc - 1) + dt3 * dq3(inc-1) + bet1*ddq3(inc-1);
    res = ddqc + w0c*sin(qc);
    iter = 0;
    for it = 1:nitermax
        if(abs(res)>=threshold)
            iter = iter + 1;
            correction = -res/(1 + w0c*bet2*cos(qc));
            ddqc = ddqc + correction;
            dqc = dqc + gam2 * correction;
            qc = qc + bet2 * correction;
            res = ddqc + w0c * sin(qc);
        end
    end
    q3(inc) = qc;
    dq3(inc) = dqc;
    ddq3(inc) = ddqc;
    niter(inc) = iter;
    residu(inc) = res;
```

```
end

energ3 = 0.5*(dq3.*dq3) - w0c * cos(q3);
figure(1)
plot(t3,q3);
figure(2)
plot(t3,energ3);
```

La solution et la quantité:



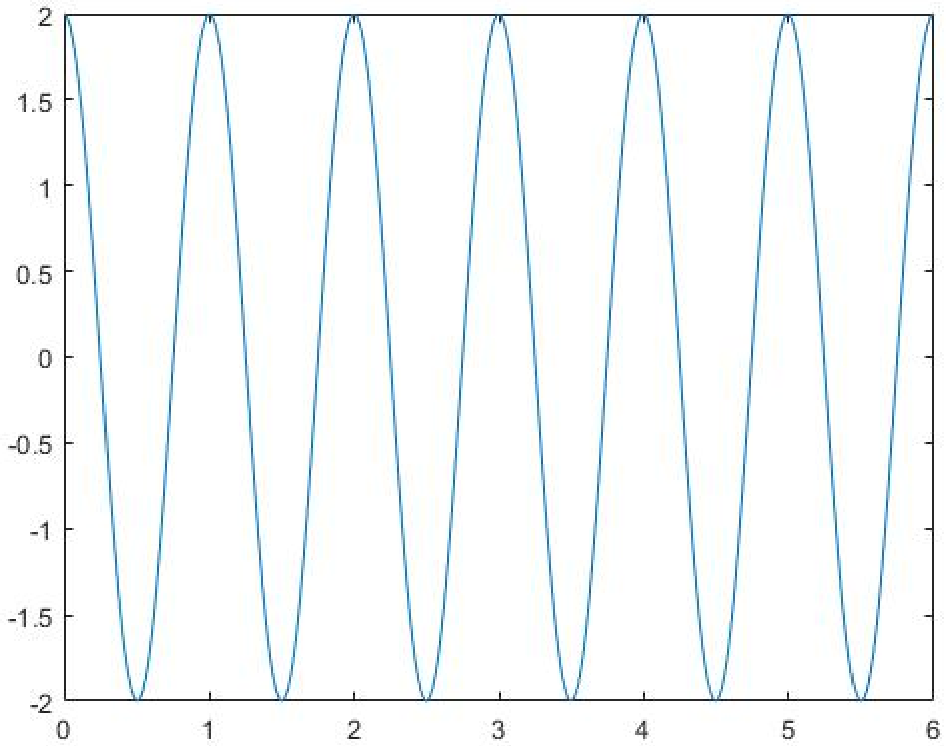


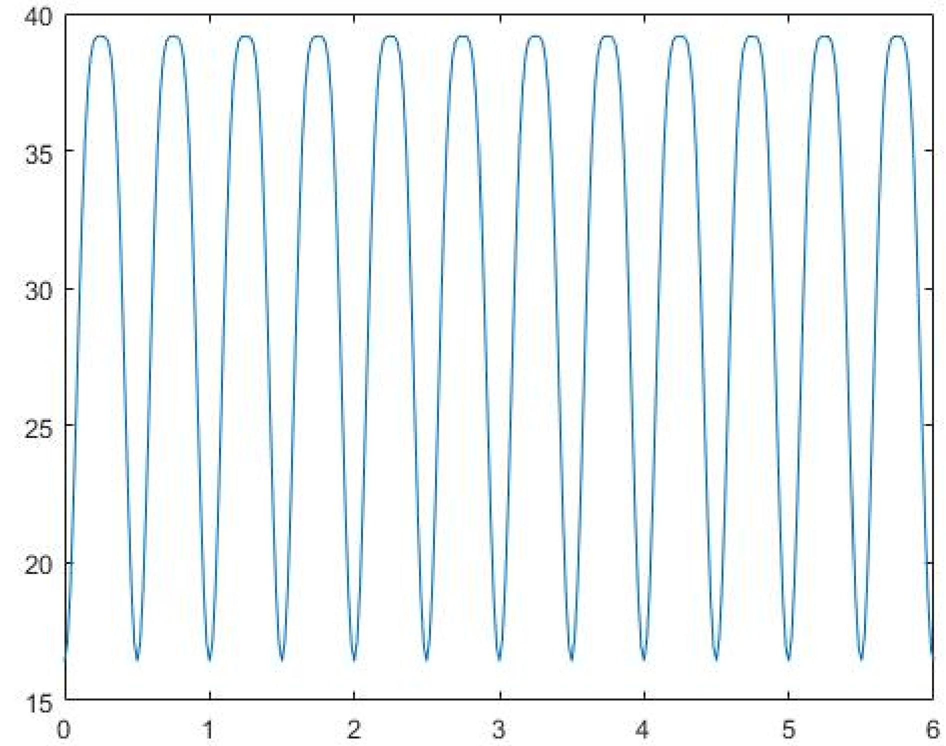
La quantité est la plus precise dans la methode NEWMARK.

Quand gamma=0.5, beta=0:

```
dt1 = 0.001;
T0 = 6;
t1 = (0:dt1:T0)';
np1 = size(t1,1);
q1 = zeros(np1,1);
dq1 = zeros(np1,1);
energ1 = zeros(np1,1);

q0 = pi/3;
dq0 = 0;
w0 = 2*pi;
w0c = w0*w0;

q1(1) = q0;
dq1(1) = dq0;
ddq0c = -w0c* (q0);

for inc = 2:np1
    q1(inc) = q1(inc-1)+dt1*dq1(inc-1)+dt1*dt1*0.5*ddq0c;
    ddqc = -w0c*sin(q1(inc));
    dq1(inc) = dq1(inc-1)+0.5*dt1*(ddq0c+ddqc);
    ddq0c = ddqc;
end

energ1 = 0.5*(dq1 .*dq1) - w0c * cos(q1);
figure(1)
plot(t1,q1);
figure(2)
plot(t1,energ1);
```

La solution et la quantité:





oscillateur linéaire amorti à un degré de liberté

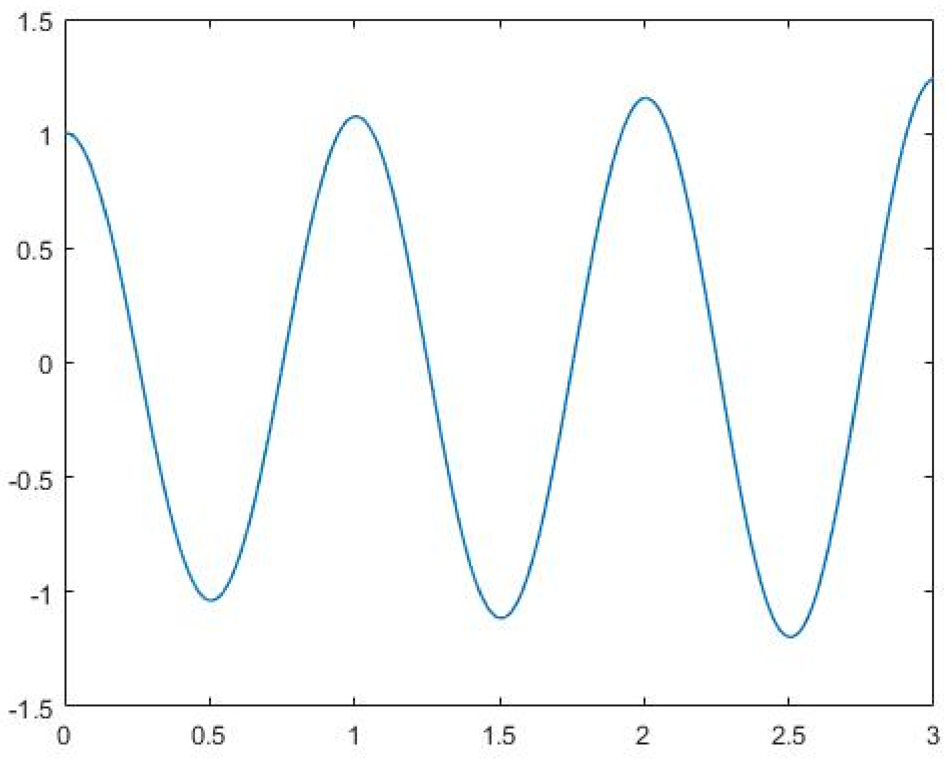
EULER explicite

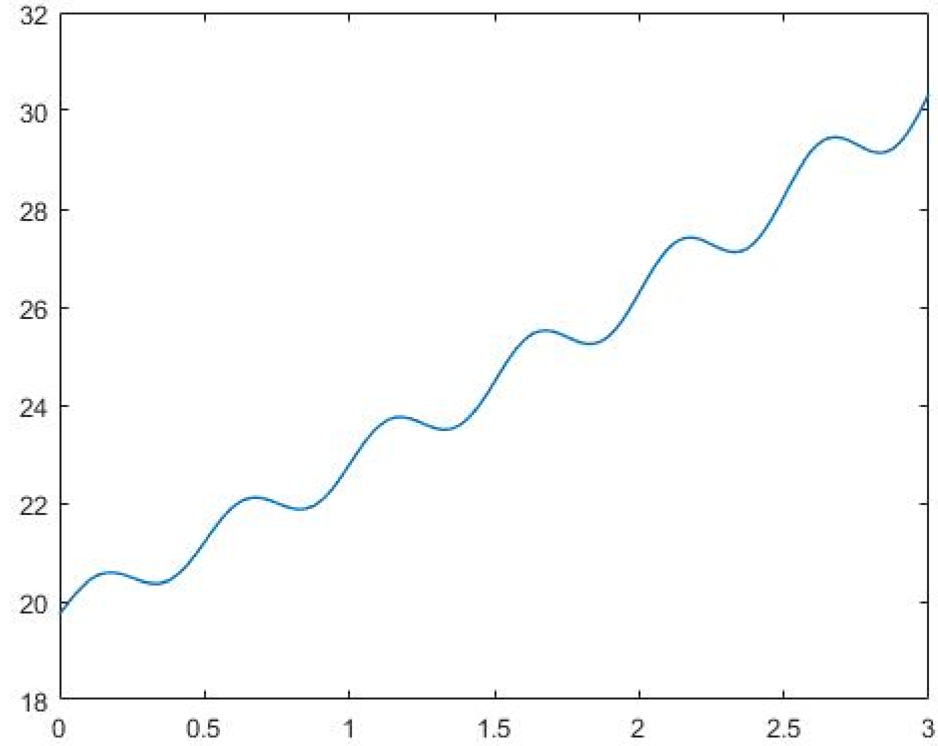
```
%x''+2*e*w0*x'+w0c*x=0
t0=3;
w0=2*pi;
w0c=w0^2;
e=0.02;
q0=1;
dq0=0.0;
dt1=0.01;
t1=(0:dt1:t0)';
np1=size(t1,1);
q1=zeros(np1,1);
dq1=zeros(np1,1);
ddq1=zeros(np1,1);
energ1=zeros(np1,1);
q1(1)=q0;
dq1(1)=dq0;
ddq1(1)=-w0c*q1(1);
%explicite
for inc=2:np1
    q1(inc)=q1(inc-1)+dt1*dq1(inc-1);
    dq1(inc)=dq1(inc-1)+dt1*ddq1(inc-1);
    ddq1(inc)=-2*e*w0*dq1(inc)-w0c*q1(inc);
end;
ener1=0.5*(dq1.*dq1+w0c*(q1.^2));
figure(1)
plot(t1,q1,'Linewidth',1);
figure(2)
plot(t1,ener1,'Linewidth',1);
```

2*e/w0=6.3662e-03

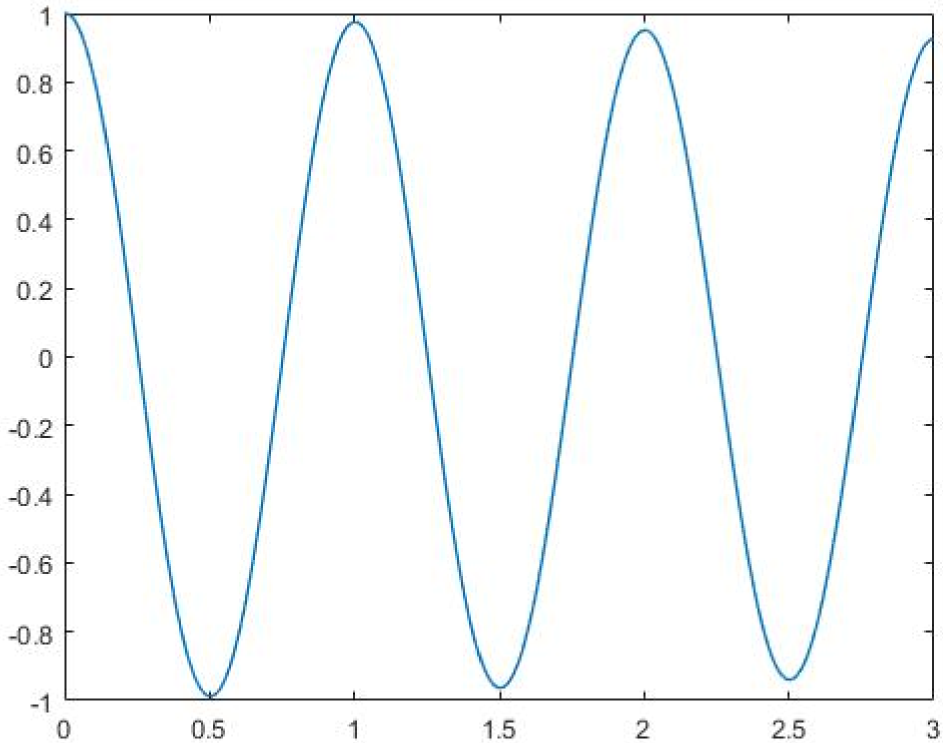
Si l'on prend dt1 = 0.01 > 2 * e / w0:

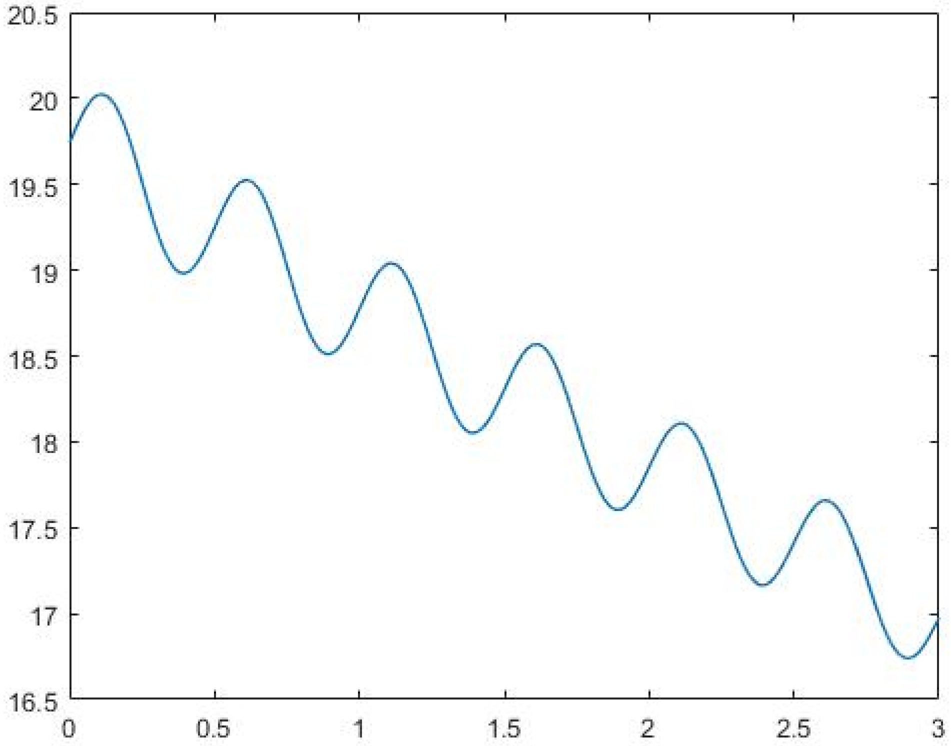
La solution et la quantité:



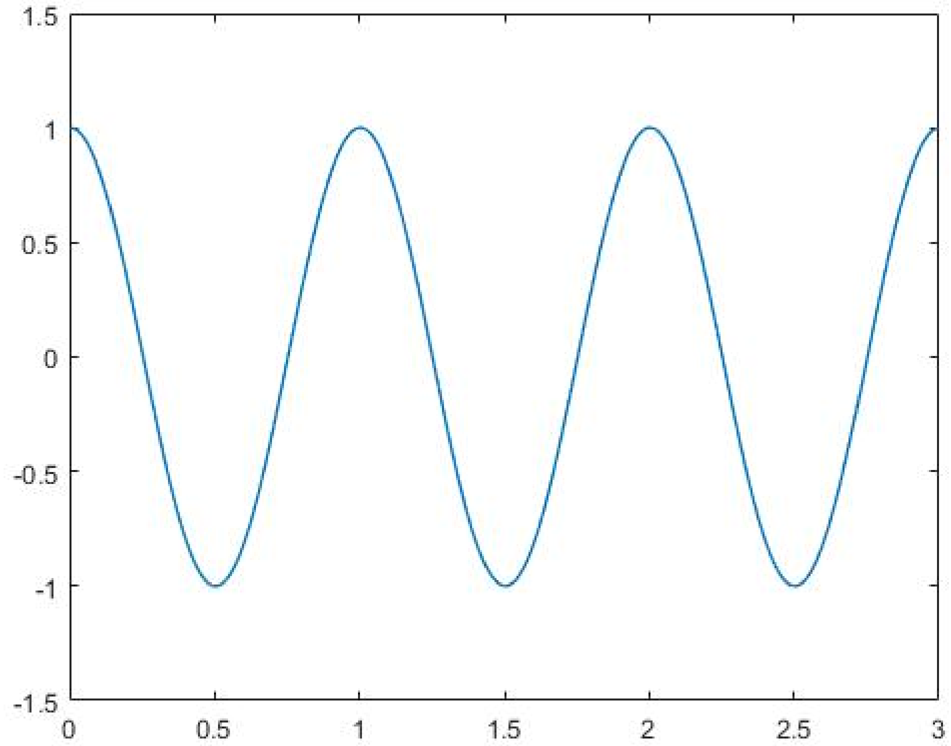


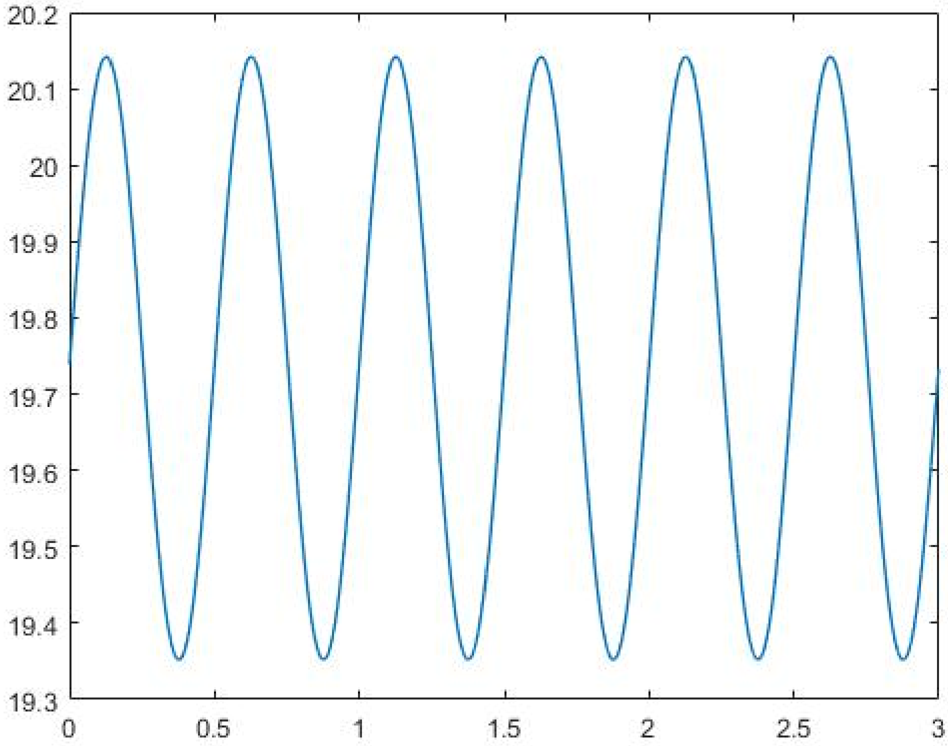
Si l'on prend dt1=0.82e/w0:





Si l'on prend dt1=2*e/w0:





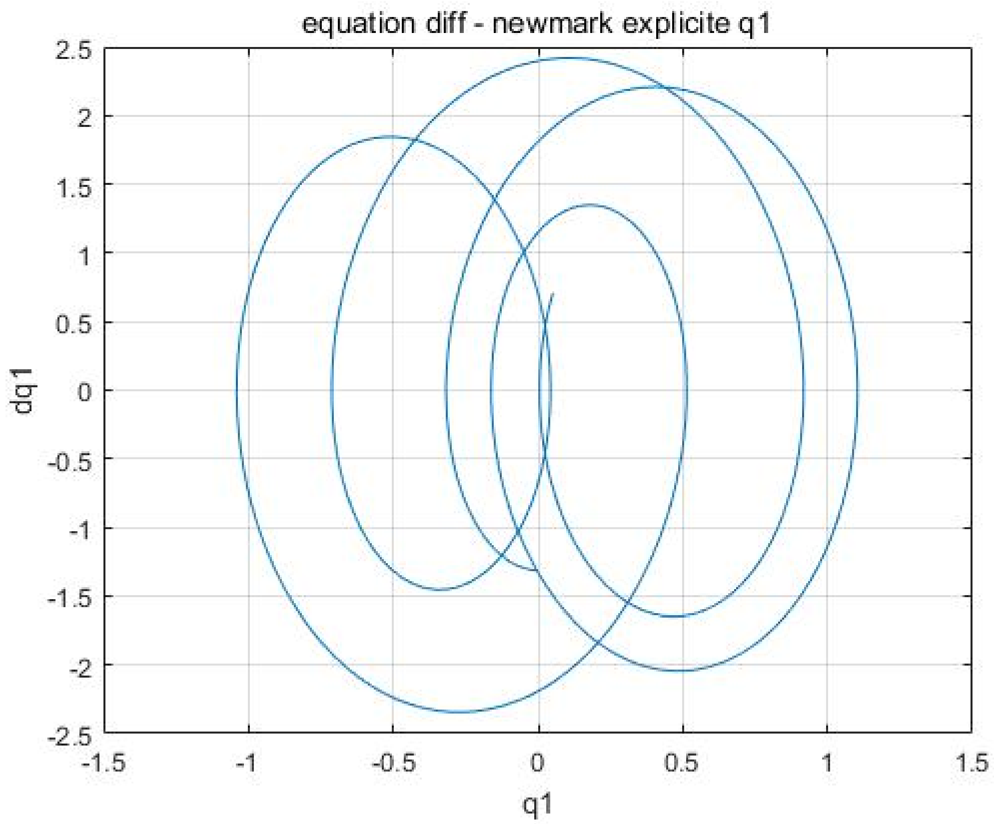
double pendule avec l'hypothèse des petits mouvements

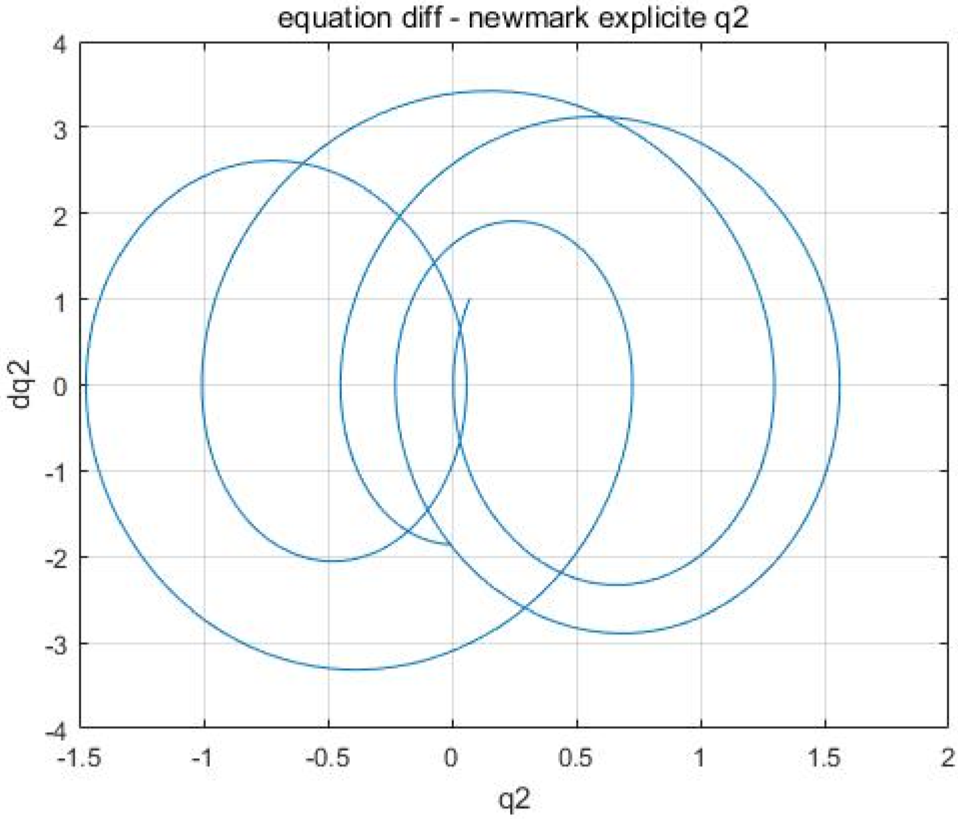
NEWMARK explicite

```
% newmark explicite
%sys lineaire diff 2e ordre
T0=8;
dt = 0.02;
t4 = (0:dt:T0)';
np4 = size(t4,1);
%
a = 0.5;
m = 2;
g = 9.81;
f0=20;
%
q1 = zeros(np4,1); dq1 = zeros(np4,1); ddq1 = zeros(np4,1);
q2 = zeros(np4,1); dq2 = zeros(np4,1); ddq2 = zeros(np4,1);
%
q1(1) = 0;
dq1(1) = -1.31519275;
%
q2(1) = 0;
dq2(1) = -1.85996342;

%
ddq5c = -2*g*q1(1)/a+g*q2(1)/a;
ddq6c = 2*g*q1(1)/a-2*g*q2(1)/a;
```

```
%sys lineaire diff 2e ordre

%...

for inc = 2:np4
    q1(inc)=q1(inc-1)+dt*dq1(inc-1)+0.5*dt*dt*ddq5c;
    q2(inc)=q2(inc-1)+dt*dq2(inc-1)+0.5*dt*dt*ddq6c;

    ddq1(inc) = -2*g*q1(inc)/a+g*q2(inc)/a+f0*(1-1/sqrt(2))*sin(inc*dt)/(m*a);
    ddq2(inc) = 2*g*q1(inc)/a-2*g*q2(inc)/a+f0*(sqrt(2)-1)*sin(inc*dt)/(m*a);

    dq1(inc)=dq1(inc-1)+0.5*dt*(ddq1(inc-1)+ddq1(inc));
    dq2(inc)=dq2(inc-1)+0.5*dt*(ddq2(inc-1)+ddq2(inc));

    ddq5c = ddq1(inc);
    ddq6c = ddq2(inc);
end
figure(1);
plot(q1,dq1)
xlabel('q1');
ylabel('dq1');
grid
title('equation diff - newmark explicite q1')

figure(2);
plot(q2,dq2)
xlabel('q2');
ylabel('dq2');
grid
title('equation diff - newmark explicite q2')
```

Resultat :   (dq en fonction de q)





oscillateur non linéaire `a un degré de liberté

NEWMARK explicite

```
dt1 = 0.02;
T0 = 6;
t1 = (0:dt1:T0)';
np1 = size(t1,1);
q1 = zeros(np1,1);
dq1 = zeros(np1,1);
energ1 = zeros(np1,1);

q0 = 2;
dq0 = 0;
w0 = 2*pi;
w0c = w0*w0;
a=0.1;

q1(1) = q0;
dq1(1) = dq0;
ddq0c = -w0c*q0-a*q0*q0;

for inc = 2:np1
    q1(inc) = q1(inc-1)+dt1*dq1(inc-1)+dt1*dt1*0.5*ddq0c;
    ddqc = -w0c*q1(inc)-a*q1(inc)*q1(inc);
    dq1(inc) = dq1(inc-1)+0.5*dt1*(ddq0c+ddqc);
    ddq0c = ddqc;
end
figure(1)
```

```
plot(t1,q1);
```