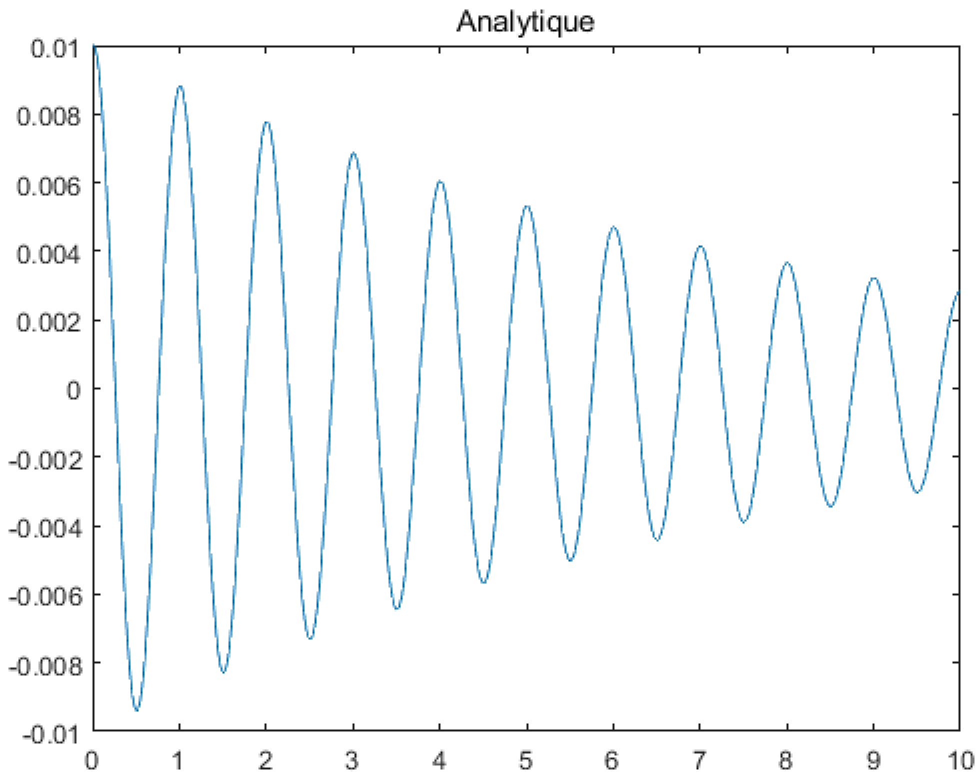
Etude d’un oscillateur lineaire amorti a un degre de liberte

Table of Contents

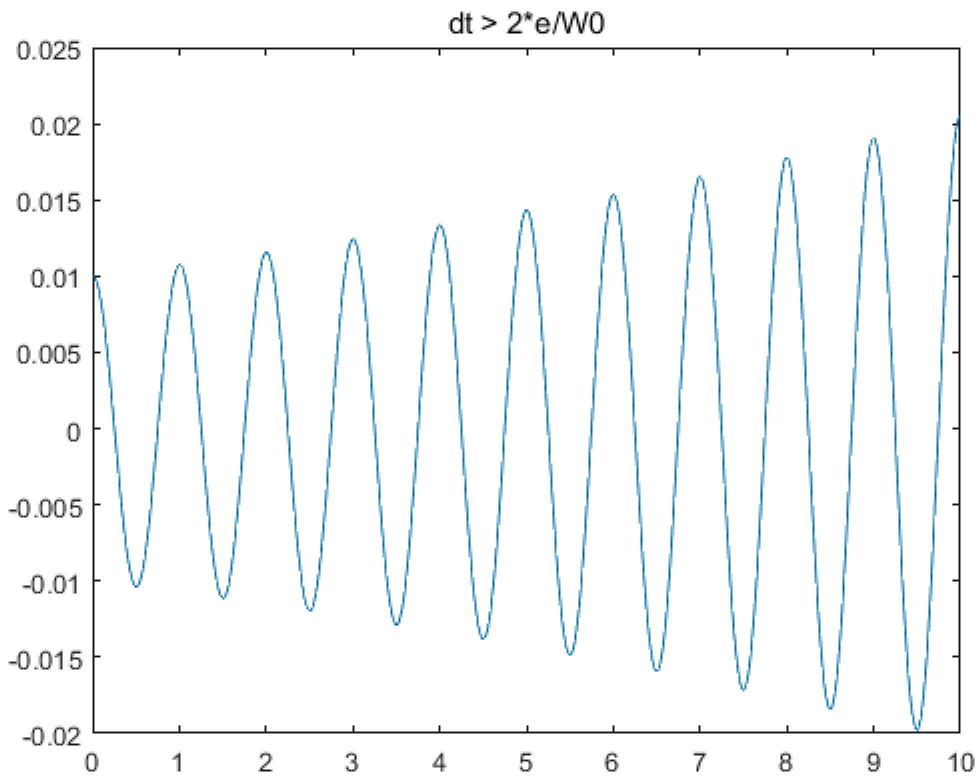
Q1

```
T0 = 1;
W0 = 2*pi/T0;
e = 0.02;
m = 1;
b = 2*e*W0*m;
X0 = 0.01;
dX0 = 0;
Ft = 0;
omiga = W0*(1-e^2)^0.5;
x = [];
x(1) = X0;
n = 1;
for t = 0:0.01:10*T0
    n = n + 1;
    x(n) = exp(-e*W0*t)*(X0*cos(omiga*t) + (e*W0*X0 + dX0)/
omiga*sin(omiga*t));
end
t = linspace(0,10*T0,n);
plot(t,x);
title('Analytique');
```



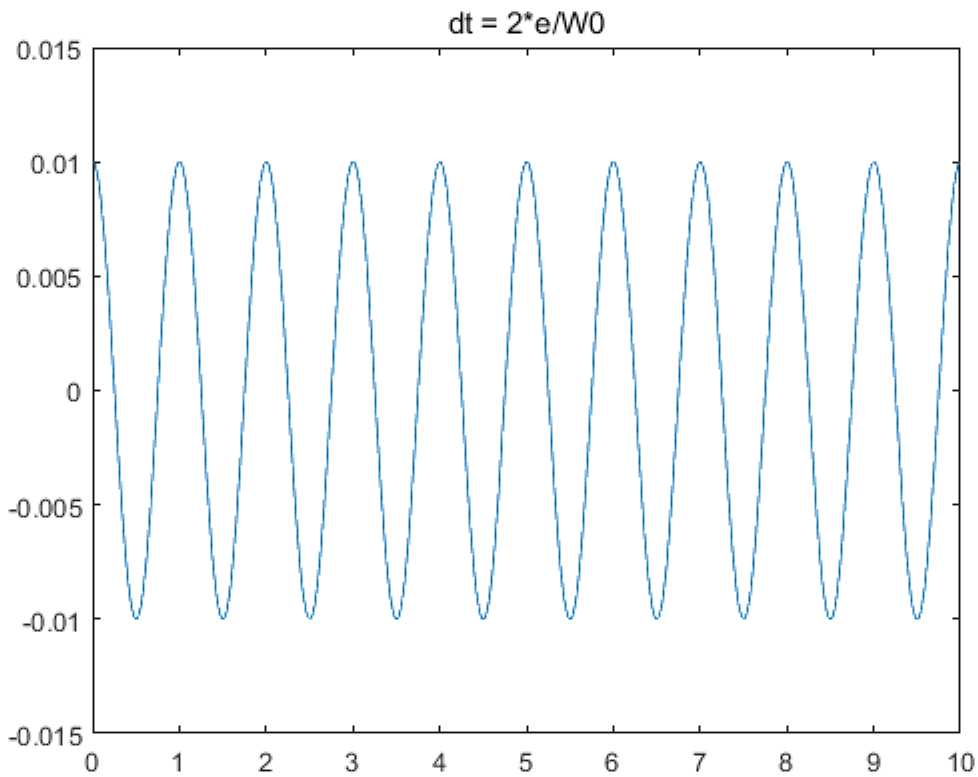
1.1a

```
t_1 = 2*e/W0;%0.0064
dt = 0.01;%>t_1
A = [1,dt;-dt*W0^2,1-2*dt*e*W0];
X = [X0;dX0];
x_ex = [];
dx_ex = [];
x_ex(1) = X0;
dx_ex(1) = dX0;
n = 1;
for t = 0:dt:10*T0
    n = n + 1;
    X = A*X;
    x_ex(n) = X(1,1);
    dx_ex(n) = X(2,1);
end
t = linspace(0,10*T0,n);
plot(t,x_ex);
title('dt > 2*e/W0');
%on peut voir que x diverge;
```



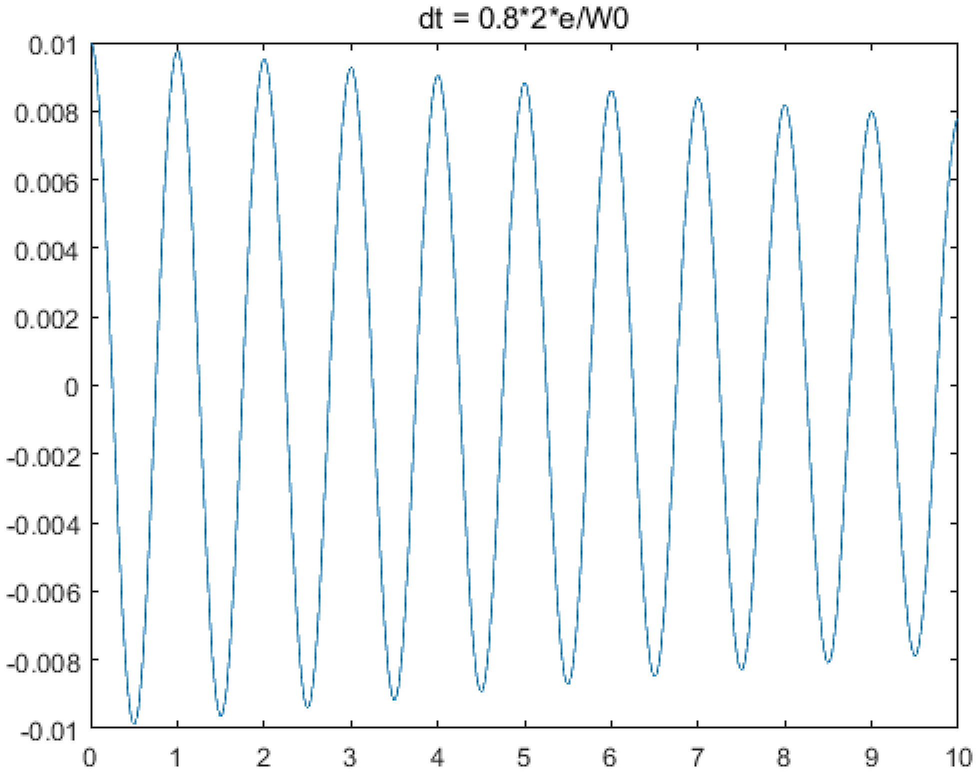
1.1b

```
dt = t_1;
A = [1,dt;-dt*W0^2,1-2*dt*e*W0];
X = [X0;dX0];
x_ex = [];
dx_ex = [];
x_ex(1) = X0;
dx_ex(1) = dX0;
n = 1;
for t = 0:dt:10*T0
    n = n + 1;
    X = A*X;
    x_ex(n) = X(1,1);
    dx_ex(n) = X(2,1);
end
t = linspace(0,10*T0,n);
plot(t,x_ex);
title('dt = 2*e/W0');
% x est sinusoidale, ni converge ni diverge.
```



1.1c

```
dt = 0.8*t_1;
A = [1,dt;-dt*W0^2,1-2*dt*e*W0];
X = [X0;dX0];
x_ex = [];
dx_ex = [];
x_ex(1) = X0;
dx_ex(1) = dX0;
n = 1;
for t = 0:dt:10*T0
    n = n + 1;
    X = A*X;
    x_ex(n) = X(1,1);
    dx_ex(n) = X(2,1);
end
t = linspace(0,10*T0,n);
plot(t,x_ex);
title('dt = 0.8*2*e/W0');
% x converge.
```



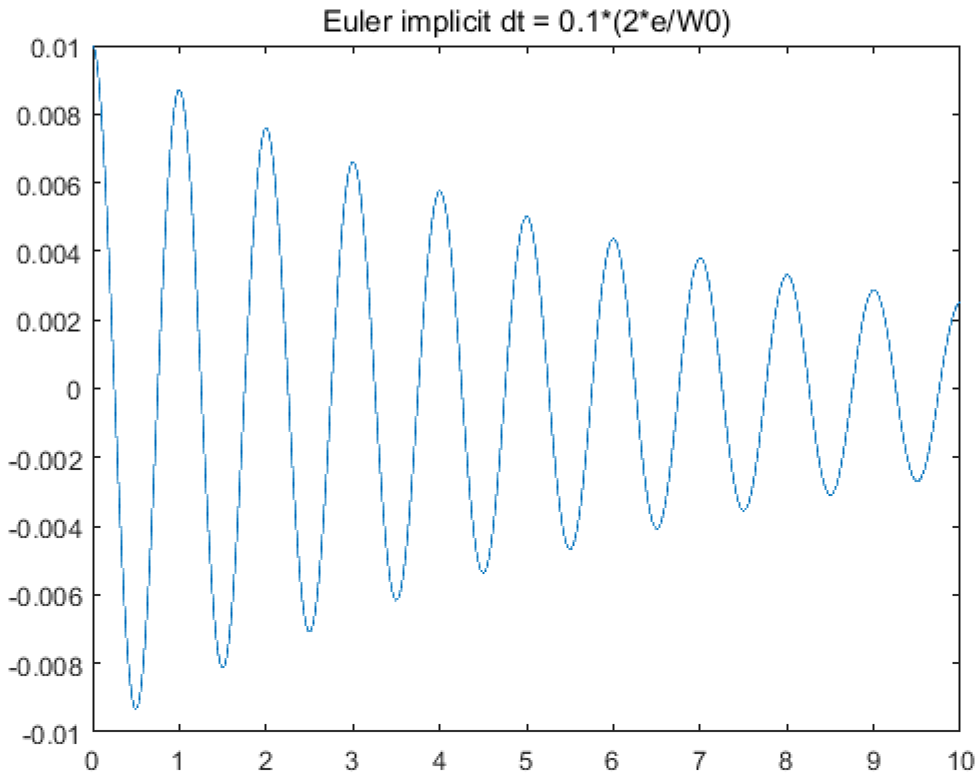
1.1d

```
%le rapport de dt/(2*e/W0) est un critere de la solution, et le
 rapport
%doit etre plus petit que 1 pour que la solution soit precis
```

1.2

```
t_1 = 2*e/W0;
dt = 0.1*t_1;
A_im = [1+2*dt*e*W0,dt;-dt*W0^2,1]/(1 + 2*dt*e*W0 + dt^2*W0^2);
X = [X0;dX0];
x_im = [];
dx_im = [];
x_im(1) = X0;
dx_im(1) = dX0;
n = 1;
for t = 0:dt:10*T0
    n = n + 1;
    X = A_im*X;
    x_im(n) = X(1,1);
    dx_im(n) = X(2,1);
end
t = linspace(0,10*T0,n);
plot(t,x_im);
```

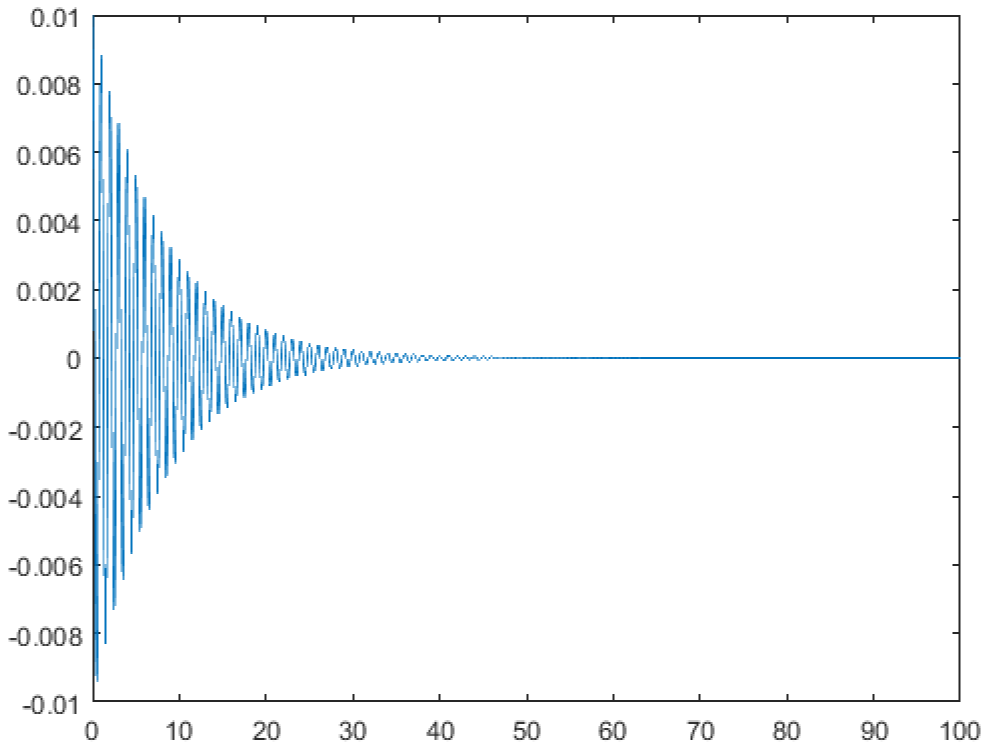
```
title('Euler implicit dt = 0.1*(2*e/W0)');
%x converge toujours en Euler implicite n'importe quel dt, si le
 rapport de dt/(2*e/W0)est plus petit, x converge moins vite, et dt
%augment, x converge plus vite.
```



1.3

```
dt = 0.01;
M = [0,1;-W0^2,-2*e*W0];
X = [X0;dX0];
x_rg = [];
dx_rg = [];
x_rg(1) = X0;
dx_rg(1) = dX0;
n = 1;
for t = 0:dt:100*T0
    n = n + 1;
    k1 = M * X;
    k2 = M * (X + k1 * dt/2);
    k3 = M * (X + k2 * dt/2);
    k4 = M * (X + k3 * dt);
    K = (k1 + 2*k2 + 2*k3 + k4)/6;
    X = X + K *dt;
    x_rg(n) = X(1,1);
    dx_rg(n) = X(2,1);
end
```

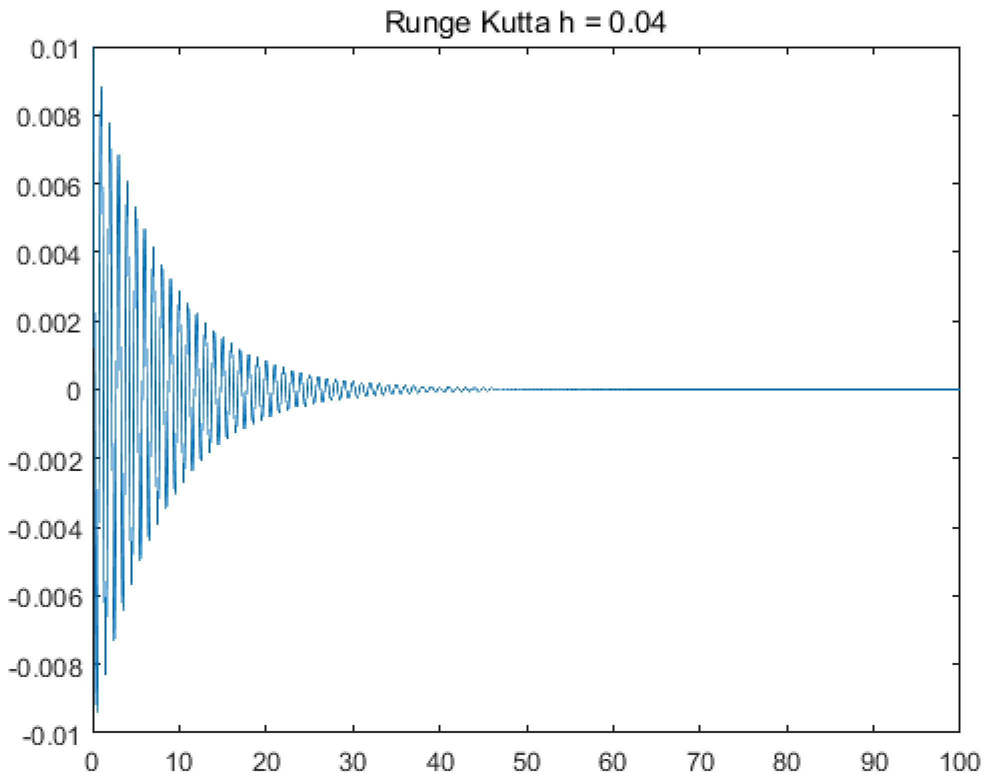
```
t = linspace(0,100*T0,n);
plot(t,x_rg);
```



1.3a h = 0.04

```
h = 0.04;
dt = h*2*2^0.5/W0;
M = [0,1;-W0^2,-2*e*W0];
X = [X0;dX0];
x_rg = [];
dx_rg = [];
x_rg(1) = X0;
dx_rg(1) = dX0;
n = 1;
for t = 0:dt:100*T0
    n = n + 1;
    k1 = M * X;
    k2 = M * (X + k1 * dt/2);
    k3 = M * (X + k2 * dt/2);
    k4 = M * (X + k3 * dt);
    K = (k1 + 2*k2 + 2*k3 + k4)/6;
    X = X + K *dt;
    x_rg(n) = X(1,1);
    dx_rg(n) = X(2,1);
end
t = linspace(0,100*T0,n);
```

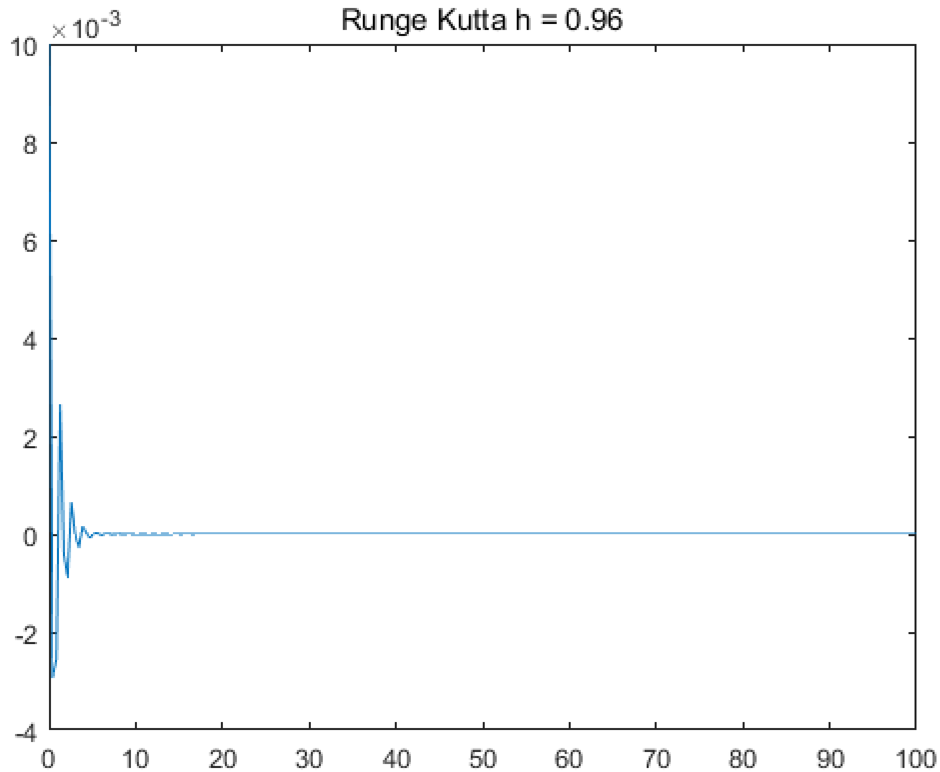
```
plot(t,x_rg);
title('Runge Kutta h = 0.04')
```



1.3a h = 0.96

```
h = 0.96;
dt = h*2*2^0.5/W0;
M = [0,1;-W0^2,-2*e*W0];
X = [X0;dX0];
x_rg = [];
dx_rg = [];
x_rg(1) = X0;
dx_rg(1) = dX0;
n = 1;
for t = 0:dt:100*T0
    n = n + 1;
    k1 = M * X;
    k2 = M * (X + k1 * dt/2);
    k3 = M * (X + k2 * dt/2);
    k4 = M * (X + k3 * dt);
    K = (k1 + 2*k2 + 2*k3 + k4)/6;
    X = X + K *dt;
    x_rg(n) = X(1,1);
    dx_rg(n) = X(2,1);
end
t = linspace(0,100*T0,n);
```

```
plot(t,x_rg);
title('Runge Kutta h = 0.96')
```

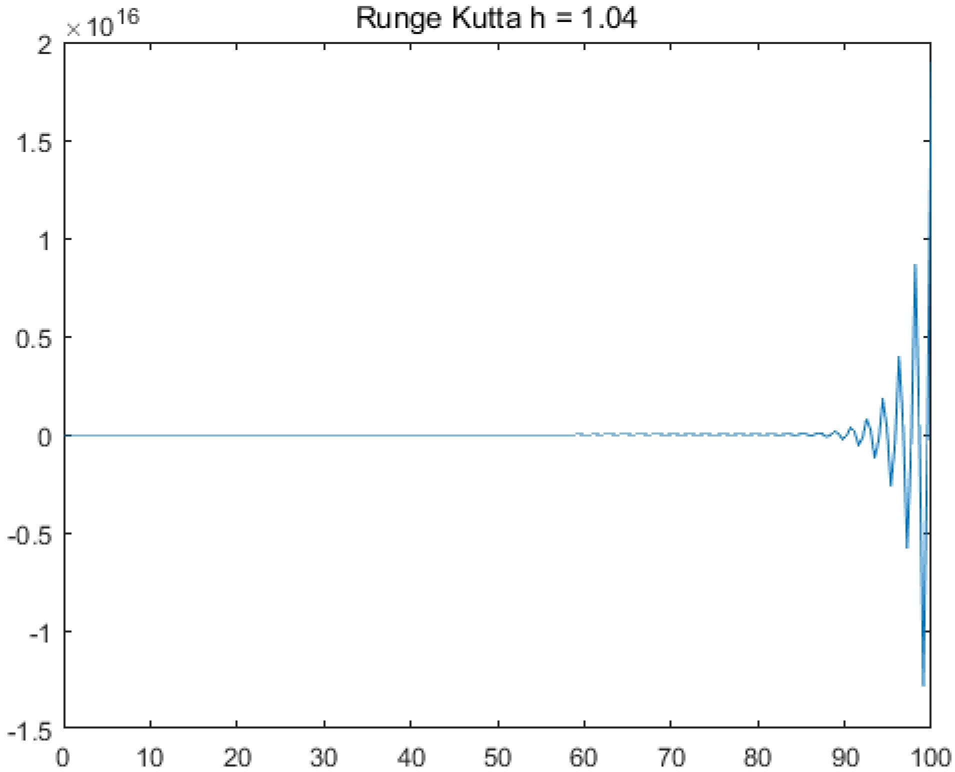


1.3a h =1.04

```
h = 1.04;
dt = h*2*2^0.5/W0;
M = [0,1;-W0^2,-2*e*W0];
X = [X0;dX0];
x_rg = [];
dx_rg = [];
x_rg(1) = X0;
dx_rg(1) = dX0;
n = 1;
for t = 0:dt:100*T0
    n = n + 1;
    k1 = M * X;
    k2 = M * (X + k1 * dt/2);
    k3 = M * (X + k2 * dt/2);
    k4 = M * (X + k3 * dt);
    K = (k1 + 2*k2 + 2*k3 + k4)/6;
    X = X + K *dt;
    x_rg(n) = X(1,1);
    dx_rg(n) = X(2,1);
end
t = linspace(0,100*T0,n);
```

```
plot(t,x_rg);
title('Runge Kutta h = 1.04');

%la stabilite de x depend de h, h augment, x est moins stable.
% h depasse un valeur critique, x diverge.
```



1.3b

```
hc = 1.0135;
%hmax = 1.0138 diverge un peu
%hmin = 1.0132 converge un peu
tc = hc * 2*2^0.5/W0; % tc = 0.4562
```

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