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Etude d'un double pendule avec l'hypothèse des petits mouvements

1. Résolution avec un schéma de NEWMARK explicite

$\beta = 0;\ \gamma = \frac{1}{2};$

D'après cela, on peut simplifier l'équation (2) (3) :

$$q_{n+1} = q_n + \Delta t * \dot{q}_n + \frac{\Delta t^2}{2} * \ddot{q}_n$$

$$\dot{q}_{n+1} = \dot{q}_n + \frac{\Delta t}{2} * \ddot{q}_n + \frac{\Delta t}{2} \ddot{q}_{n+1}$$

Selon l'équation(1), on obtient :

$$M\ddot{q}_n + Kq_n = F_0 sinwt \begin{matrix} a \\ a/\sqrt{2} \end{matrix}$$

$$\mathrm{M} = \mathrm{m}a^2 * \begin{matrix} 2 & 1 \\ 1 & 1 \end{matrix}$$

$$\mathrm{K} = \mathrm{mga} * \begin{matrix} 2 & 0 \\ 0 & 1 \end{matrix}$$

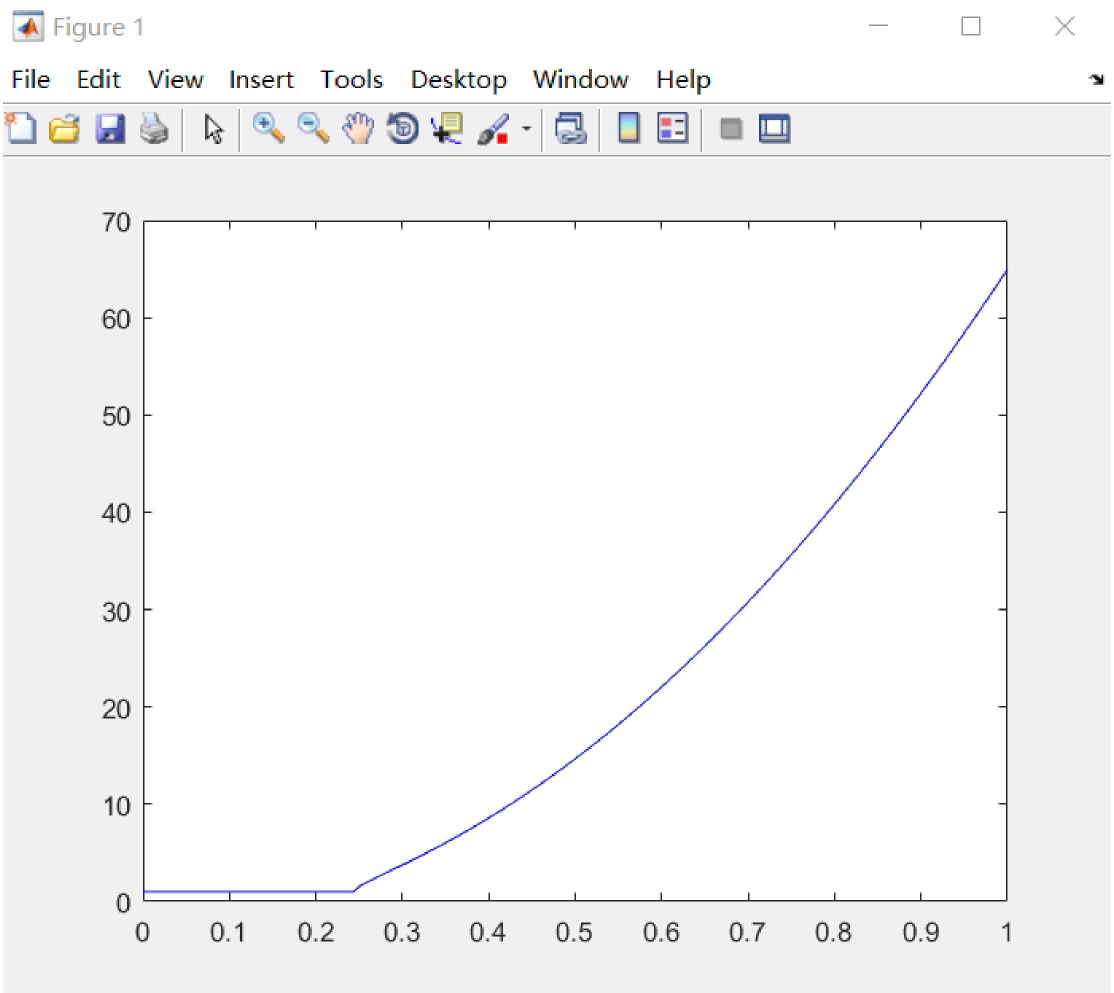
Il faut faire la discrétisation de $F_0 sinwt \rightarrow F_0 \sin(wn\Delta t)$.

1.1)

$$\mathrm{A} = \begin{matrix} Id_2 - \frac{\Delta t^2}{2} M^{-1}K & \Delta t \\ \frac{\Delta t^3}{4} M^{-1}KM^{-1}K - \Delta t M^{-1}K & Id_2 - \frac{\Delta t^2}{2} M^{-1}K \end{matrix}$$

1.2)Sur l'intervalle[0,1](pour Δt), on cherche la valeur propre plus grande :

```
A=[eye(2)-((delta_t^2)/2).*inv(M)*K,delta_t.*eye(2);
    ((delta_t^3)/4).*inv(M)*K*inv(M)*K-
delta_t.*inv(M)*K,eye(2)-((delta_t^2)/2).*inv(M)*K];
eig_max=[];
for delta_t=0:0.001:1;
    eig_max=[eig_max,max(abs(eig(eval(A))))];
end
t=0:0.001:1;
plot(t, eig_max,'b');
```



En agrandissant ce figure, je trouve le temps critique est environs 0.242s.

1.3） $M\ddot{q}_0 + Kq_0 = 0$

1.4) On combine les conditions et obtient :

$$q_{n+1} = q_n + \Delta t * \dot{q}_n + \frac{\Delta t^2}{2} * \ddot{q}_n$$

$$\dot{q}_{n+1} = \dot{q}_n + \frac{\Delta t}{2} * \ddot{q}_n + \frac{\Delta t}{2} \ddot{q}_{n+1}$$

$$M\ddot{q}_n + Kq_n = F_0 \sin(wn\Delta t) \begin{matrix} a \\ a/\sqrt{2} \end{matrix}$$

(resp. $M\ddot{q}_{n+1} + Kq_{n+1} = F_0 \sin\{w(n+1)\Delta t\} \begin{matrix} a \\ a/\sqrt{2} \end{matrix}$)

1.5)

```
dt=0.04;
n=floor(T0/dt);
nn=[1:n];
ti=dt.*nn;
vec=[a;a/sqrt(2)];
qj=zeros(2,n);
qj(:,1)=[theta10;theta20];
qj_derive=zeros(2,n);
qj_derive(:,1)=[theta10_derive;theta20_derive];
qj_derive2=zeros(2,n);
for i=1:n-1
    qj_derive2(:,i)=-
inv(M)*K*qj(:,i)+(F0*sin(w*i*dt)).*inv(M)*vec;

qj(:,i+1)=qj(:,i)+dt.*qj_derive(:,i)+((dt^2)/2).*qj_deriv
e2(:,i);
    qj_derive2(:,i+1)=-
inv(M)*K*qj(:,i+1)+(F0*sin(w*(i+1)*dt)).*inv(M)*vec;

qj_derive(:,i+1)=qj_derive(:,i)+(dt/2).*qj_derive2(:,i)+(
dt/2).*qj_derive2(:,i+1);
end
plot(ti,qj);
```

1.6)

	0	Δt	$2\Delta t$	0.5
q	0	-0.0370	-0.0730	0.0278
$\dot{q}$	-1.8600	-1.8247	-1.7610	1.7209

$\ddot{q}$	1.0383	2.4854	3.8885	-1.3582

2. Résolution avec un schéma de NEWMARK implicite

$\beta = 0.25$

$\gamma = 0.5$

$q_{n+1} = q_n + \Delta t * \dot{q}_n + 0.25 * \Delta t^2 \ddot{q}_n + 0.25 * \Delta t^2 \ddot{q}_{n+1}$

$\dot{q}_{n+1} = \dot{q}_n + 0.5 * \Delta t \ddot{q}_n + 0.5 * \Delta t \ddot{q}_{n+1}$

2.1)

la matrice d'amplification A est une matrice 2*2

$A(1,1) = inv(1 + 0.25\Delta t^2 M^{-1}K) * (1 - 0.25\Delta t^2 M^{-1}K)$

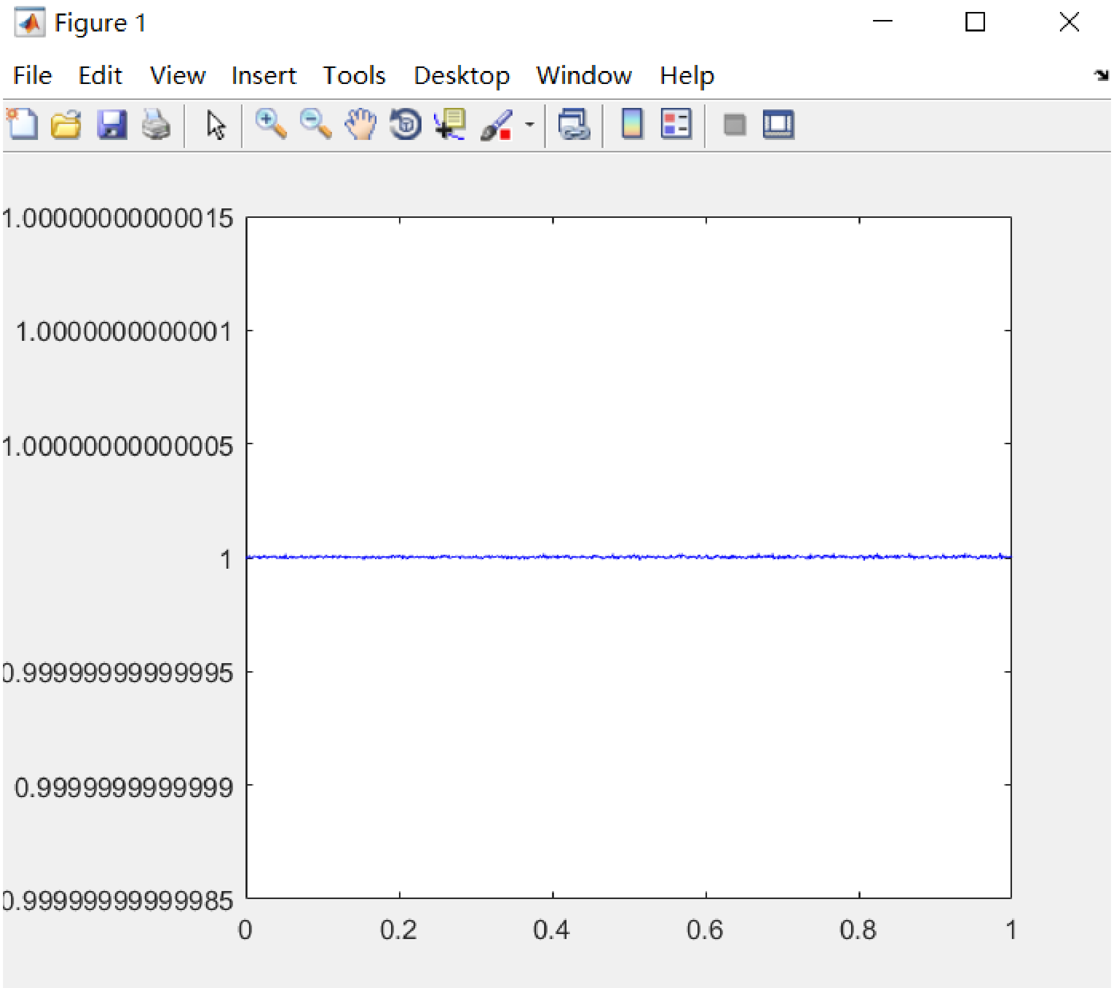
$A(1,2) = inv(1 + 0.25\Delta t^2 M^{-1}K) * \Delta t$

$$A(2,1) = -0.5\Delta t M^{-1}\ddot{K} * (1 + inv(1 + 0.25\Delta t^2 M^{-1}K) * (1 - 0.25\Delta t^2 M^{-1}K))$$

$A(2,2) = 1 - 0.5\Delta t^2 M^{-1}K * inv(1 + 0.25\Delta t^2 M^{-1}K)$

2.2)

```
for delta_t=0:0.001:1;
    eig_max=[eig_max,max(abs(eig(eval(A))))];
end
t=0:0.001:1;
plot(t, eig_max,'b');
```



Il reste Presque toujours 1.

2.3) $M\ddot{q}_0 + Kq_0 = 0$

2.4)

$$q_{n+1} = q_n + \Delta t * \dot{q}_n + 0.25 * \Delta t^2 \ddot{q}_n + 0.25 * \Delta t^2 \ddot{q}_{n+1}$$

$$\dot{q}_{n+1} = \dot{q}_n + 0.5 * \Delta t \ddot{q}_n + 0.5 * \Delta t \ddot{q}_{n+1}$$

$$M\ddot{q}_n + Kq_n = F_0 \sin(wn\Delta t)_{a/\sqrt{2}}^{a}$$

(resp. $M\ddot{q}_{n+1} + Kq_{n+1} = F_0 \sin\{w(n+1)\Delta t\}_{a/\sqrt{2}}^{a}$)

D'après la matrice a'amplification, les valeurs vont converger plus rapide que schéma explicite. c'est facile de voir qu'il est stable.

2.5)

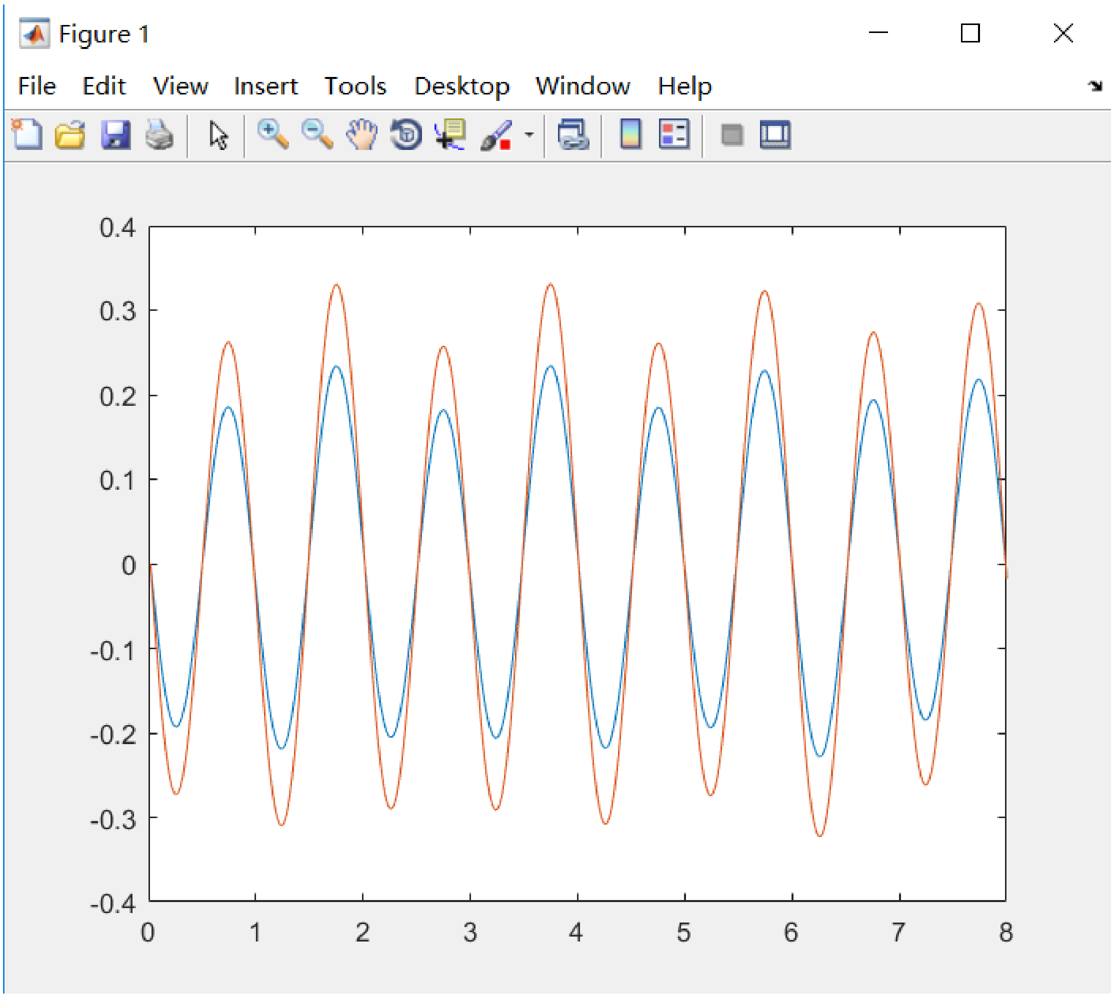
```
clear all
m=2;
a=0.5;
g=9.81;
F0=20;
w=2*pi;
theta10=0;
theta20=0;
theta10_derive=-1.31519275;
theta20_derive=-1.85996342 ;
T0=8;
M=(m*a^2).*[2,1;1,1];
K=(m*g*a).*[2,0;0,1];
eig_max=[];
syms delta_t
A=[inv(eye(2)+(0.25*delta_t^2).*inv(M)*K)*(eye(2)-
(0.25*(delta_t^2)).*inv(M)*K),delta_t.*inv(eye(2)+(0.25*d
elta_t^2).*inv(M)*K);
    (-
0.5*delta_t).*inv(M)*K*(eye(2)+inv(eye(2)+(0.25*delta_t^2
).*inv(M)*K)*(eye(2)-(0.25*delta_t^2).*inv(M)*K)),eye(2)-
(0.5*delta_t^2).*inv(M)*K*inv(eye(2)+(0.25*delta_t^2).*in
v(M)*K)];
% for delta_t=0:0.001:1;
%     eig_max=[eig_max,max(abs(eig(eval(A))))];
% end
% t=0:0.001:1;
% plot(t, eig_max,'b');
dt=0.02;
n=floor(T0/dt);
nn=[1:n];
ti=dt.*nn;
vec=[a;a/sqrt(2)];
qj=zeros(2,n);
qj(:,1)=[theta10;theta20];
qj_derive=zeros(2,n);
qj_derive(:,1)=[theta10_derive;theta20_derive];
qj_derive2=zeros(2,n);
for i=1:n-1
%     qj_derive2(:,i)=-
inv(M)*K*qj(:,i)+(F0*sin(w*i*dt)).*inv(M)*vec;
%
```

```
qj(:,i+1)=qj(:,i)+dt.*qj_derive(:,i)+((dt^2)/2).*qj_deriv
e2(:,i);
%     qj_derive2(:,i+1)=-
inv(M)*K*qj(:,i+1)+(F0*sin(w*(i+1)*dt)).*inv(M)*vec;
%
qj_derive(:,i+1)=qj_derive(:,i)+(dt/2).*qj_derive2(:,i)+(
dt/2).*qj_derive2(:,i+1);
     qj_derive2(:,i)=-
inv(M)*K*qj(:,i)+(F0*sin(w*i*dt)).*inv(M)*vec;

qj(:,i+1)=inv(eye(2)+(0.25*dt^2).*inv(M)*K)*(qj(:,i)+dt.*
qj_derive(:,i)+(0.25*(dt^2)).*qj_derive2(:,i)+(0.25*(dt^2
)*F0*sin(w*(i+1)*dt)).*inv(M)*vec);
     qj_derive2(:,i+1)=-
inv(M)*K*qj(:,i+1)+(F0*sin(w*(i+1)*dt)).*inv(M)*vec;

qj_derive(:,i+1)=qj_derive(:,i)+(dt/2).*qj_derive2(:,i)+(
dt/2).*qj_derive2(:,i+1);
end
plot(ti,qj);
```

2.6）



	0	Δt	$2\Delta t$	0.5
q	0	-0.0368	-0.0727	0.0268
$\dot{q}$	-1.8600	-1.8247	-1.7611	1.7189
$\ddot{q}$	1.0383	2.4837	3.8853	-1.3465