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Oscillateur non linéaire à une degré de liberté

1.Résolution avec un schéma de NEWMARK explicite

1.1

$\beta = 0; \gamma = \frac{1}{2}$; Donc on a :

$$q_{j+1} = q_j + \Delta t * \dot{q}_j + \frac{\Delta t^2}{2} * \ddot{q}_j$$

$$\dot{q}_{j+1} = \dot{q}_j + \frac{\Delta t}{2} * \ddot{q}_j + \frac{\Delta t}{2} \ddot{q}_{j+1}$$

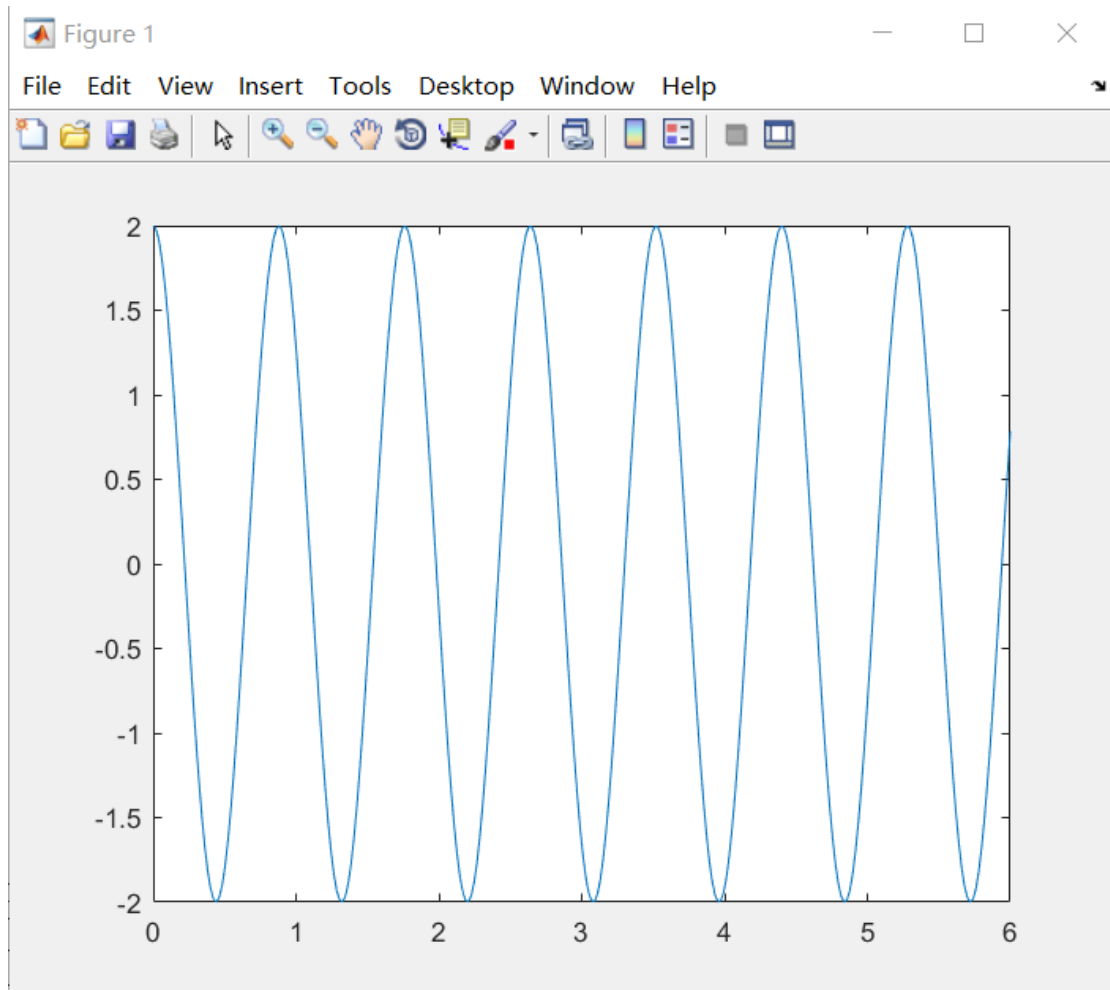
De plus, $\ddot{q}_{j+1} = -w_0^2 q_{j+1} (1 + a * q_{j+1}^2)$

1.2

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1 - clear all;
2 - w0=2*pi;
3 - a=0.1;
4 - T0=6;
5 - delta_t=0.02;
6 - m=1;% on suppose m=1kg
7 - n=floor(T0/delta_t);
8 - ti=linspace(0,T0,n);
9 - ti=ti';
10 - qj=zeros(n,1);
11 - qj_derive=zeros(n,1);
12 - qj_derive2=zeros(n,1);
13 - qj(1,1)=2;
14 - qj_derive(1,1)=0;
15 - qj_derive2(1,1)=-w0*w0*qj(1,1)*(1+a*qj(1,1)^2);
16 - for i=1:n-1
17 -     qj(i+1,1)=qj(i,1)+delta_t*qj_derive(i,1)+(delta_t^2)*qj_derive2(i,1)/2;
18 -     qj_derive2(i+1,1)=-w0*w0*qj(i+1,1)*(1+a*qj(i+1,1)^2);
19 -     qj_derive(i+1,1)=qj_derive(i,1)+delta_t*qj_derive2(i,1)/2+delta_t*qj_derive2(i+1,1)/2;
20 - end
21 - plot(ti,qj);
22
23
24

```



1.3

	0	Δt	$2\Delta t$	T_0
$q(t)$	2	1.9779	1.9123	0.7837

2. Résolution avec un schéma de NEWMARK implicite

$$q_{j+1} = q_j + \Delta t * \dot{q}_j + 0.25 * \Delta t^2 \ddot{q}_j + 0.25 * \Delta t^2 \ddot{q}_{j+1}$$

$$\dot{q}_{j+1} = \dot{q}_j + 0.5 * \Delta t \ddot{q}_j + 0.5 * \Delta t \ddot{q}_{j+1}$$

2.1. il faut minimiser la valeur :

$$w_0^2 q_{j+1} (1 + a * q_{j+1}^2)$$

2.2

$$\Delta q_{j+1} = \beta * \Delta t^2 * \Delta \ddot{q}_{j+1} \text{ et } \Delta \dot{q}_{j+1} = \gamma * \Delta t * \Delta \ddot{q}_{j+1}$$

En considérant la correction, l'expression devient :

$$\ddot{q}_{j+1}^* + \Delta \ddot{q}_{j+1} + \omega^2 (q_{j+1}^* + \Delta q_{j+1}) (1 + a (q_{j+1}^* + \Delta q_{j+1})^2) = 0$$

2.3 En fonction de ppt, on sait qu'il faut estimer en continu la valeur de :

$$w_0^2 q_{j+1}^* (1 + a * q_{j+1}^{*2})$$

Et

$$\Delta \ddot{q}_{n+1} = - \frac{f(\ddot{q}_{n+1}^*, \dot{q}_{n+1}^*, q_{n+1}^*)}{\frac{\partial f}{\partial \ddot{q}_{n+1}^*} + \frac{\partial f}{\partial q_{n+1}^*} \beta \Delta t^2}$$

Où $f(\ddot{q}_{j+1}^*, \dot{q}_{j+1}^*, q_{j+1}^*) = \ddot{q}_{j+1}^* + \omega^2 * q_{j+1}^* * (1 + a * q_{j+1}^{*2})$

En simplifiant l'expression, on obtient :

$$\Delta \ddot{q}_{j+1} = \frac{-\ddot{q}_{j+1}^* - \omega^2 * q_{j+1}^* * (1 + a * q_{j+1}^{*2})}{(1 + \omega^2 * (1 + 3a q_{j+1}^{*2}) * \beta * \Delta t^2)}$$

```
clear all;  
w0=2*pi;  
a=0.1;
```

```

T0=6;
delta_t=0.02;
beta=0.5;
gamma=0.25;
m=1;
k=m*(w0^2);
n=floor(T0/delta_t);
t=linspace(0,T0,n);
ti=t';
qj=zeros(n,1);
qj_derive=zeros(n,1);
qj_derive2=zeros(n,1);
delta_qj=zeros(n,1);
delta_qj_derive=zeros(n,1);
delta_qj_derive2=zeros(n,1);
qj(1,1)=2;
qj_derive(1,1)=0;
qj_derive2(1,1)=0;
for i=1:n-1
    qj_derive2(i+1,1)=0;
    qj_derive(i+1,1)=qj_derive(i,1)+delta_t*(1-
gamma)*qj_derive2(i,1);

qj(i+1,1)=qj(i,1)+delta_t*qj_derive(i,1)+(delta_t^2)*(0.5
-beta)*qj_derive2(i,1);

while(abs(qj_derive2(i+1,1)+(w0^2)*qj(i+1,1)*(1+a*qj(i+1,
1)^2))>0.01)
    delta_qj_derive2(i+1,1)=(-qj_derive2(i+1,1)-
(w0^2)*qj(i+1,1)*(1+a*qj(i+1,1)^2))/(1+(w0^2)*(1+3*a*qj(i
+1,1)^2)*beta*delta_t^2);

delta_qj_derive(i+1,1)=gamma*delta_t*delta_qj_derive2(i+1
,1);

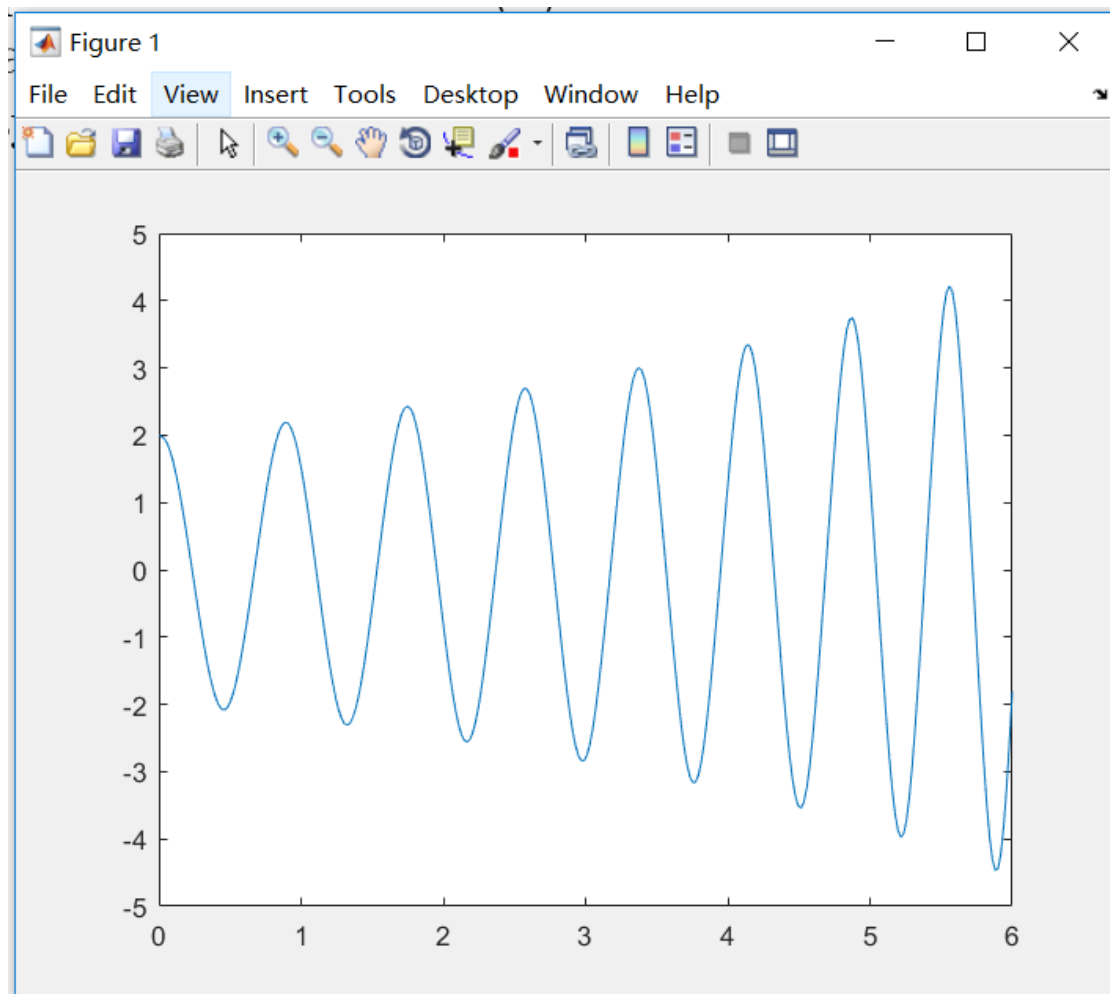
delta_qj(i+1,1)=beta*(delta_t^2)*delta_qj_derive2(i+1,1);

qj_derive2(i+1,1)=qj_derive2(i+1,1)+delta_qj_derive2(i+1,
1);

qj_derive(i+1,1)=qj_derive(i+1,1)+delta_qj_derive(i+1,1);
    qj(i+1,1)=qj(i+1,1)+delta_qj(i+1,1);
end
end

```

```
plot(ti,qj);
```



2.4

	0	Δt	$2\Delta t$	T_0
q(t)	2	1.9783	1.9462	-1.8076

3. Energie mécanique

3.1. $E = m * \dot{q}^2 / 2 + \frac{kq^2}{2} + \frac{kaq^4}{4}$

3.2 schéma explicite

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8 - n=floor(T0/delta_t);
9 - ti=linspace(0, T0, n);
0 - ti=ti';
1 - qj=zeros(n, 1);
2 - qj_derive=zeros(n, 1);
3 - qj_derive2=zeros(n, 1);
4 - qj(1, 1)=2;
5 - qj_derive(1, 1)=0;
6 - qj_derive2(1, 1)=-w0*w0*qj(1, 1)*(1+a*qj(1, 1)^2);
7 - E=zeros(n, 1);
8 - E(1, 1)=0;
9 - for i=1:n-1
0 -     qj(i+1, 1)=qj(i, 1)+delta_t*qj_derive(i, 1)+(delta_t^2)*qj_derive2(i, 1)/2;
1 -     qj_derive2(i+1, 1)=-w0*w0*qj(i+1, 1)*(1+a*qj(i+1, 1)^2);
2 -     qj_derive(i+1, 1)=qj_derive(i, 1)+delta_t*qj_derive2(i, 1)/2+delta_t*qj_derive2(i+1, 1)/2;
3 -     E(i+1)=0.5*m*(qj_derive(i+1)^2)+0.5*k*(qj(i+1)^2)+0.25*k*a*(qj(i+1)^4);
4 - end
5 - plot(ti(2:n), E(2:n));
6
7

```

Schéma implicite

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22 - E=zeros(n, 1);
23 - E(1, 1)=0;
24 - for i=1:n-1
25 -     qj_derive2(i+1, 1)=0;
26 -     qj_derive(i+1, 1)=qj_derive(i, 1)+delta_t*(1-gamma)*qj_derive2(i, 1);
27 -     qj(i+1, 1)=qj(i, 1)+delta_t*qj_derive(i, 1)+(delta_t^2)*(0.5-beta)*qj_derive2(i, 1);
28 -     while(abs(qj_derive2(i+1, 1)+(w0^2)*qj(i+1, 1)*(1+a*qj(i+1, 1)^2))>0.01)
29 -         delta_qj_derive2(i+1, 1)=(-qj_derive2(i+1, 1)-(w0^2)*qj(i+1, 1)*(1+a*qj(i+1, 1)^2))/(1+(w0^2)*(1+3*a*qj(i+1, 1)^2)*beta*delta_t^2);
30 -         delta_qj_derive(i+1, 1)=gamma*delta_t*delta_qj_derive2(i+1, 1);
31 -         delta_qj(i+1, 1)=beta*(delta_t^2)*delta_qj_derive2(i+1, 1);
32 -         qj_derive2(i+1, 1)=qj_derive2(i+1, 1)+delta_qj_derive2(i+1, 1);
33 -         qj_derive(i+1, 1)=qj_derive(i+1, 1)+delta_qj_derive(i+1, 1);
34 -         qj(i+1, 1)=qj(i+1, 1)+delta_qj(i+1, 1);
35 -     end
36 -     E(i+1)=0.5*m*(qj_derive(i+1)^2)+0.5*k*(qj(i+1)^2)+0.25*k*a*(qj(i+1)^4);
37 - end
38 - % plot(ti(2:n), E(2:n));
39 - plot(ti, qj);|
40
41

```

3.3

Si on prend $\Delta t = 0.02s$:

Schéma explicite

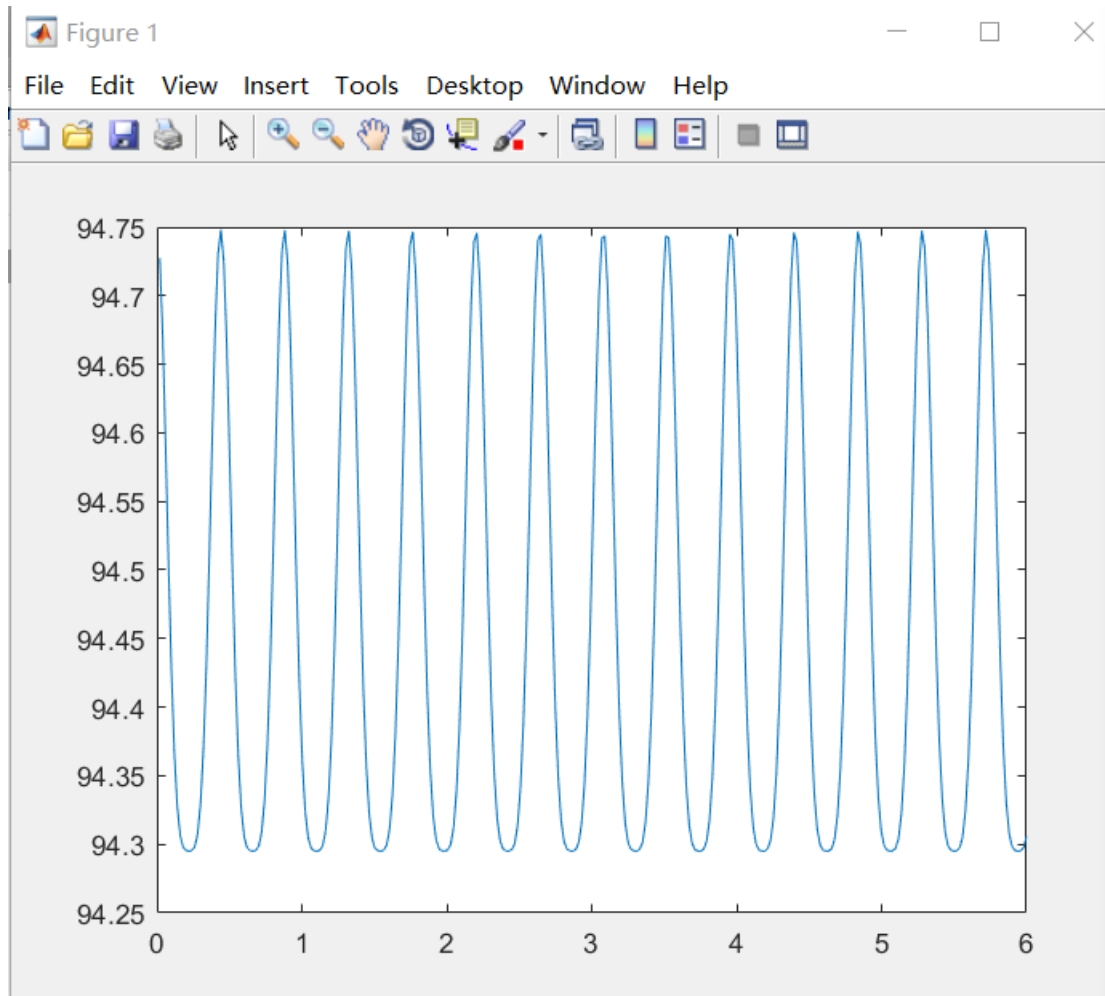
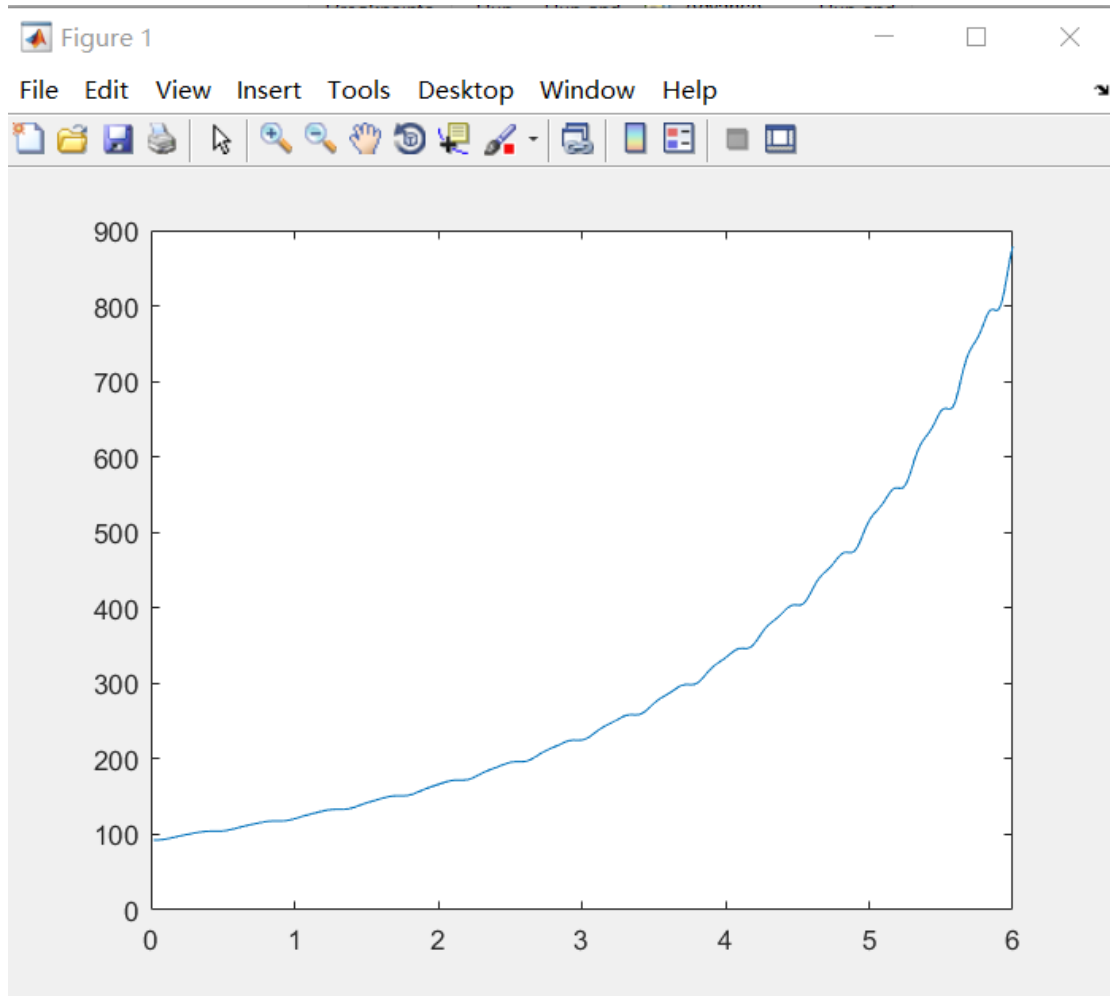


Schéma implicite



La valeur absolue de l'énergie de schéma implicite va devenir de plus en plus grande.