**Nom :Sacha**

**Numéro :SY1924144**

Oscillateur non linéaire à une degré de liberté

1.Résolution avec un schéma de NEWMARK explicite

1.1

$\beta = 0;\ \gamma = \frac{1}{2};$ Donc on a :

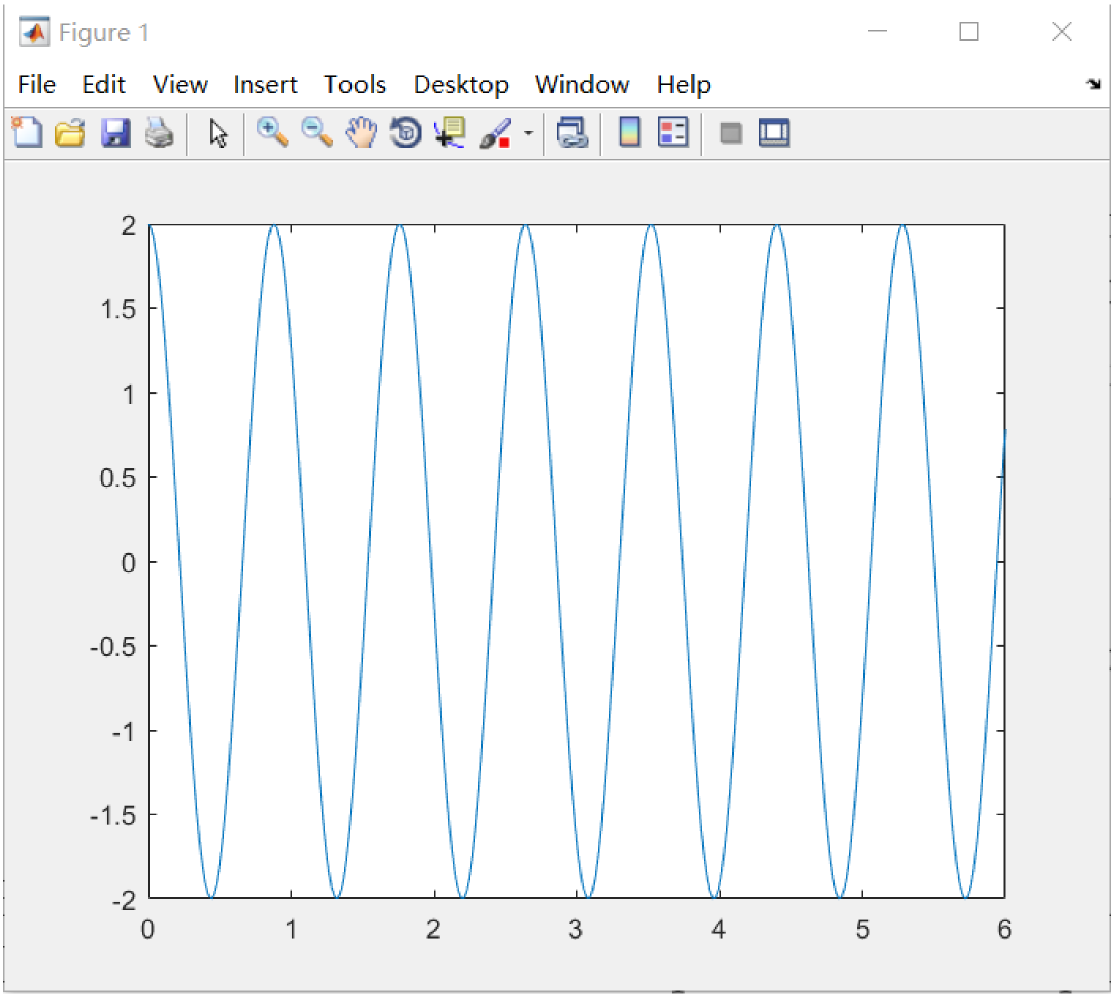
$$q_{j+1} = q_j + \Delta t * \dot{q}_j + \frac{\Delta t^2}{2} * \ddot{q}_j$$

$$\dot{q}_{j+1} = \dot{q}_j + \frac{\Delta t}{2} * \ddot{q}_j + \frac{\Delta t}{2} \ddot{q}_{j+1}$$

De plus, $\ddot{q}_{j+1} = -{w_0}^2 q_{j+1}(1 + a * {q_{j+1}}^2)$

1.2

```matlab
clear all;
w0=2*pi;
a=0.1;
T0=6;
delta_t=0.02;
m=1;% on suppose m=1kg
n=floor(T0/delta_t);
ti=linspace(0,T0,n);
ti=ti';
qj=zeros(n,1);
qj_derive=zeros(n,1);
qj_derive2=zeros(n,1);
qj(1,1)=2;
qj_derive(1,1)=0;
qj_derive2(1,1)=-w0*w0*qj(1,1)*(1+a*qj(1,1)^2);
for i=1:n-1
    qj(i+1,1)=qj(i,1)+delta_t*qj_derive(i,1)+(delta_t^2)*qj_derive2(i,1)/2;
    qj_derive2(i+1,1)=-w0*w0*qj(i+1,1)*(1+a*qj(i+1,1)^2);
    qj_derive(i+1,1)=qj_derive(i,1)+delta_t*qj_derive2(i,1)/2+delta_t*qj_derive2(i+1,1)/2;
end
plot(ti,qj);
```




1.3

| | 0 | Δt | 2Δt | $T_0$ |
|---|---|---|---|---|
| q(t) | 2 | 1.9779 | 1.9123 | 0.7837 |

## 2.Résolution avec un schéma de NEWMARK implicite

$$q_{j+1} = q_j + \Delta t * \dot{q}_j + 0.25 * \Delta t^2 \ddot{q}_j + 0.25 * \Delta t^2 \ddot{q}_{j+1}$$

$$\dot{q}_{j+1} = \dot{q}_j + 0.5 * \Delta t \ddot{q}_j + 0.5 * \Delta t \ddot{q}_{j+1}$$

2.1.il faut minimiser la valeur :

$$w_0{}^2 q_{j+1}(1 + a * q_{j+1}{}^2)$$

2.2

$\Delta q_{j+1} = \beta * \Delta t^2 * \Delta\ddot{q}_{j+1}$ *et* $\Delta\dot{q}_{j+1} = \gamma * \Delta t * \Delta\ddot{q}_{j+1}$

En considérant la correction,l'expression devient :

$\ddot{q}_{j+1}{}^{*} + \Delta\ddot{q}_{j+1} + \omega^2(q_{j+1}{}^{*} + \Delta q_{j+1})(1 + a(q_{j+1}{}^{*} +$

$\Delta q_{j+1})^2)$=*0*

2.3 En fonction de ppt, on sait qu'il faut estimer en continu la valeur de :

$w_0{}^2 q_{j+1}^{*}(1 + a * q_{j+1}^{*}{}^2)$

Et

$$\Delta\ddot{q}_{n+1} = -\frac{f\left(\ddot{q}^{*}_{n+1}, \dot{q}^{*}_{n+1}, q^{*}_{n+1}\right)}{\dfrac{\partial f}{\partial \ddot{q}^{*}_{n+1}} + \dfrac{\partial f}{\partial q^{*}_{n+1}}\beta\Delta t^2}$$

Où $f\left(\ddot{q}_{j+1}{}^{*}, \dot{q}_{j+1}{}^{*}, q_{j+1}{}^{*}\right) = \ddot{q}_{j+1}{}^{*} + \omega^2 * q_{j+1}{}^{*} * (1 + a * q_{j+1}{}^{*2})$

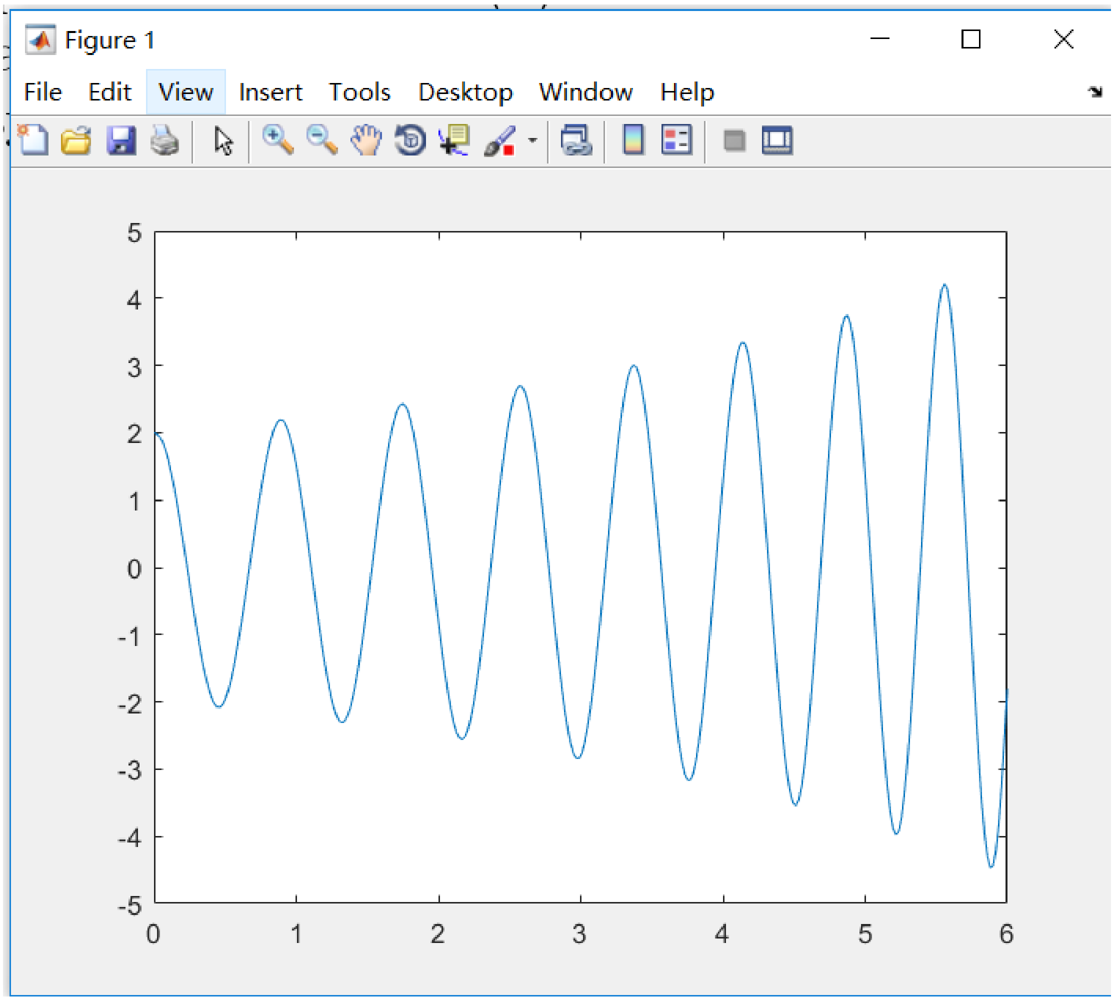
En simplifiant l'expression, on obtient :

$$\Delta\ddot{q}_{j+1} = \frac{-\ddot{q}_{j+1}{}^{*} - \omega^2 * q_{j+1}{}^{*} * (1 + a * q_{j+1}{}^{*2})}{(1 + \omega^2 * (1 + 3a q_{j+1}{}^{*2}) * \beta * \Delta t^2)}$$

```
clear all;
w0=2*pi;
a=0.1;
```

```
T0=6;
delta_t=0.02;
beta=0.5;
gamma=0.25;
m=1;
k=m*(w0^2);
n=floor(T0/delta_t);
t=linspace(0,T0,n);
ti=t';
qj=zeros(n,1);
qj_derive=zeros(n,1);
qj_derive2=zeros(n,1);
delta_qj=zeros(n,1);
delta_qj_derive=zeros(n,1);
delta_qj_derive2=zeros(n,1);
qj(1,1)=2;
qj_derive(1,1)=0;
qj_derive2(1,1)=0;
 for i=1:n-1
   qj_derive2(i+1,1)=0;
   qj_derive(i+1,1)=qj_derive(i,1)+delta_t*(1-
gamma)*qj_derive2(i,1);

qj(i+1,1)=qj(i,1)+delta_t*qj_derive(i,1)+(delta_t^2)*(0.5
-beta)*qj_derive2(i,1);

while(abs(qj_derive2(i+1,1)+(w0^2)*qj(i+1,1)*(1+a*qj(i+1,
1)^2))>0.01)
        delta_qj_derive2(i+1,1)=(-qj_derive2(i+1,1)-
(w0^2)*qj(i+1,1)*(1+a*qj(i+1,1)^2))/(1+(w0^2)*(1+3*a*qj(i
+1,1)^2)*beta*delta_t^2);

delta_qj_derive(i+1,1)=gamma*delta_t*delta_qj_derive2(i+1
,1);

delta_qj(i+1,1)=beta*(delta_t^2)*delta_qj_derive2(i+1,1);

qj_derive2(i+1,1)=qj_derive2(i+1,1)+delta_qj_derive2(i+1,
1);

qj_derive(i+1,1)=qj_derive(i+1,1)+delta_qj_derive(i+1,1);
        qj(i+1,1)=qj(i+1,1)+delta_qj(i+1,1);
    end
 end
```

```
plot(ti,qj);
```




2.4

| | 0 | Δt | 2Δt | $T_0$ |
|---|---|---|---|---|
| q(t) | 2 | 1.9783 | 1.9462 | -1.8076 |

3. Energie mécanique

3.1. E=$m * \dot{q}^2/2 + \frac{kq^2}{2} + \frac{kaq^4}{4}$

3.2 schéma explicite

```
n=floor(T0/delta_t);
ti=linspace(0,T0,n);
ti=ti';
qj=zeros(n,1);
qj_derive=zeros(n,1);
qj_derive2=zeros(n,1);
qj(1,1)=2;
qj_derive(1,1)=0;
qj_derive2(1,1)=-w0*w0*qj(1,1)*(1+a*qj(1,1)^2);
E=zeros(n,1);
E(1,1)=0;
for i=1:n-1
    qj(i+1,1)=qj(i,1)+delta_t*qj_derive(i,1)+(delta_t^2)*qj_derive2(i,1)/2;
    qj_derive2(i+1,1)=-w0*w0*qj(i+1,1)*(1+a*qj(i+1,1)^2);
    qj_derive(i+1,1)=qj_derive(i,1)+delta_t*qj_derive2(i,1)/2+delta_t*qj_derive2(i+1,1)/2;
    E(i+1)=0.5*m*(qj_derive(i+1)^2)+0.5*k*(qj(i+1)^2)+0.25*k*a*(qj(i+1)^4);
end
plot(ti(2:n),E(2:n));
```

## Schéma implicite

```
E=zeros(n,1);
E(1,1)=0;
for i=1:n-1
  qj_derive2(i+1,1)=0;
  qj_derive(i+1,1)=qj_derive(i,1)+delta_t*(1-gamma)*qj_derive2(i,1);
  qj(i+1,1)=qj(i,1)+delta_t*qj_derive(i,1)+(delta_t^2)*(0.5-beta)*qj_derive2(i,1);
   while(abs(qj_derive2(i+1,1)+(w0^2)*qj(i+1,1)*(1+a*qj(i+1,1)^2))>0.01)
       delta_qj_derive2(i+1,1)=(-qj_derive2(i+1,1)-(w0^2)*qj(i+1,1)*(1+a*qj(i+1,1)^2))/(1+(w0^2)*(1+3*a*qj(i+1,1)^2)*beta*delta_t^2);
       delta_qj_derive(i+1,1)=gamma*delta_t*delta_qj_derive2(i+1,1);
       delta_qj(i+1,1)=beta*(delta_t^2)*delta_qj_derive2(i+1,1);
       qj_derive2(i+1,1)=qj_derive2(i+1,1)+delta_qj_derive2(i+1,1);
       qj_derive(i+1,1)=qj_derive(i+1,1)+delta_qj_derive(i+1,1);
       qj(i+1,1)=qj(i+1,1)+delta_qj(i+1,1);
   end
    E(i+1)=0.5*m*(qj_derive(i+1)^2)+0.5*k*(qj(i+1)^2)+0.25*k*a*(qj(i+1)^4);
end
% plot(ti(2:n),E(2:n));
plot(ti,qj);
```

# 3.3

Si on prend $\Delta t = 0.02s$ :

## Schéma explicite

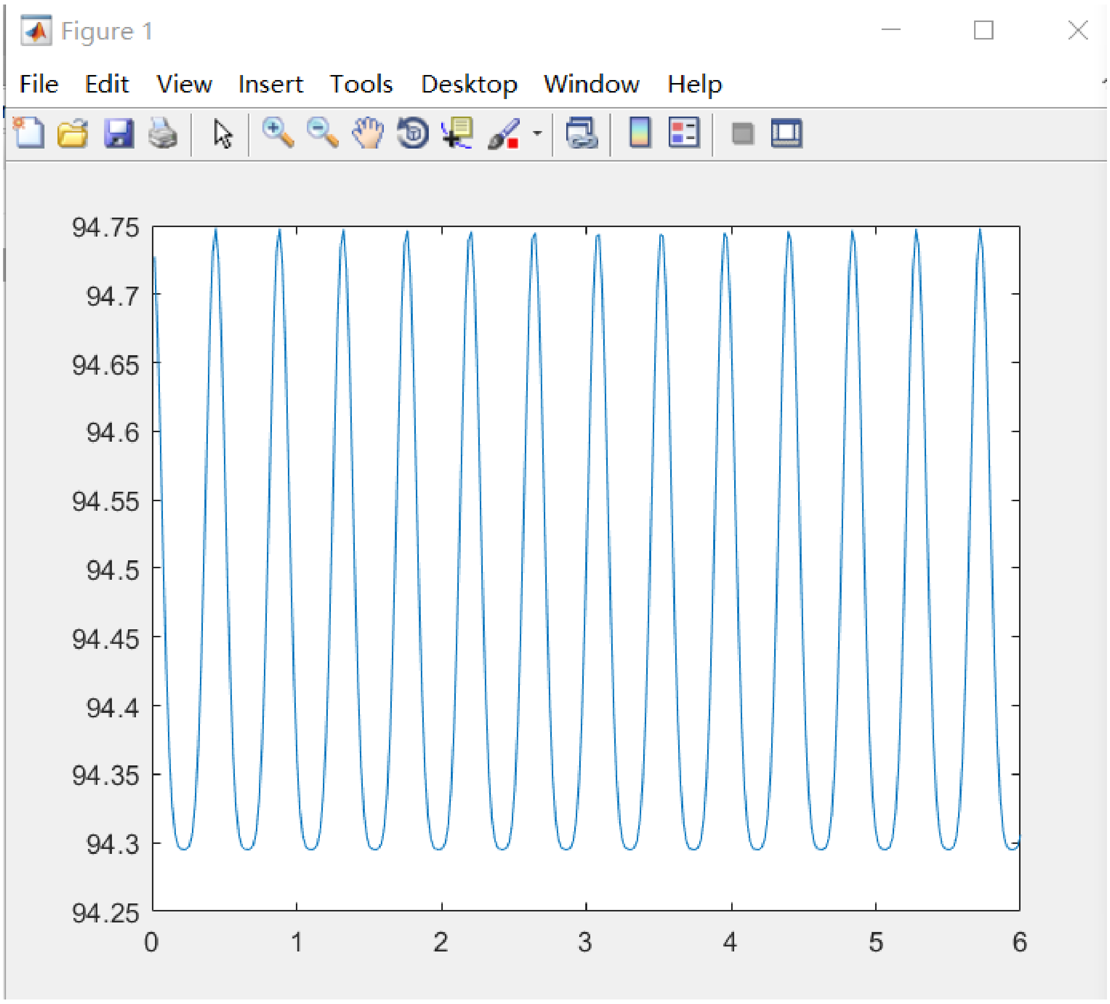
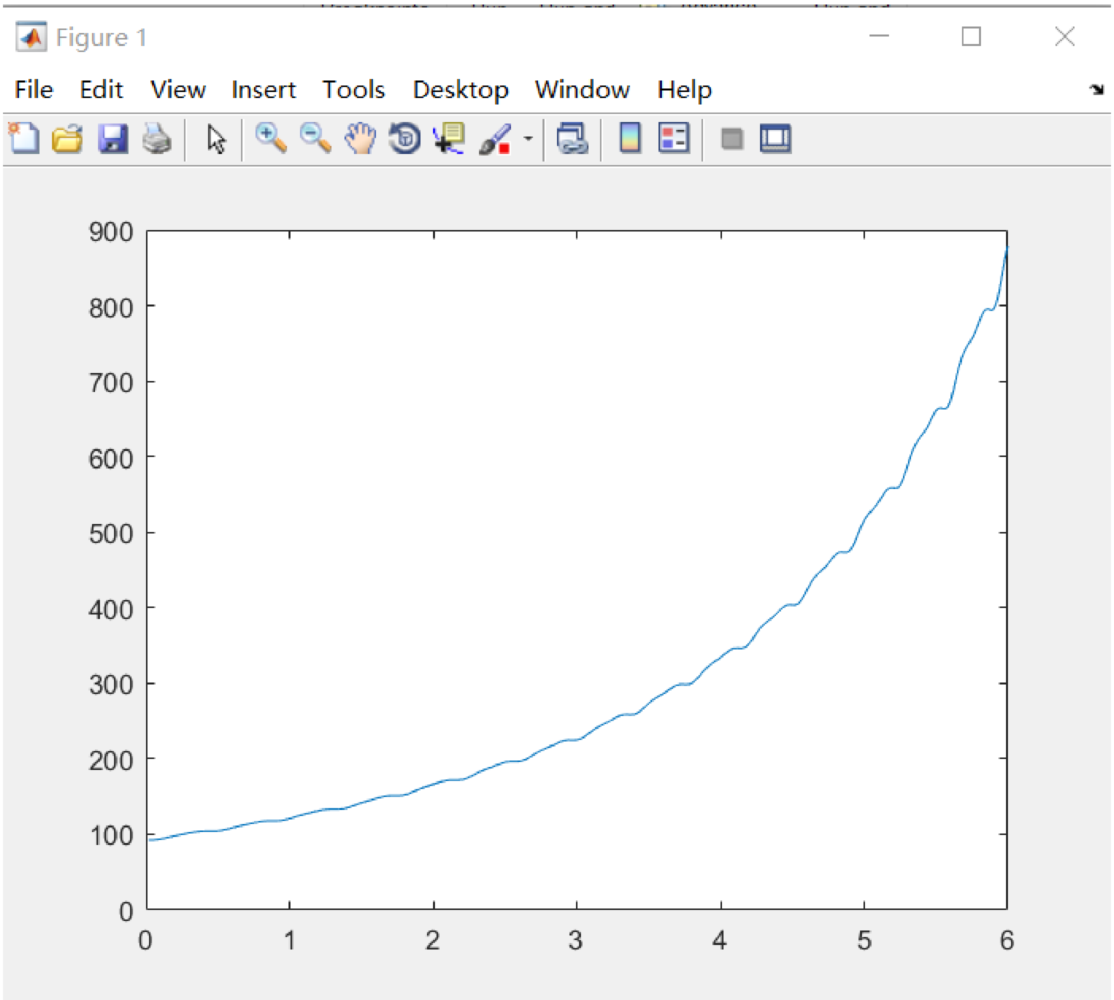



Schéma implicite




La valeur absolue de l'énergie de schéma implicite va devenir de plus en plus grande.