

## Oscillateur conservatif linéaire à un degré de liberté:

1.1 Pour l'équation (1), on peut obtenir:

$$q = A \cos(\omega_0 t + B)$$

avec les conditions aux limites (4), on peut obtenir:

$$q = \cos(\omega_0 t)$$

1.2 On peut calculer  $E^*$  avec la résultat de 1.1:

$$E^* = \frac{1}{2} (\omega_0^2 \sin^2(\omega_0 t) + \omega_0^2 \cos^2(\omega_0 t)) = \frac{1}{2} \omega_0^2 = 2\pi^2$$

On trouve que  $E^*$  est toujours une constante.

2.1 On peut utiliser l'équation (1) dans l'équation (5):

$$\begin{vmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{vmatrix} = \begin{vmatrix} q_j \\ \dot{q}_j \end{vmatrix} + \Delta t \times \begin{vmatrix} \dot{q}_j \\ -\omega_0^2 q_j \end{vmatrix} = \begin{vmatrix} q_j + \Delta t \dot{q}_j \\ -\Delta t \omega_0^2 q_j + \dot{q}_j \end{vmatrix}$$

Alors, on peut l'écrire du matrice, et on peut obtenir l'équation (6).

```
2.2 a)
w0 = 2*pi;
T0 = 3;
dt = 0.01;

q0 = [1];
q1 = [0];
q2 = [-w0^2*q0(1)];

count = 1;
t = [0];
while t(count) <= T0
    q0(count+1) = q0(count) + dt*q1(count);
    q1(count+1) = q1(count) + dt*q2(count);
    q2(count+1) = -w0^2*q0(count+1);
    t = [t, dt*count];
    count = count + 1;
end
2.2 b)
w0 = 2*pi;
T0 = 3;
dt = 0.01;
```

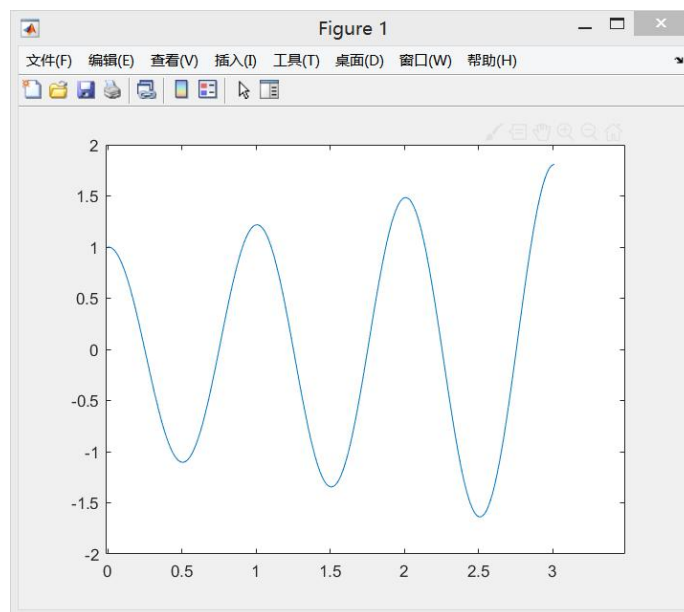
```

q0 = [1];
q1 = [0];
A = [1 dt; -w0^2*dt 1];
U = [q0;q1];

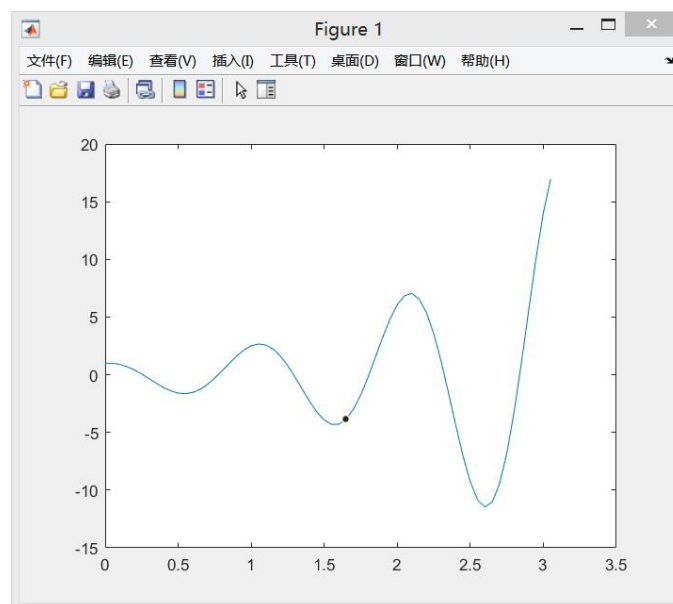
count = 1;
t = [0];
while t(count) < T0
    U(:,count+1) = A*U(:,count);
    t = [t,dt*count];
    count = count+1;
end

```

2.3 pour  $dt = 0.01$ , on peut obtenir la résultat:



Pour  $dt = 0.05$ , on peut obtenir la résultat:



On trouve que la divergence est plus lente si  $dt$  est plus petit.

2.4 On peut trouver que  $E^*$  augmente avec le temps, mais il ne change pas en fait.

2.5 On utilise  $[U,V]=\text{eig}(A)$  pour obtenir des valeurs propres, et on peut trouver des valeurs propres sont des nombres complexes. Donc, la résultat est divergente.

3.1

Méthode1:

```
w0 = 2*pi;
T0 = 3;
dt = 0.01;

q = [1];
dq = [0];
ddq = [-w0^2*q(1)];

count = 1;
t = [0];
E = [1/2*(dq(1)^2+w0^2*q(1)^2)];
while t(count) <= T0
    q(count+1) = (q(count)+dt*dq(count))/(1+dt^2*w0^2);
    ddq(count+1) = -w0^2*q(count+1);
    dq(count+1) = dq(count)+dt*ddq(count+1);
    E = [E, 1/2*(dq(count+1)^2+w0^2*q(count+1)^2)];
    t = [t, dt*count];
    count = count + 1;
end
```

end

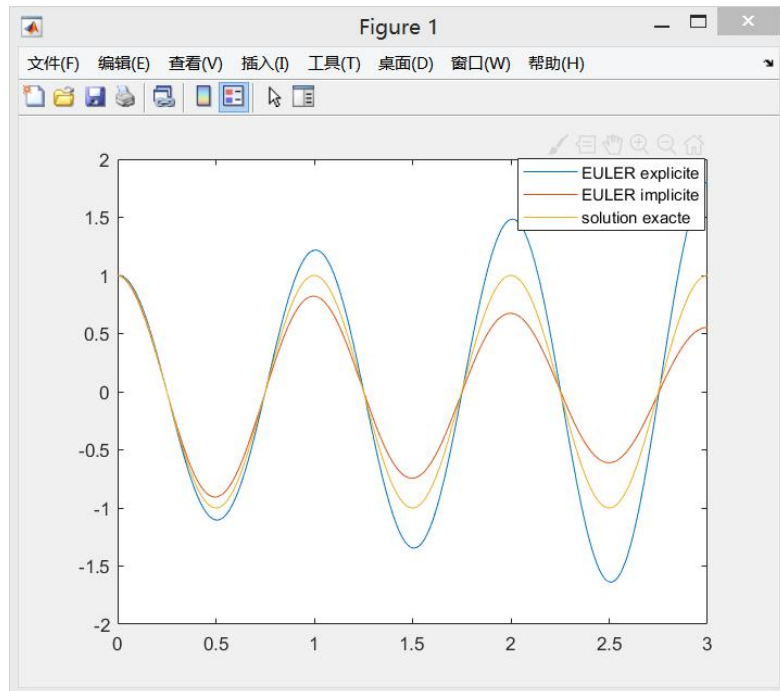
Méthode2:

```
w0 = 2*pi;
T0 = 3;
dt = 0.01;

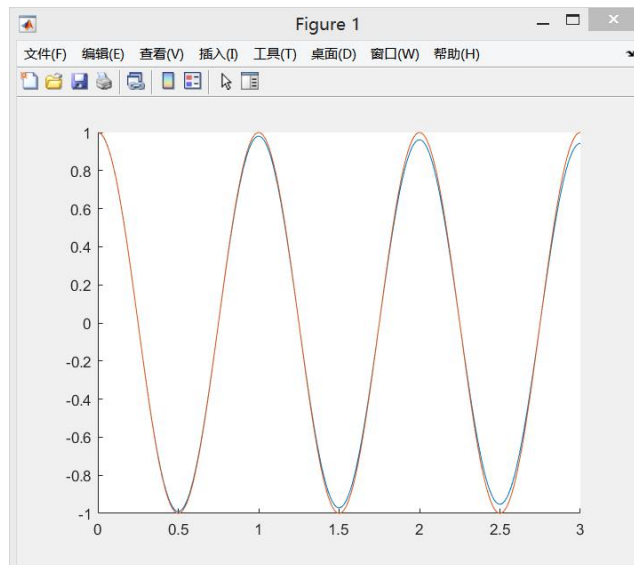
q = [1];
dq = [0];
A = 1/(1+w0^2*dt^2)*[1 dt;-w0^2*dt 1];
U = [q;dq];

count = 1;
t = [0];
while t(count) < T0
    U(:,count+1) = A*U(:,count);
    t = [t,dt*count];
    count = count+1;
end
```

end  
3.2

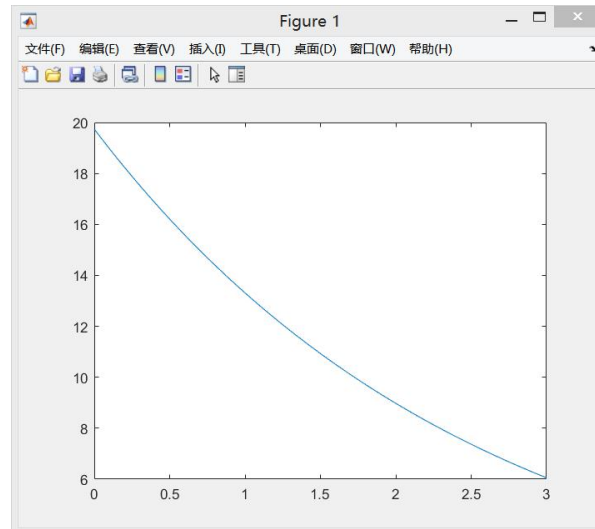


3.3  
Quand  $dt=0.001$ , on trouve que l'erreur est plus petite:



### 3.4

$E^*$  varie comme la figure quand  $dt=0.01$ :



On trouve que il est plus en plus petit, lors la solution exacte est une constante. Cependant, comparé avec la solution d'EULER explicite, il change moins rapide. Quand  $dt$  est plus petit, il change moins rapide.

### 3.5

Les valeurs propres de la matrice  $A$  sont  $1-i*w_0*dt$  et  $1+i*w_0*dt$ . On trouve module de  $(1-i*w_0*dt)^p$  tanvers 0. Donc, on trouve la résultat.

### 4.1

Soit  $u = \begin{pmatrix} q \\ \dot{q} \end{pmatrix}$ , alors, on a  $du = \begin{pmatrix} \dot{q} \\ q \end{pmatrix}$ . Donc, on a une équation au premier ordre:

$$du = \begin{pmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{pmatrix} u$$

### 4.2

```
clear all
w0 = 2*pi;
T0 = 3;
dt = 0.01;

t = (0:dt:3)';
count = size(t,1);
q = zeros(count,1);
dq = zeros(count,1);

q0 = 1;
dq0 = 0;

q(1) = q0;
```

```

dq(1) = dq0;

u = [q0; dq0];
for i = 2:count
    tc = t(i-1);
    xc = u;
    k1 = cal_fc(xc,tc,w0);
    xc = u + k1/2*dt;
    k2 = cal_fc(xc,tc+dt/2,w0);
    xc = u + k2/2*dt;
    k3 = cal_fc(xc,tc+dt/2,w0);
    xc = u + k3*dt;
    k4 = cal_fc(xc,tc+dt,w0);
    k = (k1+2*k2+2*k3+k4)/6;
    u = u + k*dt;

    q(i) = u(1);
    dq(i) = u(2);
end

```

#### 4.3

Quand on prend  $dt=0.01s$ , on trouve que la résultat est plus précise.

#### 4.4

On trouve que  $E^*$  augment avec le temps, mais plus lentement que on a vu dans un schéma d'EULER explicite.

#### 5.1.1

```

gama = 0.5;
beta = 0.25;

w0 = 2*pi;
T0 = 3;
dt = 0.01;

t = (0:dt:3)';
count = size(t,1);
q = zeros(count,1);
dq = zeros(count,1);
E = zeros(count,1);

B = [1+beta*dt^2*w0^2 0; gama*dt*w0^2 1];
C = [1-(0.5-beta)*dt^2*w0^2 dt; -(1-gama)*dt*w0^2 1];
A = B^(-1)*C;

q0 = 1;

```

```

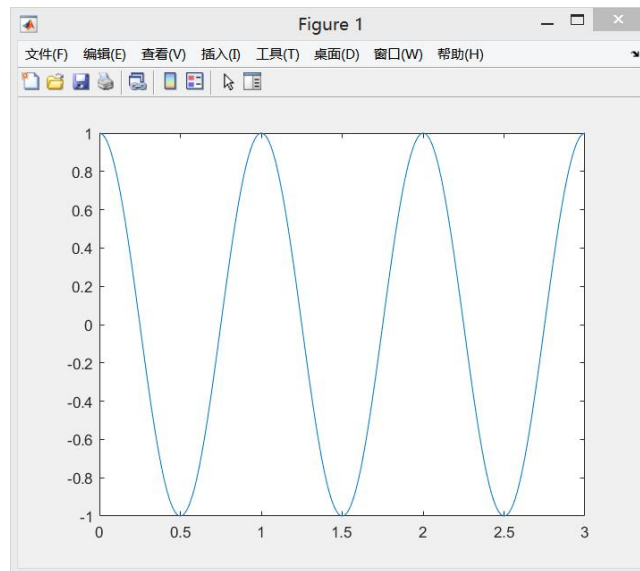
dq0 = 0;

q(1) = q0;
dq(1) = dq0;
E(1) = 1/2*(dq(1)^2+w0^2*q(1)^2);

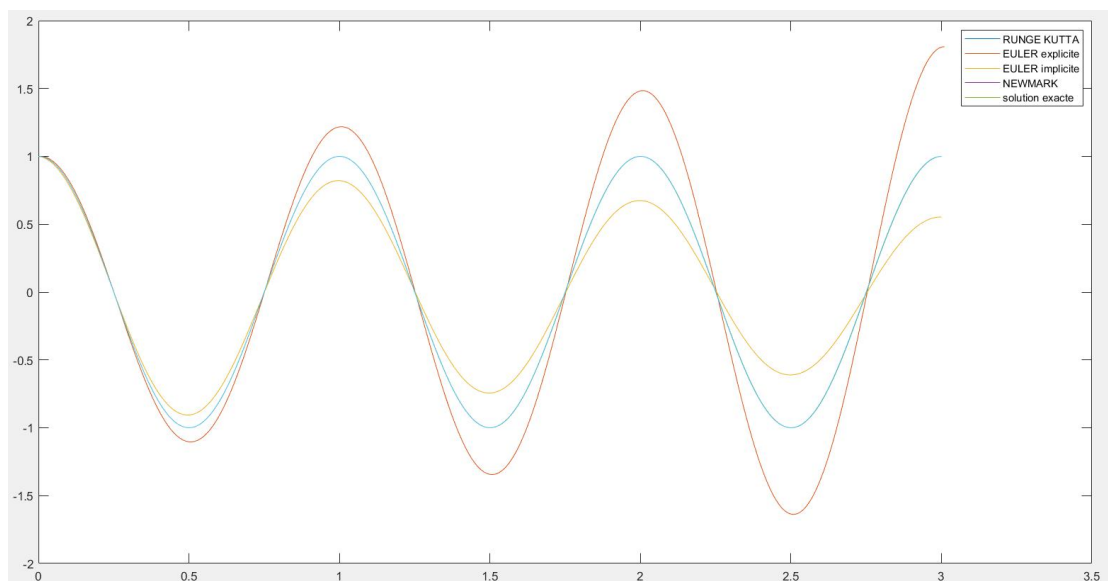
u = [q0;dq0];
for i = 2:count
    u = A*u;
    q(i) = u(1);
    dq(i) = u(2);
    E(i) = 1/2*(dq(i)^2+w0^2*q(i)^2);
end

```

On trouve le figure:

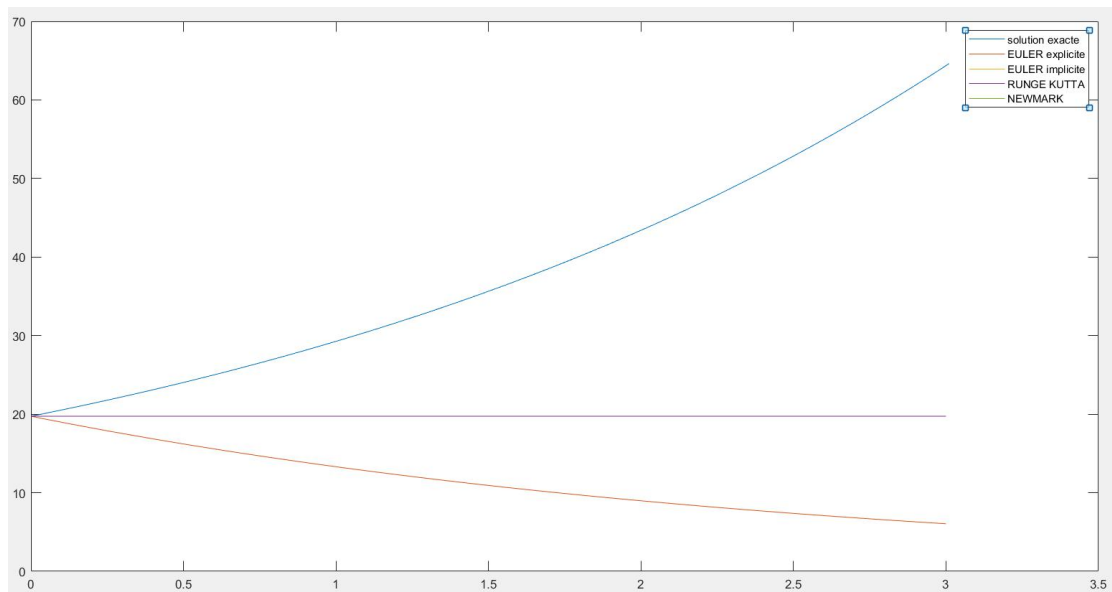


### 5.1.2



On trouve que la solution NEWMARK est la plus proche de la solution exacte, presque la même.

### 5.1.3



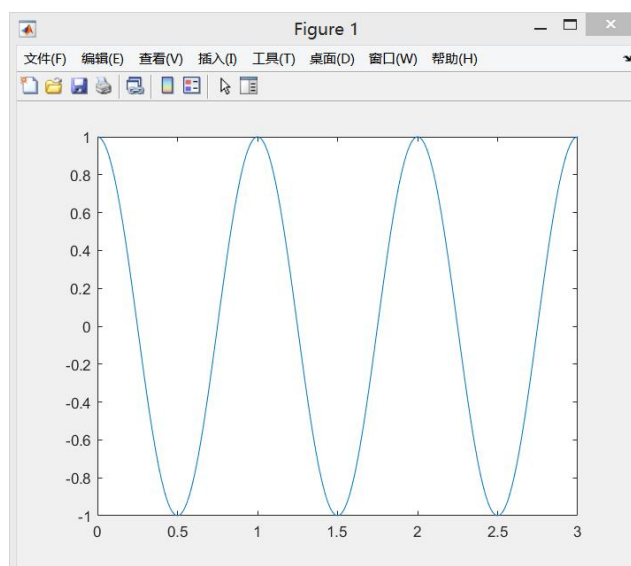
On trouve que  $E^*$  de la solution NEWMARK est la plus proche de la solution exacte.

### 5.1.4

dt(s)	Valeurs propres	
1	$-0.8160 + 0.5781i$	$-0.8160 - 0.5781i$
0.1	$0.8203 + 0.5719i$	$0.8203 - 0.5719i$
0.01	$0.9980 + 0.0628i$	$0.9980 - 0.0628i$
0.001	$1.0000 + 0.0063i$	$1.0000 - 0.0063i$
0.0001	$1.0000 + 0.0006i$	$1.0000 - 0.0006i$

Quand dt est plus petit, alors, la partie réelle est plus proche de 1, et la partie imaginaire est plus proche de 0. Donc, la solution est plus stable.

### 5.2.1



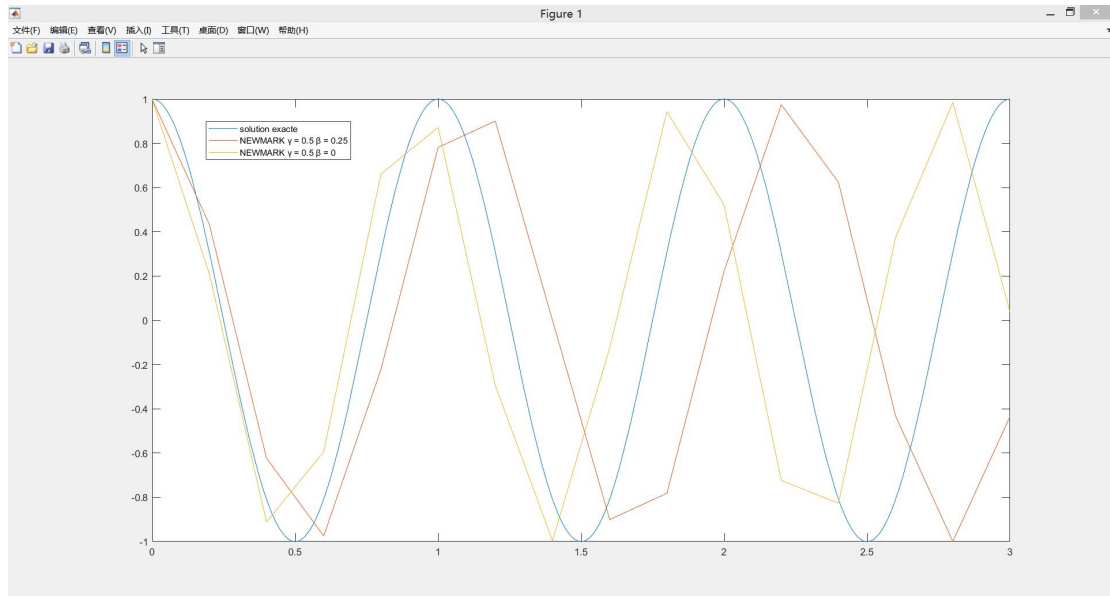


## 5.2.2

Prèsque la même de 5.1.2

## 5.2.3

$dt = 0.2s$ :



$dt = 0.5s$ :

La solution de NEWMARK  $\gamma = 0.5 \beta = 0$  est beaucoup grand que les deux autres quand  $t > 2.7s$ , il ne converge pas.

On trouve que la solution de NEWMARK  $\gamma = 0.5 \beta = 0.25$  est plus lente, mais plus convergeable quand  $dt$  est grand.

## 5.2.4

On trouve que quand  $dt = 0.0015s$ , alors la valeur propre est  $1.0000 + 0.0094i$ . On pense que  $1.5ms$  est le temp critique. Alors,  $\alpha = 0.0047$ .

## Etude d'un oscillateur linéaire amorti à un degré de liberté

### 1.1.a

```
clear all
T0 = 1;
w0 = 2*pi;
e = 0.02;
dt = 0.05*2*e/w0;
omega = w0*sqrt(1-e^2);

t = (0:dt:10*T0)';
count = size(t,1);
x = zeros(count,1);
dx = zeros(count,1);

x0 = 0.01;
```

```

dx0 = 0;
tx =
exp(-e*w0.*t) .* (x0*cos(omega.*t) + (e*w0*x0+dx0) ./omega
.*sin(omega.*t));

```

```

A=[1 dt;-w0^2*dt 1-2*e*w0*dt];

```

```

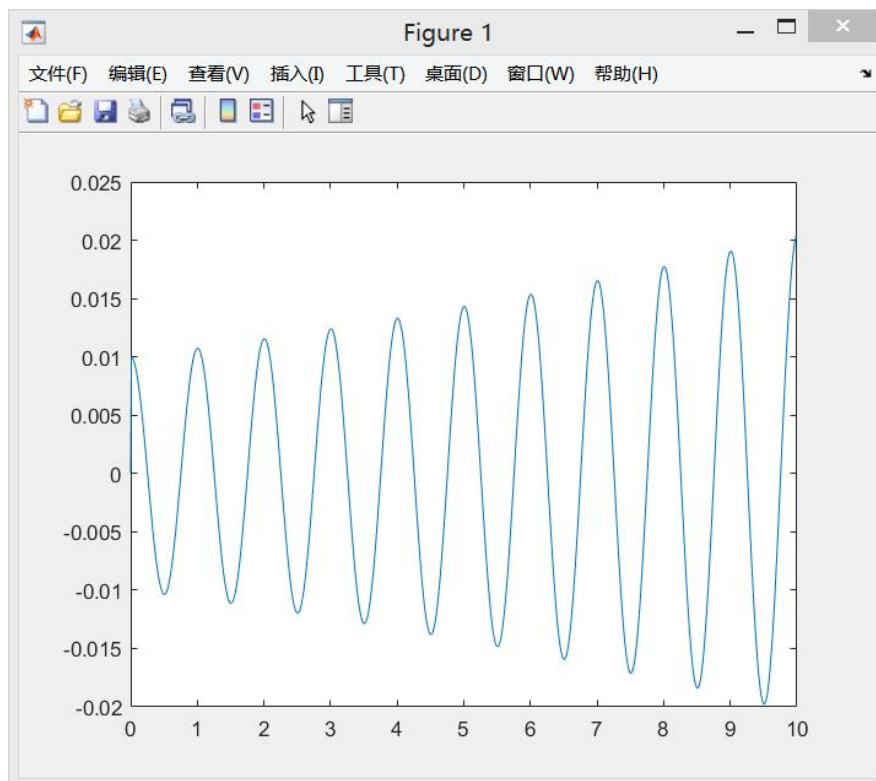
u = [x0;dx0];
for i = 2:count
    u = A*u;
    x(i) = u(1);
    dx(i) = u(2);
end

```

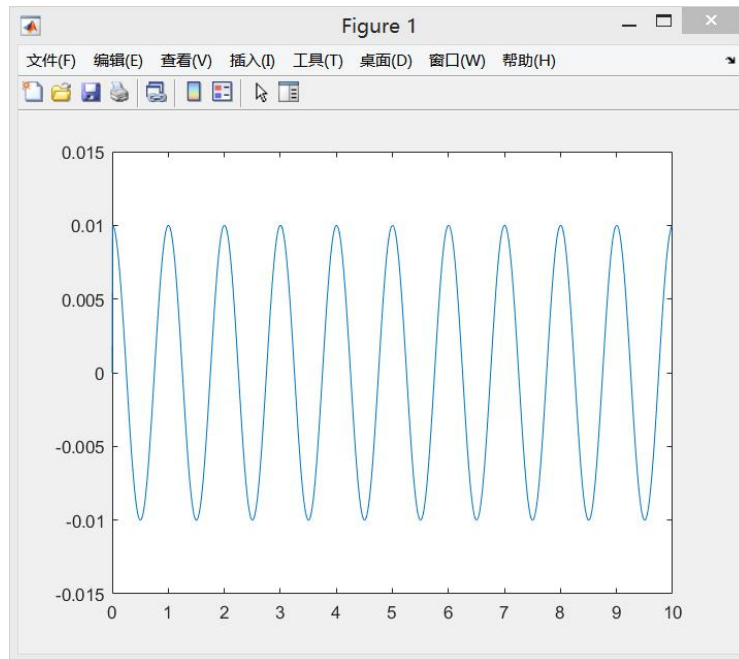
```

(tx(count)-x(count))/tx(count)

```

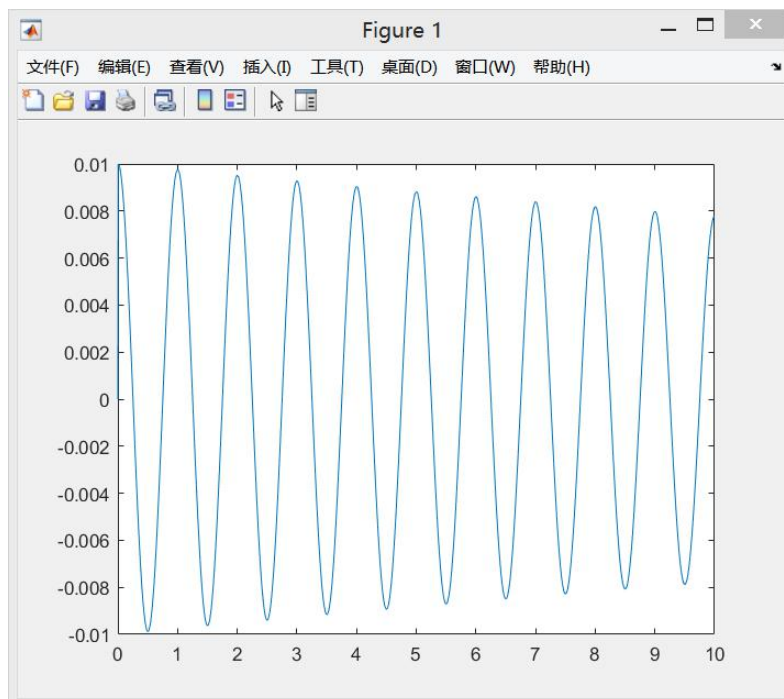


1.1. b  
On change  $dt = 2*e/w_0$ ;



1.1. c

On change  $\Delta t = 0.8 \cdot 2 \cdot e / \omega_0$ ;



1.1.d

On peut comparer  $x(10 \cdot T_0)$  s'ils sont égaux.

Quand  $\frac{\Delta t}{2\xi \omega_0} = 0.0758$ ,  $(tx(\text{count}) - x(\text{count})) / tx(\text{count}) < 0.01$ .

1.2

```
clear all
T0 = 1;
```

```

w0 = 2*pi;
e = 0.02;
dt = 0.3*2*e/w0;
omega = w0*sqrt(1-e^2);

t = (0:dt:10*T0)';
count = size(t,1);
x = zeros(count,1);
dx = zeros(count,1);

x0 = 0.01;
dx0 = 0;

A=[1 -dt;w0^2*dt 1+2*e*w0*dt];
A=A^(-1);

u = [x0;dx0];
for i = 2:count
    u = A*u;
    x(i) = u(1);
    dx(i) = u(2);
end

```

Il n'a pas de temps critique dans cette situation.

1.3

La fonction: cal\_f2.m:

```

function [dU] = cal_f2(xc,tc,w0)
dU = zeros(2,1);
dU(1) = xc(2);
dU(2) = -w0^2*xc(1) - 2*0.02*w0*xc(2);
end

```

Alors,

```

clear all
w0 = 2*pi;
T0 = 1;
h = 0.04;
dt = h*2*sqrt(2)/w0;

t = (0:dt:100*T0)';
count = size(t,1);
q = zeros(count,1);
dq = zeros(count,1);

q0 = 0.01;

```

```

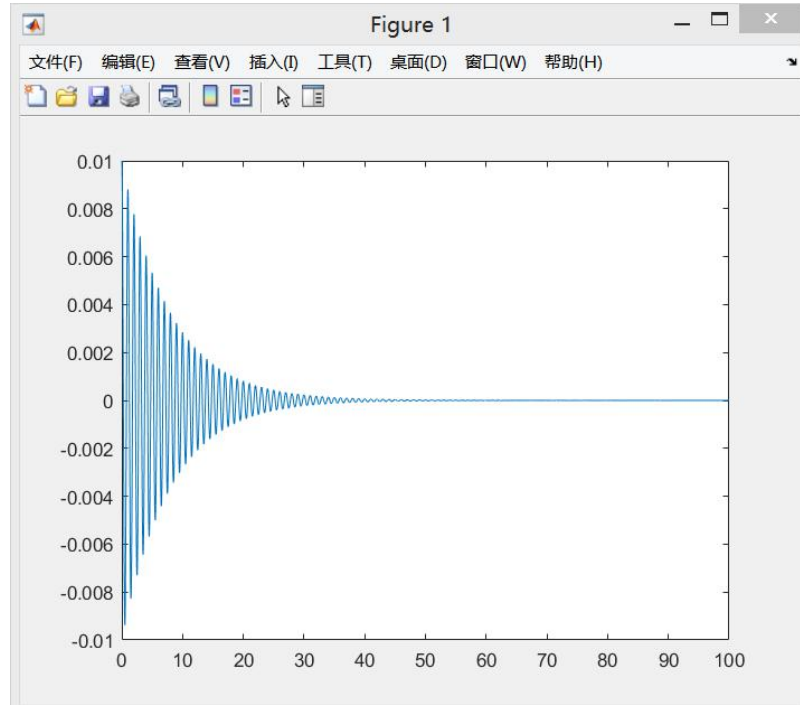
dq0 = 0;

q(1) = q0;
dq(1) = dq0;

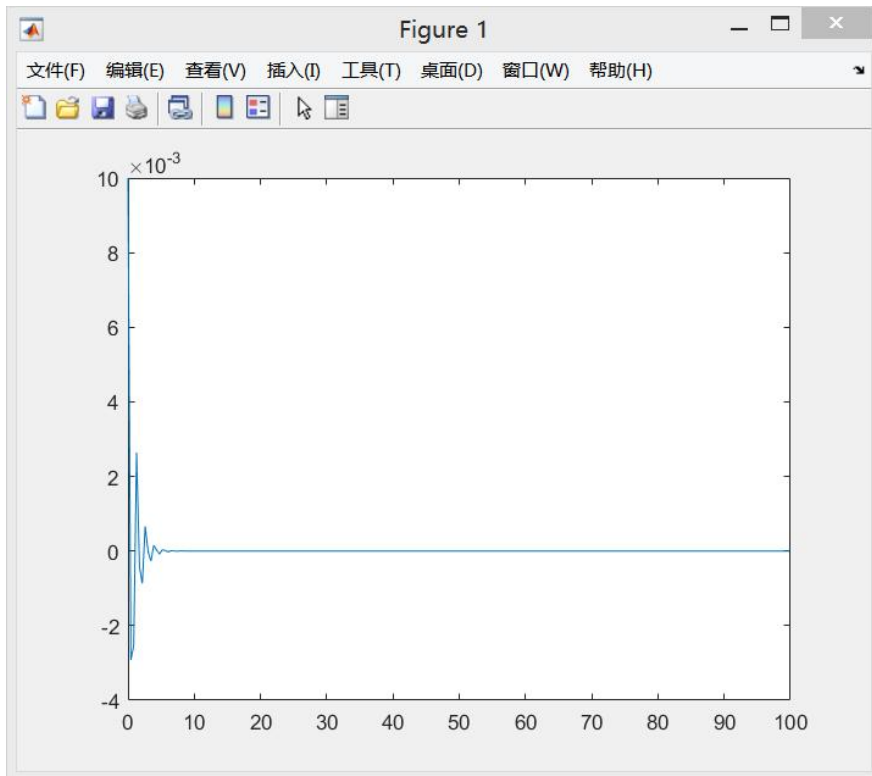
u = [q0; dq0];
for i = 2:count
    tc = t(i - 1);
    xc = u;
    k1 = cal_f2(xc,tc,w0);
    xc = u + k1/2*dt;
    k2 = cal_f2(xc,tc+dt/2,w0);
    xc = u + k2/2*dt;
    k3 = cal_f2(xc,tc+dt/2,w0);
    xc = u + k3*dt;
    k4 = cal_f2(xc,tc+dt,w0);
    k = (k1+2*k2+2*k3+k4)/6;
    u = u + k*dt;

    q(i) = u(1);
    dq(i) = u(2);
end

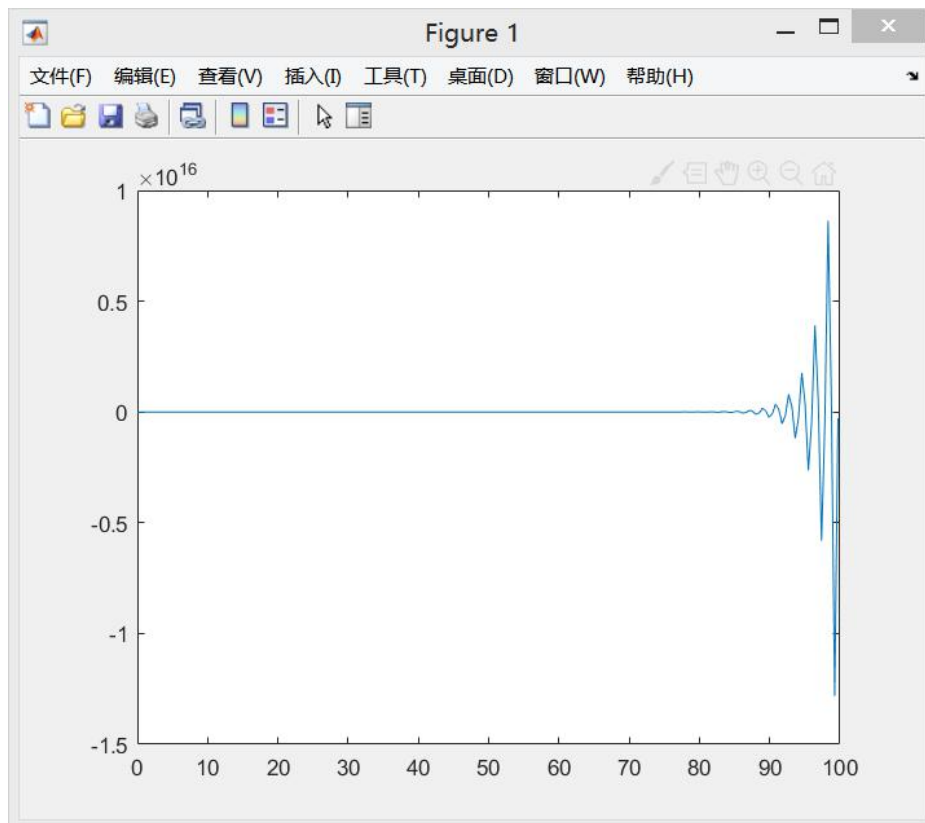
```



$h = 0.04$



$h = 0.96$



$h = 1.04$

On trouve le temps critique est environs  $2 \cdot \sqrt{2} / \omega_0$ , Si  $h$  est plus grand que 1, alors, ce n'est pas stable. Quand  $dt$  est plus petit, alors la solution est plus précise.

## Étude d'un double pendule avec l'hypothèse des petits mouvements

1.1

Après le calcul, on trouve que:

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & -\beta\Delta t^2 & 0 & 0 & 0 \\ 0 & 1 & -\gamma\Delta t & 0 & 0 & 0 \\ 2g & 0 & 2a & 0 & 0 & a \\ 0 & 0 & 0 & 1 & 0 & -\beta\Delta t^2 \\ 0 & 0 & 0 & 0 & 1 & -\gamma\Delta t \\ 0 & 0 & a & 0 & g & a \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & \Delta t & (0.5 - \beta)\Delta t^2 & 0 & 0 & 0 \\ 0 & 1 & (1 - \gamma)\Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta t & (0.5 - \beta)\Delta t^2 \\ 0 & 0 & 0 & 0 & 1 & (1 - \gamma)\Delta t \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Alors, matrice d'amplification  $A = \text{inv}(\mathbf{B}) * \mathbf{C} : (\text{dt} = 0.01)$

A =

```
1.0000    0.0100    0.0001         0         0         0
-0.1801    0.9982    0.0050         0    0.0820    0.0004
-36.0219  -0.3602   -0.0018         0   16.4019    0.0820
         0         0         0    1.0000    0.0100    0.0001
 0.1640    0.0016    0.0000         0    0.8360    0.0042
32.8039    0.3280    0.0016         0  -32.8039   -0.1640
```

1.2

```
clear all
m = 2;
a = 0.5;
g = 9.81;
F0 = 20;
w = 2*pi;
beta = 0;
gama = 0.5;
%dt = 0.01;

theta10 = 0;
theta20 = 0;
```

```

dtheta10 = -1.31519275;
dtheta20 = -1.85996342;

for dt = 0.001:0.001:1
    B=[1,0,-beta*dt^2,0,0,0;
    0,1,-gama*dt,0,0,0;
    2*g,0,2*a,0,0,a;
    0,0,0,1,0,-beta*dt^2;
    0,0,0,0,1,-gama*dt;
    0,0,a,0,g,a];
    C=[1,dt,dt^2*(0.5-beta),0,0,0;
    0,1,dt*(1-gama),0,0,0;
    0,0,0,0,0,0;
    0,0,0,1,dt,dt^2*(0.5-beta);
    0,0,0,0,1,dt*(1-gama);
    0,0,0,0,0,0];
    A = inv(B)*C;
    E = eig(A);
    count = 1;
    for i = 1:1:6
        E(i) = abs(real(E(i)));
        if E(i)>1
            count = 0;
        end
    end
    if count == 0
        break;
    end
end
dt

```

On trouve que le temps critique égale à 0.320s.

1.3

$$\ddot{\theta}_{10} = \frac{1}{a}(-2g\theta_{10} + g\theta_{20})$$

$$\ddot{\theta}_{20} = \frac{1}{a}(2g\theta_{10} - 2g\theta_{20})$$

1.4



$$\begin{bmatrix} 1 & 0 & -\beta\Delta t^2 & 0 & 0 & 0 \\ 0 & 1 & -\gamma\Delta t & 0 & 0 & 0 \\ 2g & 0 & 2a & 0 & 0 & a \\ 0 & 0 & 0 & 1 & 0 & -\beta\Delta t^2 \\ 0 & 0 & 0 & 0 & 1 & -\gamma\Delta t \\ 0 & 0 & a & 0 & g & a \end{bmatrix} \begin{bmatrix} \theta_{1,j+1} \\ \dot{\theta}_{1,j+1} \\ \ddot{\theta}_{1,j+1} \\ \theta_{2,j+1} \\ \dot{\theta}_{2,j+1} \\ \ddot{\theta}_{2,j+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & (0.5-\beta)\Delta t^2 & 0 & 0 & 0 \\ 0 & 1 & (1-\gamma)\Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta t & (0.5-\beta)\Delta t^2 \\ 0 & 0 & 0 & 0 & 1 & (1-\gamma)\Delta t \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_{1,j} \\ \dot{\theta}_{1,j} \\ \ddot{\theta}_{1,j} \\ \theta_{2,j} \\ \dot{\theta}_{2,j} \\ \ddot{\theta}_{2,j} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{F}{m}\sin(\omega t_{j+1}) \\ 0 \\ 0 \\ \frac{F}{\sqrt{2}m}\sin(\omega t_{j+1}) \end{bmatrix}$$

1.5

```
clear all
```

```
m = 2;
```

```
a = 0.5;
```

```
g = 9.81;
```

```
F0 = 20;
```

```
w = 2*pi;
```

```
beta = 0;
```

```
gama = 0.5;
```

```
dt = 0.02;
```

```
T0 = 8;
```

```
t = (0:dt:T0)';
```

```
count = size(t,1);
```

```
q1 = zeros(count,1);
```

```
dq1 = zeros(count,1);
```

```
ddq1 = zeros(count,1);
```

```
q2 = zeros(count,1);
```

```
dq2 = zeros(count,1);
```

```
ddq2 = zeros(count,1);
```

```
theta10 = 0;
```

```
theta20 = 0;
```

```
dtheta10 = -1.31519275;
```

```
dtheta20 = -1.85996342;
```

```
q1(1) = theta10;
```

```
dq1(1) = dtheta10;
```

```
q2(1) = theta20;
```

```
dq2(1) = dtheta20;
```

```
ddq1(1) = (-2*g*theta10+g*theta20)/2;
```

```
ddq2(1) = (2*g*theta10-2*g*theta20)/2;
```

```
q = [q1 dq1 ddq1 q2 dq2 ddq2]';
```

```
B=[1,0,-beta*dt^2,0,0,0;
```

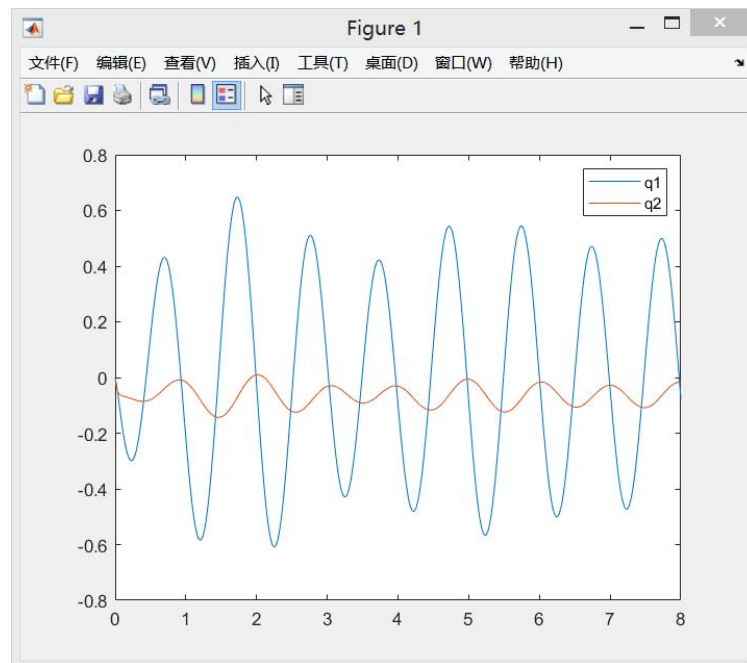
```

0,1,-gama*dt,0,0,0;
2*g,0,2*a,0,0,a;
0,0,0,1,0,-beta*dt^2;
0,0,0,0,1,-gama*dt;
0,0,a,0,g,a];
C=[1,dt,dt^2*(0.5-beta),0,0,0;
0,1,dt*(1-gama),0,0,0;
0,0,0,0,0,0;
0,0,0,1,dt,dt^2*(0.5-beta);
0,0,0,0,1,dt*(1-gama);
0,0,0,0,0,0];
A = inv(B)*C;

for i = 2:count
    q =
A*q+inv(B)*[0;0;F0*sin(w*t(i))/m;0;0;F0*sin(w*t(i))/m
/sqrt(2)];
    q1(i) = q(1);
    dq1(i) = q(2);
    ddq1(i) = q(3);
    q2(i) = q(4);
    dq2(i) = q(5);
    ddq2(i) = q(6);
end

```

1.6



t=0, q1=0 dq1=-1.13 ddq1=0, q2=0 dq2=-1.86 ddq2=0. t=dt, q1=-0.0263 dq1=-1.56  
ddq1=-24.4, q2=-0.0372 dq2=-1.34 ddq2=52.4. t=2dt, q1=-0.0624 dq1=-1.88  
ddq1=-7.59, q2=-0.0534 dq2=-0.583 ddq2=22.6. t=0.5, q1=0.145 dq1=2.39

ddq1=-3.49, q2=-0.0794 dq2=0.121 ddq2=1.12.

2.1

Après le calcul, on trouve

$$\mathbf{B} = \begin{bmatrix} 2a + 2g\beta\Delta t^2 & a \\ a & a + g\beta\Delta t^2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} -2a & -a \\ -a & -a \end{bmatrix},$$

Alors,  $\mathbf{A} = \mathbf{B}^{-1} * \mathbf{C}$ .

2.2

Il n'y a pas de temps critique car les valeurs propres sont toujours inférieure que 1, qui indique que la solution est toujours stable.

2.3

On trouve la même résultat de la question 1.3.

2.4

On trouve la même résultat de la question 1.4.

2.5

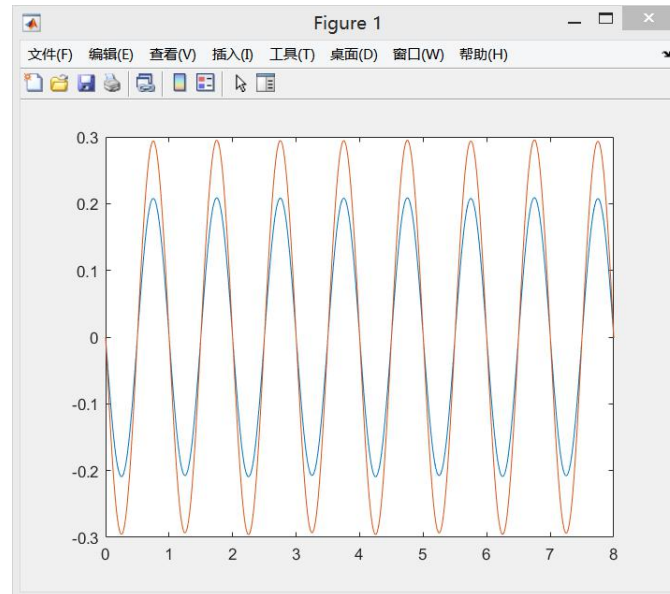
```
clear all;
dt2 = 0.02;
T0 = 8;
t2 = (0 : dt2 : T0)';
np2 = size(t2, 1);
m = 2;
a = 0.5;
g = 9.81;
beta= 0.25;
gamma= 0.5;
F0 = 20;
w = 2 * pi;
B = [2 * a + 2 * g * beta * dt2 * dt2, a;
     a, a + g * beta * dt2 * dt2];
C = [-2 * a , -a;
     -a, -a];
A = inv(B) * C;
theta10 = 0;
theta20 = 0;
dtheta10 = -1.31519275;
dtheta20 = -1.85996342;
ddtheta10 = (- 2 * g * theta10 + g * theta20) / a;
ddtheta20 = (2 * g * theta10 - 2 * g * theta20) / a;
q = [theta10; dtheta10; ddtheta10; theta20; dtheta20;
     ddtheta20];
q1b = zeros(np2, 1);
dq1b = zeros(np2, 1);
ddq1b = zeros(np2, 1);
q2b = zeros(np2, 1);
```

```

dq2b = zeros(np2, 1);
ddq2b = zeros(np2, 1);
q1b(1) = theta10;
dq1b(1) = dtheta10;
ddq1b(1) = ddtheta10;
q2b(1) = theta20;
dq2b(1) = dtheta20;
ddq2b(1) = ddtheta20;
deltatheta = zeros(2, 1);
for inc = 2 : np2
    q(1) = q(1) + dt2 * q(2) + dt2 * dt2 * (0.5 -beta) * q(3);
    q(2) = q(2) + dt2 * (1 - gamma) * q(3);
    q(3) = 0;
    q(4) = q(4) + dt2 * q(5) + dt2 * dt2 * (0.5 -beta) * q(6);
    q(5) = q(5) + dt2 * (1 - gamma) * q(6);
    q(6) = 0;
    while abs(a * (2 * q(3) + q(6)) + 2 * g * q(1) - ((F0 *
sin(w * t2(inc))) / m)) >= 0.01 || abs(a * (q(3) + q(6))
+ g * q(4) - ((F0 * sin(w * t2(inc))) / ((2 ^ 0.5) * m))) >=
0.01
        deltatheta = A * [q(3); q(6)] + (inv(B) * [(((F0 *
sin(w * t2(inc))) / m) - 2 * g * q(1)); ((F0 * sin(w *
t2(inc))) / ((2 ^ 0.5) * m) - g * q(4))]);
        q = q + [beta * dt2 * dt2 * deltatheta(1);
        gamma * dt2 * deltatheta(1);
        deltatheta(1);
        beta * dt2 * dt2 * deltatheta(2);
        gamma * dt2 * deltatheta(2);
        deltatheta(2)];
    end
    q1b(inc) = q(1);
    dq1b(inc) = q(2);
    ddq1b(inc) = q(3);
    q2b(inc) = q(4);
    dq2b(inc) = q(5);
    ddq2b(inc) = q(6);
end

```

2.6



Les résultats sont plus moins dans cette solution:

$t=0, q_1=0 \quad dq_1=-1.32 \quad ddq_1=0, \quad q_2=0 \quad dq_2=-1.86 \quad ddq_2=0.$   
 $t=dt, q_1=-0.0262 \quad dq_1=-1.30 \quad ddq_1=1.04, \quad q_2=-0.0371 \quad dq_2=-1.85 \quad ddq_2=1.46.$   
 $t=2dt, q_1=-0.0520 \quad dq_1=-1.27 \quad ddq_1=2.05, \quad q_2=-0.0735 \quad dq_2=-1.80 \quad ddq_2=2.91.$   
 $t=0.5, q_1=9.22e-4 \quad dq_1=1.31 \quad ddq_1=0.0106, \quad q_2=-0.00130 \quad dq_2=1.82 \quad ddq_2=0.0105.$

## Oscillateur non linéaire à un degré de liberté

### 1.1

Par calculer, on obtient:

$$\begin{bmatrix} 1 & 0 & -\beta\Delta t^2 \\ 0 & 1 & -\gamma\Delta t \\ -\omega_0^2(1+q_{j+1}^2) & 0 & 1 \end{bmatrix} \begin{bmatrix} q_{j+1} \\ \dot{q}_{j+1} \\ \ddot{q}_{j+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & \Delta t^2(0.5-\beta) \\ 0 & 1 & \Delta t(1-\gamma) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_j \\ \dot{q}_j \\ \ddot{q}_j \end{bmatrix}$$

### 1.2

```
clear all;
T0 = 6;
a = 0.1;
w0 = 2 * pi;
y0 = 2;
dt = 0.02;

q0 = 2;
dq0 = 0;

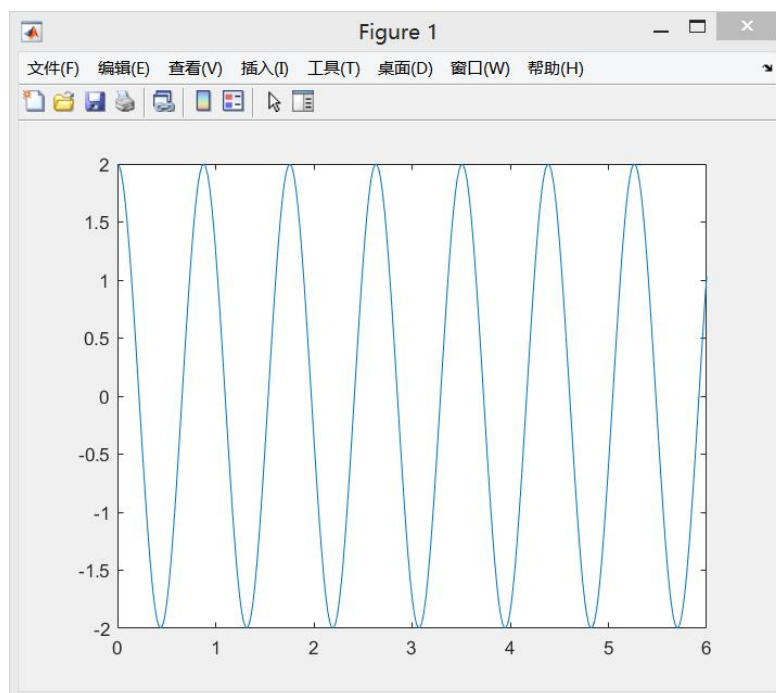
t = (0:dt:T0)';
count = size(t,1);
q1 = zeros(count,1);
dq1 = zeros(count,1);
```

```

q1(1) = q0;
dq1(1) = dq0;
ddq0c = - w0^2*q0*(1+a*q0^2);
for inc = 2 : count
    q1(inc) = q1(inc-1)+dt*dq1(inc-1)+dt^2*0.5*ddq0c ;
    ddqc = - w0^2*q1(inc)*(1+a*q1(inc)*q1(inc));
    dq1(inc) = dq1(inc-1)+0.5*dt*(ddq0c+ddqc);
    ddq0c = ddqc;
end

```

1.3



$q(0)=2, q(dt)=1.98, q(2dt)=1.91, q(T0)=1.03$

2.1

On minimise l'équation de  $q'' + \omega^2 q (1 + a q^2) = 0$ .

2.2

$$\ddot{q}_{j+1}^* = \ddot{q}_j^* + \Delta \ddot{q}_{j+1}^*$$

Avec :

$$\ddot{q}_{j+1}^* + \Delta \ddot{q}_{j+1}^* + \omega_0^2 (q_{j+1}^* + \beta \Delta t^2 \Delta \ddot{q}_{j+1}^*) (1 + a (q_{j+1}^* + \beta \Delta t^2 \Delta \ddot{q}_{j+1}^*)^2) = 0$$

2.3

```

clear all;
T0 = 6;
a = 0.1;
w0 = 2*pi;
y0 = 2;

```

```

dt = 0.02;

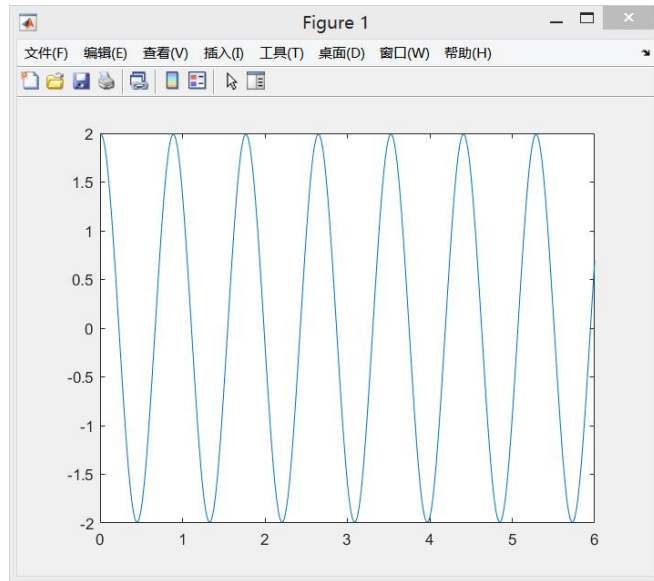
q0 = 2;
dq0 = 0;

t = (0:dt:T0)';
count = size(t,1);
q = zeros(count,1);
dq = zeros(count,1);
ddq = zeros(count,1);

q(1) = q0;
dq(1) = dq0;
ddq(1) = -w0*q0*(1+a*q0^2);

for i = 2:count
    qb = q(i-1)+dt*dq(i-1)+0.25*dt^2*ddq(i-1);
    dqb = dq(i-1)+0.5*dt*ddq(i-1);
    ddqb = 0;
    while abs(ddqb+w0^2*qb*(1+a*qb*qb)) >= 0.01
        syms deltaq2;
        eq =
ddqb+deltaq2+w0^2*(qb+0.25*dt^2*deltaq2)*(1+a*(qb+0.2
5*dt^2*deltaq2)*(qb+0.25*dt^2*deltaq2));
        deltaq2 = solve(eq,deltaq2);
        deltaq2 = double(deltaq2);
        qb = qb+0.25*dt^2*deltaq2(1);
        dqb = dqb+0.5*dt*deltaq2(1);
        ddqb = ddqb+deltaq2(1);
    end
    q(i) = qb;
    dq(i) = dqb;
    ddq(i) = ddqb;
end
2.4

```



On peut obtenir  $q(0)=2$ ,  $q(dt)=1.99$ ,  $q(2dt)=1.94$ ,  $q(T0)=0.692$ .

3.1

$$E = \frac{1}{2} \omega_0^2 q^2 + \frac{1}{4} a \omega_0^2 q^4 + \frac{1}{2} \dot{q}^2$$

3.2

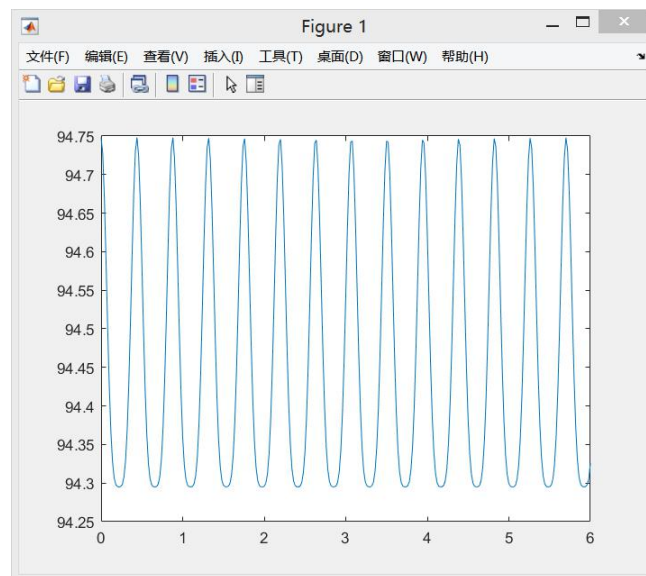
On prend  $E(i) =$

$$\frac{1}{2} * (\omega_0^2 * q(i)^2 + \dot{q}(i)^2) + \frac{1}{4} * \omega_0^2 * a * q(i)^4;$$

Pour calculer.

3.3

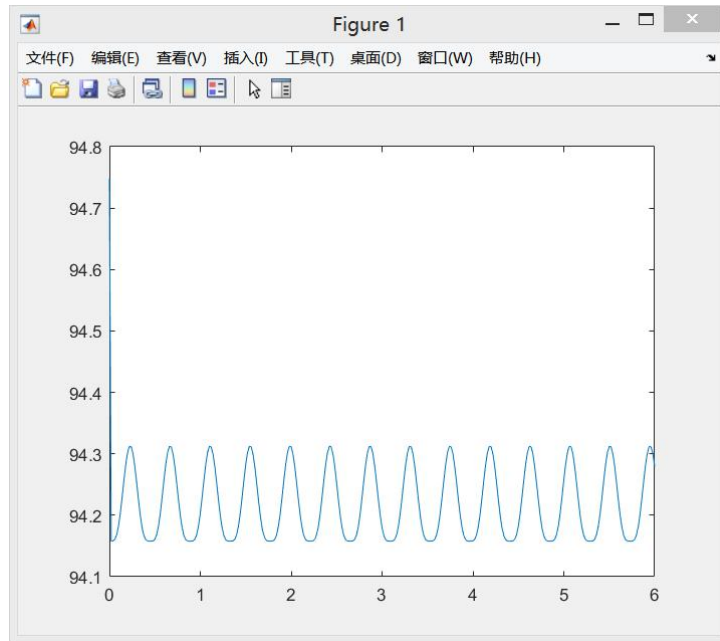
Pour la première solution:



L'Énergie ne change pas très grand, et il est toujours environs 94.5.

Pour la deuxième solution:





L'Énergie est plus petit que l'énergie original, il change moins rapide que la première solution.