

## *Etude d'un oscillateur linéaire amorti à un degré de liberté*

### **1.1.**

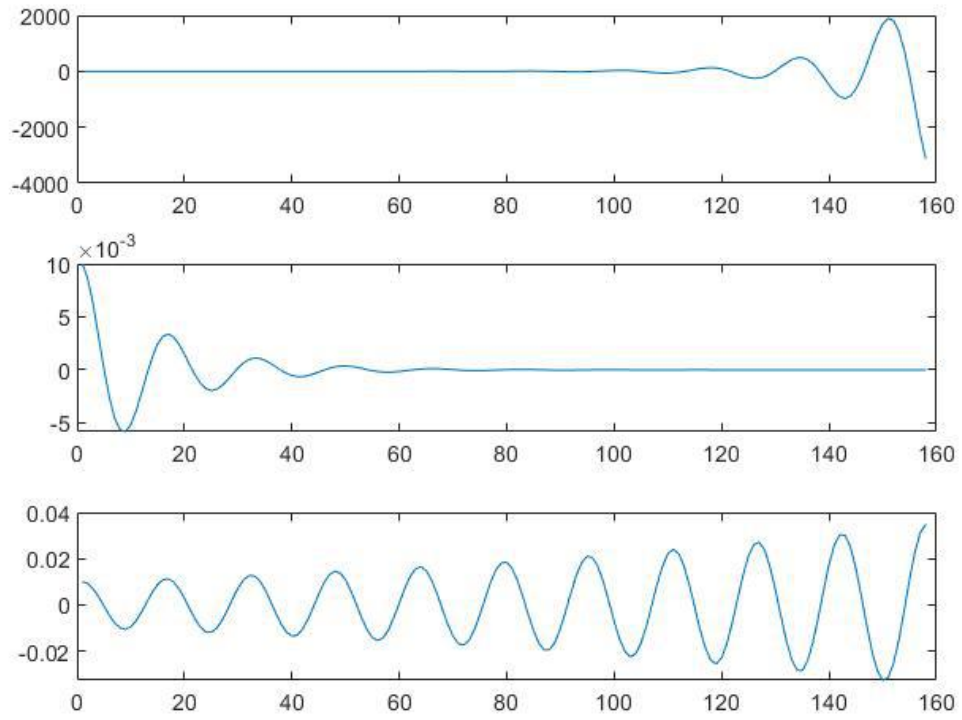
```
clear all;
T0=1;
w0=2*pi/T0;
e=0.02;
m=1;
b=2*w0*m;
x0=0.01;
dx0=0;
v=2*e/w0;
F=0;
dt=v;
n=fix(10*T0/dt);

%1.1
U1(:,1)=[x0,dx0];
A1=[0 1;-w0^2 2*e*w0];
B1=[0; F/m];
for i=1:n
    U1(:,i+1)=U1(:,i)+A1*U1(:,i)*dt+B1;
end
subplot(3,1,1)
plot(1:n+1,U1(1,:))
```

a)

$$dt=2*e/w0*10$$

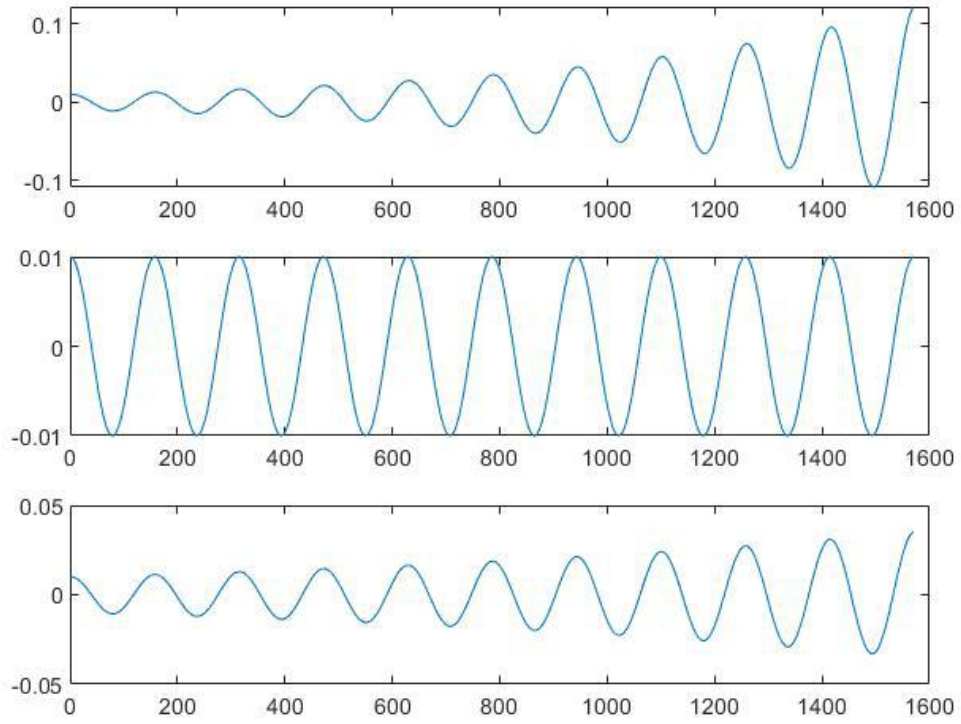
première figure



b)

$$dt=2*e/w_0$$

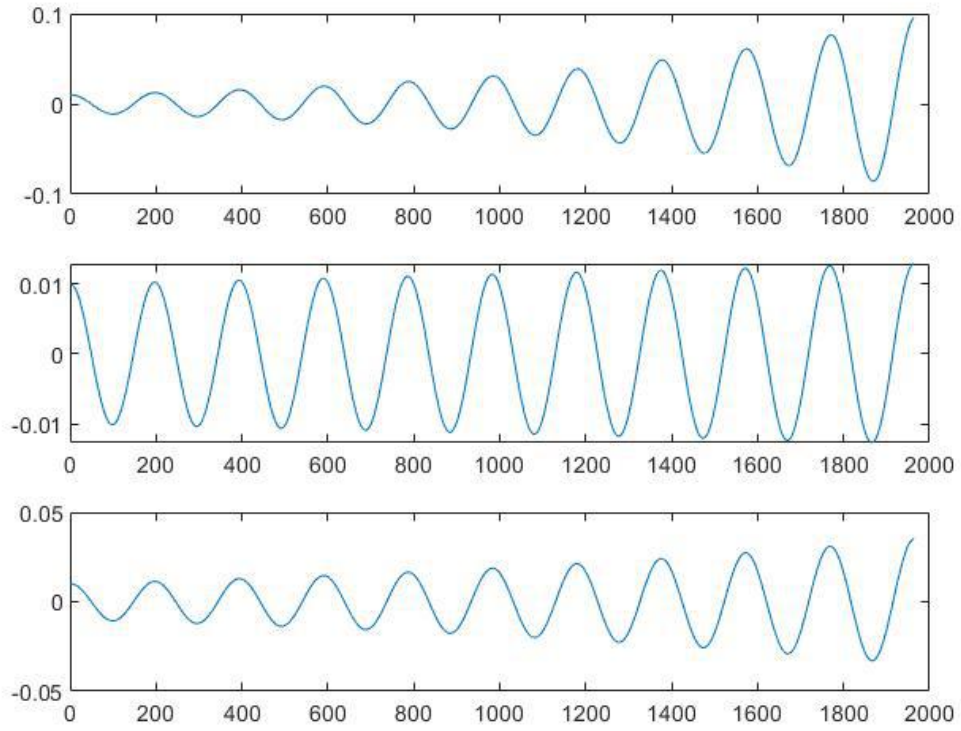
première figure



c)

$$dt=2*e/w0*0.8$$

première figure



d)

la convergence.

S'il est converge, on peut calculer l'erreur entre le résultat infini et le résultat exacte.

À partir la valeur de 0.05

## 1.2

```

clear all;
T0=1;
w0=2*pi/T0;
e=0.02;
m=1;
b=2*w0*m;
x0=0.01;
dx0=0;
v=2*e/w0;
F=0;
dt=v;
n=fix(10*T0/dt);

%1.1
U1(:,1)=[x0,dx0];
A1=[0 1;-w0^2 2*e*w0];
B1=[0; F/m];
for i=1:n
    U1(:,i+1)=U1(:,i)+A1*U1(:,i)*dt+B1;
end
subplot(3,1,1)
plot(1:n+1,U1(1,:))

%1.2
U2(:,1)=[x0,dx0];
A2=[0 1;-w0^2 2*e*w0];
B2=[0; F/m];
for i=1:n
    U2(:,i+1)=inv(eye(2)-A2*dt)*(U2(:,i)+B2);
end
subplot(3,1,2)
plot(1:n+1,U2(1,:))

```

*dt=0.0003s*

### 1.3

```

clear all;
T0=1;
w0=2*pi/T0;
e=0.02;
m=1;
b=2*w0*m;
x0=0.01;
dx0=0;
v=2*e/w0;
F=0;
dt=v;
n=fix(100*T0/dt);

%1.1
U1(:,1)=[x0,dx0];
A1=[0 1;-w0^2 2*e*w0];
B1=[0; F/m];
for i=1:n
    U1(:,i+1)=U1(:,i)+A1*U1(:,i)*dt+B1;
end
subplot(3,1,1)
plot(1:n+1,U1(1,:))

%1.2
U2(:,1)=[x0,dx0];
A2=[0 1;-w0^2 2*e*w0];
B2=[0; F/m];
for i=1:n
    U2(:,i+1)=inv(eye(2)-A2*dt)*(U2(:,i)+B2);
end
subplot(3,1,2)
plot(1:n+1,U2(1,:))

%1.3
U3(:,1)=[x0,dx0];
A3=[0 1;-w0^2 2*e*w0];
B3=[0; F/m];
for i=1:n
    K1=A3*U3(:,i);
    K2=A3*(U3(:,i)+K1*dt/2);
    K3=A3*(U3(:,i)+K2*dt/2);
    K4=A3*(U3(:,i)+K3*dt);

```

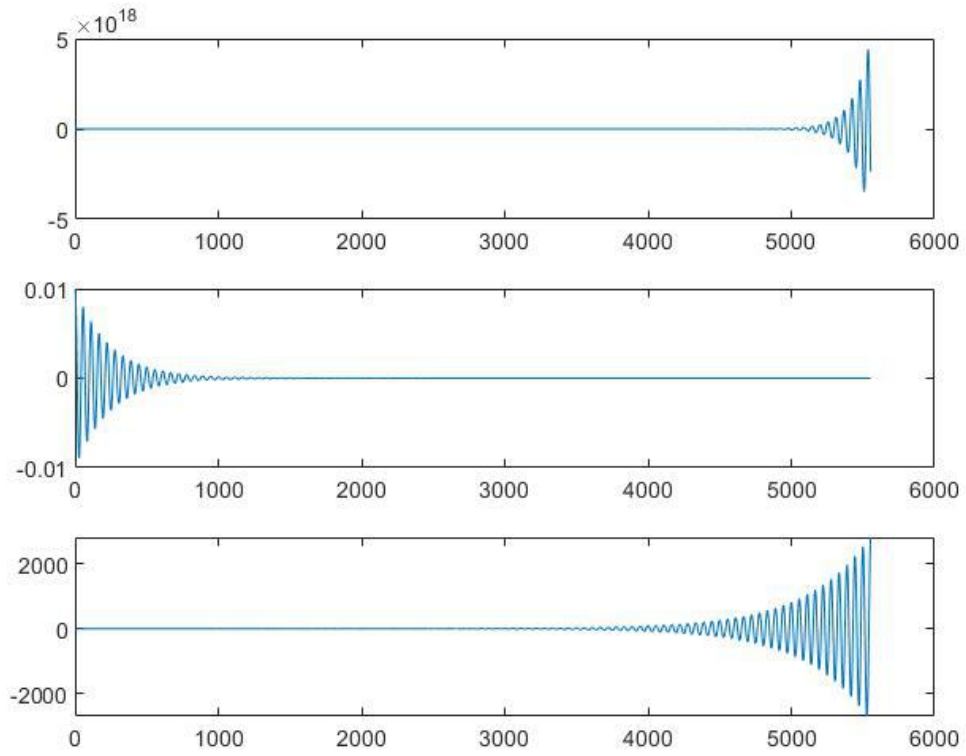
```
U3(:, i+1) = U3(:, i) + dt * (K1 + 2 * K2 + 2 * K3 + K4) / 6;  
end  
subplot(3, 1, 3)  
plot(1:n+1, U3(1, :))
```



1.2. a

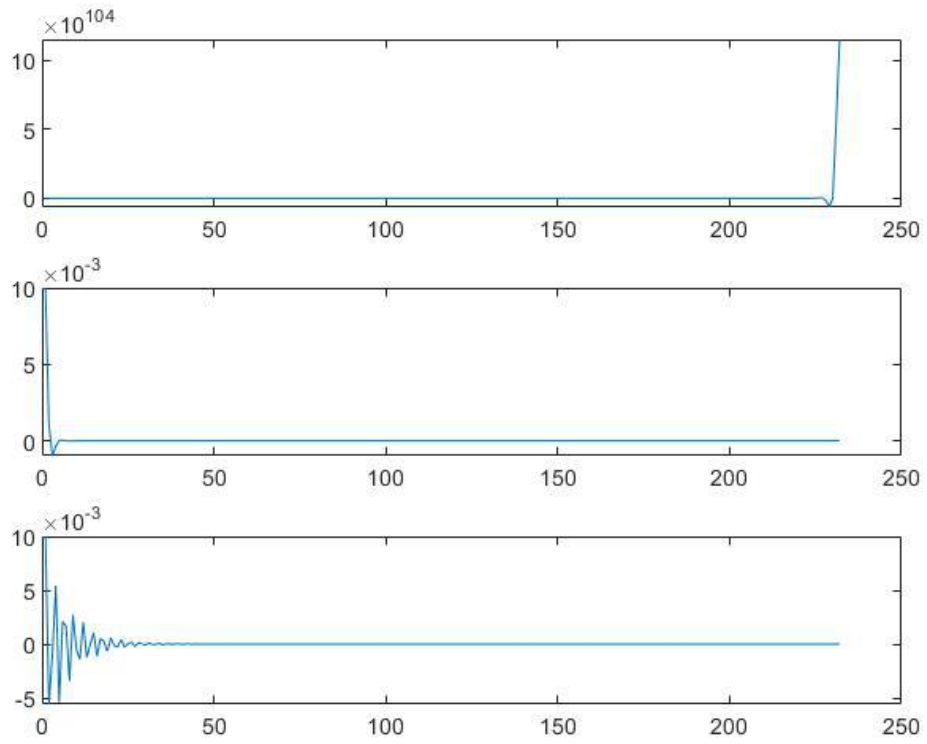
$h=0.04$

troisième figure



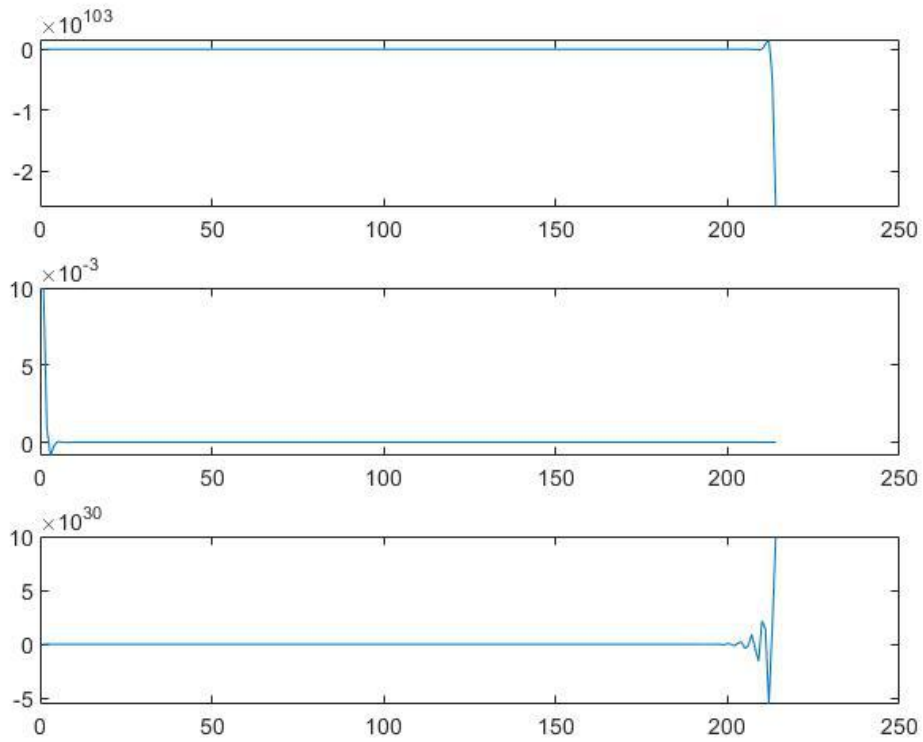
$h=0.96$

troisième figure



$h=1.04$

troisième figure



quand  $h$  augmente, la précision diminue mais la stabilité augmente.

1.3. b

$h=1.001$

Etude d'un double pendule avec l'hypothèse des petits mouvements

### 1.1

Matrice de l'amplification  $C1 = \text{inv}(A1) * B1$

$$A1 = \begin{pmatrix} 1 & 0 & -\beta\Delta t^2 \\ 0 & 1 & -\gamma\Delta t \\ \omega_0^2 & 0 & 1 \end{pmatrix}$$

$$B1 = \begin{pmatrix} 1 & \Delta t & \Delta t^2(0.5 - \beta) \\ 0 & 1 & \Delta t(1 - \gamma) \\ 0 & 0 & 0 \end{pmatrix}$$

et  $\ddot{q} = -\omega_0^2 q$

Après avoir simplifié, on a

$$C11 = \begin{pmatrix} 1 - \frac{\omega_0^2 \Delta t^2}{2(1 + \beta\omega_0^2 \Delta t^2)} & \frac{\Delta t}{1 + \beta\omega_0^2 \Delta t^2} \\ -\omega_0^2 \Delta t \left[ 1 - \frac{\gamma\omega_0^2 \Delta t^2}{2(1 + \beta\omega_0^2 \Delta t^2)} \right] & 1 - \frac{\gamma\omega_0^2 \Delta t^2}{1 + \beta\omega_0^2 \Delta t^2} \end{pmatrix}$$

1.2

quand  $dt=0.05s$ , les valeurs propres sont

$$0.950651977994553 + 0.310259273407006i$$

$$0.950651977994553 - 0.310259273407006i$$

Donc, les modules des deux valeurs propres sont 1.

Donc, le temps critique est  $0.05s$

### 1.3

$$\ddot{q}_0 = -\omega_0^2 q_0$$

## 1.4

$$\begin{pmatrix} 1 & 0 & -\beta\Delta t^2 \\ 0 & 1 & -\gamma\Delta t \\ mga \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} & 0 & ma^2 \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \end{pmatrix} \begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \\ \ddot{q}_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & \Delta t & \Delta t^2(0.5-\beta) \\ 0 & 1 & \Delta t(1-\gamma) \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} q_n \\ \dot{q}_n \\ \ddot{q}_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ F_0 \sin \omega t \begin{pmatrix} a \\ \frac{a}{\sqrt{2}} \end{pmatrix} \end{pmatrix}$$



## 1.5

```

clear all;
a=0.5
m=2;
g=9.81;
F0=20;
w=2*pi;
seita10=0;
seita20=0;
dseita10=-1.31519275;
dseita20=-1.85996342;
T0=8;
dt=0.05;
n=fix(T0/dt);
xm=[2 1;1 1];
xk=[2 0;0 1];

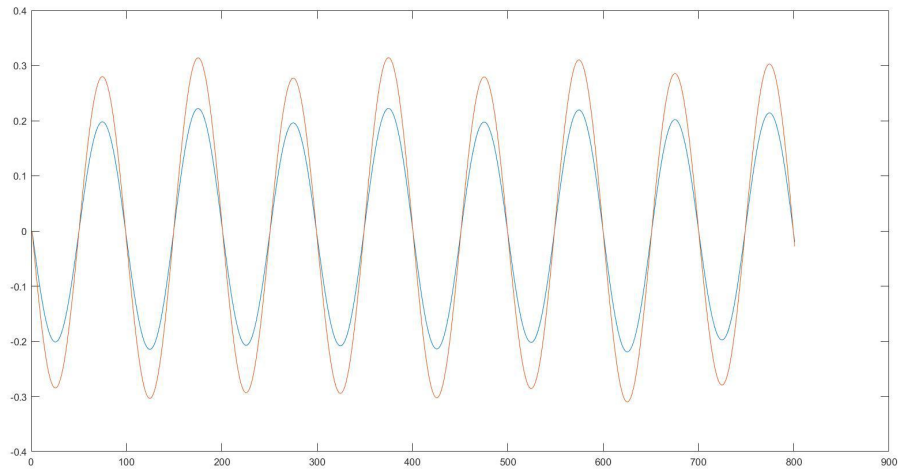
%1.5
b1=0;
y1=0.5;
A1=[1 0 -b1*dt^2;0 1 -y1*dt;w^2 0 1];
B1=[1 dt dt^2*(0.5-b1);0 1 dt*(1-y1);0 0 0];
C1=inv(A1)*B1;
X1=eig(C1);
C11=[1-(w^2*dt^2/2/(1+b1*w^2*dt^2))
dt/(1+b1*w^2*dt^2);-w^2*dt*(1-y1*w^2*dt^2/2/(1+b1*w^2
*dt^2)) 1-y1*w^2*dt^2/(1+b1*w^2*dt^2)];
X11=eig(C11);
Xm=real(X11(1))^2+imag(X11(1))^2;
q0=[seita10;seita20];
dq0=[dseita10;dseita20];
q(:,1)=[q0;dq0;-g/a*inv(xm)*xk*q0];

for i=1:n
    A111={eye(2) zeros(2) -b1*dt^2*eye(2);zeros(2)
eye(2) -y1*dt*eye(2);m*g*a*xk zeros(2) m*a^2*xm};
    A1111=cell2mat(A111);
    B111={eye(2) dt*eye(2) dt^2*(0.5-b1)*eye(2);zeros(2)
eye(2) dt*(1-y1)*eye(2);zeros(2) zeros(2) zeros(2)};
    B1111=cell2mat(B111);
    t=(i+1)*dt;
    C111={ [0;0]; [0;0]; F0*sin(w*t)*[a;a/sqrt(2)]};
    C1111=cell2mat(C111);

```

```
q(:, i+1)=inv(A1111)*(B1111*q(:, i)+C1111);  
end
```

```
plot(1:(n+1),q(1,:),1:(n+1),q(2,:))
```



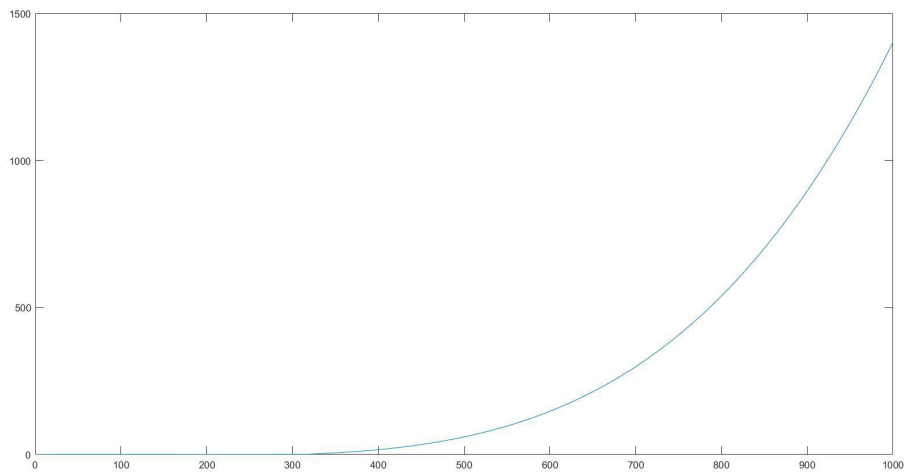
1.6

	$t=0s$	$t=0.02s$	$t=0.04s$	$t=5s$
$q$	0	-0.02630385500 00000	-0.05190406789 80187	0.017533234020 3781
	0	-0.03719926840 00000	-0.07340343659 85639	0.024795736925 0144
$dq$	-1.3151  9275000  0000	-1.29760169745 047	-1.25248100588 637	1.217773671653 70
	-1.8599  6342000  0000	-1.83508591496 410	-1.77127562117 719	1.722192039937 30
	0	1.759105254953 37	2.752963901455 98	-0.93569682854 9802
	0	2.487750503590 35	3.893278875099 84	-1.32327511714 612

## 2.1

$$C_{21} = \begin{pmatrix} 1 - \frac{\omega_0^2 \Delta t^2}{2(1 + \beta \omega_0^2 \Delta t^2)} & \frac{\Delta t}{1 + \beta \omega_0^2 \Delta t^2} \\ -\omega_0^2 \Delta t \left[ 1 - \frac{\gamma \omega_0^2 \Delta t^2}{2(1 + \beta \omega_0^2 \Delta t^2)} \right] & 1 - \frac{\gamma \omega_0^2 \Delta t^2}{1 + \beta \omega_0^2 \Delta t^2} \end{pmatrix}$$

## 2.2



*quand  $dt$  est supérieure que  $0.3s$ , il diverge.*

## 2.3

$$\ddot{q}_0 = -\omega_0^2 q_0$$

## 2.4

$$\begin{pmatrix} 1 & 0 & -\beta\Delta t^2 \\ 0 & 1 & -\gamma\Delta t \\ mga \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} & 0 & ma^2 \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \end{pmatrix} \begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \\ \ddot{q}_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & \Delta t & \Delta t^2(0.5-\beta) \\ 0 & 1 & \Delta t(1-\gamma) \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} q_n \\ \dot{q}_n \\ \ddot{q}_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ F_0 \sin \omega t \begin{pmatrix} a \\ \frac{a}{\sqrt{2}} \end{pmatrix} \end{pmatrix}$$

## 2.5

```

clear all;
a=0.5;
m=2;
g=9.81;
F0=20;
w=2*pi;
seita10=0;
seita20=0;
dseita10=-1.31519275;
dseita20=-1.85996342;
T0=8;
dt=0.02;
n=fix(T0/dt);
xm=[2 1;1 1];
xk=[2 0;0 1];

%1.5
b1=0;
y1=0.5;
A1=[1 0 -b1*dt^2;0 1 -y1*dt;w^2 0 1];
B1=[1 dt dt^2*(0.5-b1);0 1 dt*(1-y1);0 0 0];
C1=inv(A1)*B1;
X1=eig(C1);
C11=[1-(w^2*dt^2/2/(1+b1*w^2*dt^2))
dt/(1+b1*w^2*dt^2);-w^2*dt*(1-y1*w^2*dt^2/2/(1+b1*w^2
*dt^2)) 1-y1*w^2*dt^2/(1+b1*w^2*dt^2)];
X11=eig(C11);
Xm=real(X11(1))^2+imag(X11(1))^2;
q0=[seita10;seita20];
dq0=[dseita10;dseita20];
q(:,1)=[q0;dq0;-g/a*inv(xm)*xk*q0];

for i=1:n
    A111={eye(2) zeros(2) -b1*dt^2*eye(2);zeros(2)
eye(2) -y1*dt*eye(2);m*g*a*xk zeros(2) m*a^2*xm};
    A1111=cell2mat(A111);
    B111={eye(2) dt*eye(2) dt^2*(0.5-b1)*eye(2);zeros(2)
eye(2) dt*(1-y1)*eye(2);zeros(2) zeros(2) zeros(2)};
    B1111=cell2mat(B111);
    t=(i+1)*dt;
    C111={ [0;0]; [0;0]; F0*sin(w*t)*[a;a/sqrt(2)]};
    C1111=cell2mat(C111);

```



```

    q(:, i+1)=inv(A1111)*(B1111*q(:, i)+C1111);
end

%plot(1:(n+1),q(1,:),1:(n+1),q(2,:))

%2
b2=0;
y2=0.5;
n2=1000;
for i=1:n2
    dt=1/n2*i;
    C21=[1-(w^2*dt^2/2/(1+b1*w^2*dt^2))
dt/(1+b1*w^2*dt^2);-w^2*dt*(1-y1*w^2*dt^2/2/(1+b1*w^2
*dt^2)) 1-y1*w^2*dt^2/(1+b1*w^2*dt^2)];
    X2(:, i)=eig(C21);
    X2(1, i)=real(X2(1, i))^2+imag(X2(1, i));
    X2(2, i)=real(X2(2, i))^2+imag(X2(2, i));
end
plot(1:n2,max(X2(1,:),X2(2,:)))

%2.5

q0=[seita10;seita20];
dq0=[dseita10;dseita20];
q2(:, 1)=[q0;dq0;-g/a*inv(xm)*xk*q0];

for i=1:n
    A2={eye(2) zeros(2) -b2*dt^2*eye(2);zeros(2) eye(2)
-y2*dt*eye(2);m*g*a*xk zeros(2) m*a^2*xm};
    A22=cell2mat(A2);
    B2={eye(2) dt*eye(2) dt^2*(0.5-b2)*eye(2);zeros(2)
eye(2) dt*(1-y2)*eye(2);zeros(2) zeros(2) zeros(2)};
    B22=cell2mat(B2);
    t=(i+1)*dt;
    C2={[0;0];[0;0];F0*sin(w*t)*[a;a/sqrt(2)]};
    C22=cell2mat(C2);
    q2(:, i+1)=inv(A22)*(B22*q2(:, i)+C22);
end

plot(1:(n+1),q2(1,:),1:(n+1),q2(2,:))

```

## 2.6

	$dt=0s$	$dt=0.02s$	$dt=0.04s$	$dt=0.5s$
$q$	0	-1.3151927500000 0	12.485295709600 0	-4.73778408252833e+3 4
	0	-1.8599634200000 0	17.656874250800 0	6.70023850502457e+3 4
$dq$	-1.3151927500000 0	6.2426478548000 0	-57.947076962404 1	1.53873958368167e+3 6
	-1.8599634200000 0	8.8284371254000 0	-81.949533307543 9	-2.17610638819912e+3 6
$dd$ $q$	0	15.115681209600 0	-143.49513084400 8	3.17369326866994e+3 6
	0	21.376801090800 0	-202.93274195668 8	-4.48828006335576e+3 6

## *oscillateur non linéaire à un degré de liberté*

### **1.1**

$$q_{n+1} - \beta \Delta t^2 \ddot{q}_{n+1} = q_n + \Delta t \dot{q} + \Delta t^2 (0.5 - \beta) \ddot{q}_n$$

$$\dot{q}_{n+1} - \gamma \Delta t \ddot{q}_{n+1} = \dot{q}_n + \Delta t (1 - \gamma) \ddot{q}_n$$

$$\ddot{q}_{n+1} + \omega_0^2 q_{n+1} + \omega_0^2 q_{n+1}^3 = 0$$

## 1.2

```

clear all;
q0=2;
dq0=0;
w0=2*pi;
a=0.1;
T0=6;
dt=0.02;
n=fix(T0/dt);

%1
y1=0.5;
b1=0;
ddq0=-w0^2*q0-w0^2*q0^3;
q(:,1)=[q0;dq0;ddq0];

for i=1:n
    x1=q(1,i);
    x2=q(2,i);
    x3=q(3,i);
    syms x11 x22 x33;

    [q(1,i+1),q(2,i+1),q(3,i+1)]=solve(x11-b1*dt^2*x33==x
1+dt*x2+dt^2*(0.5-b1)*x3,x22-y1*dt*x33==x2+dt*(1-y1)*
x3,x33+w0^2*x11+w0^2*x11^3==0,x11,x22,x33);
end

plot(1:n+1,q(1,:))

```

## 1.3

	$t=0s$	$t=0.02s$	$t=0.04s$	$t=6s$
$q$	2	1.92104316479129	1.69979862739727	1.35093027615946
$dq$	0	-7.50503431506814	-13.6721659549249	18.0473401015936
$ddq$	-394.784176043574	-355.719255463240	-260.993908522431	-150.665238190023

## 2.1

*L'erreur entre la solution estimée et la solution exacte.*

## 2.2

$$\ddot{q}_{j+1}^* = \ddot{q}_{j+1} + \Delta\ddot{q}_{j+1}$$

$$\Delta\ddot{q}_{j+1} + \omega_0^2 \Delta q_{j+1} + 3\omega_0^2 \Delta q_{j+1} q_{j+1}^{2*} = -(\ddot{q}_{j+1}^* + \omega_0^2 q_{j+1}^* + \omega_0^2 q_{j+1}^{3*})$$

## 2.3

```

clear all;
q0=2;
dq0=0;
w0=2*pi;
a=0.1;
T0=6;
dt=0.02;
n=fix(T0/dt);

%1.2
% y1=0.5;
% b1=0;
% ddq0=-w0^2*q0-w0^2*q0^3;
% q(:,1)=[q0;dq0;ddq0];
%
% for i=1:n
%     x1=q(1,i);
%     x2=q(2,i);
%     x3=q(3,i);
%     syms x11 x22 x33;
%
[q(1,i+1),q(2,i+1),q(3,i+1)]=solve(x11-b1*dt^2*x33==x
1+dt*x2+dt^2*(0.5-b1)*x3,x22-y1*dt*x33==x2+dt*(1-y1)*
x3,x33+w0^2*x11+w0^2*x11^3==0,x11,x22,x33);
% end

% plot(1:n+1,q(1,:))

%2.3
y2=0.5;
b2=0.25;
e=0.01;
ddq0=-w0^2*q0-w0^2*q0^3;
q2(:,1)=[q0;dq0;ddq0];
for i=1:n
    qe=[1 dt dt^2*(0.5-b2);0 1 dt*(1-y2);0 0 0] *q2(:,i);
    z1=qe(1);
    z2=qe(2);
    z3=qe(3);
    while (abs(z3+w0^2*z1+w0^2*z1^3)>e)
        syms z11 z22 z33;

```



```
[z111, z222, z333]=solve(z11-b2*dt^2*z33==0, z22-y2*dt*z  
33==0, z33+w0^2*z11+3*w0^2*z11*z1^2==-(z3+w0^2*z1+w0^2  
*z1^3), z11, z22, z33);  
    qe=qe+[z111; z222; z333];  
    z1=qe(1);  
    z2=qe(2);  
    z3=qe(3);  
end  
    q2(:, i+1)=qe;  
end  
plot(1:n+1, q2(1, :))
```

## 2.4

	$t=0s$	$t=0.02s$	$t=0.04s$	$t=6s$
$q$	2	1.92477165802698	1.71199707168963	0.66395960988462 5
$dq$	0	-7.5228341973016 5	-13.754624436433 7	21.4448995141960
$dd$	-394.78417604357 4	-357.49924368659 1	-265.67978022661 4	-37.7663418997773
$q$				

### 3.1

$$E = \frac{1}{2}mv^2$$

## 3.2

```

clear all;
q0=2;
dq0=0;
w0=2*pi;
a=0.1;
T0=6;
dt=0.02;
n=fix(T0/dt);

%1.2
y1=0.5;
b1=0;
ddq0=-w0^2*q0-w0^2*q0^3;
q(:,1)=[q0;dq0;ddq0];

for i=1:n
    x1=q(1,i);
    x2=q(2,i);
    x3=q(3,i);
    syms x11 x22 x33;

    [q(1,i+1),q(2,i+1),q(3,i+1)]=solve(x11-b1*dt^2*x33==x
1+dt*x2+dt^2*(0.5-b1)*x3,x22-y1*dt*x33==x2+dt*(1-y1)*
x3,x33+w0^2*x11+w0^2*x11^3==0,x11,x22,x33);
end
E1=1/2*max(q(2,:))^2;
subplot(2,1,1)
plot(1:n+1,q(1,:))

```

## %2.3

```

y2=0.5;
b2=0.25;
e=0.01;
ddq0=-w0^2*q0-w0^2*q0^3;
q2(:,1)=[q0;dq0;ddq0];
for i=1:n
    qe=[1 dt dt^2*(0.5-b2);0 1 dt*(1-y2);0 0 0] *q2(:,i);
    z1=qe(1);
    z2=qe(2);
    z3=qe(3);
    while (abs(z3+w0^2*z1+w0^2*z1^3)>e)
        syms z11 z22 z33;
    end
end

```

```

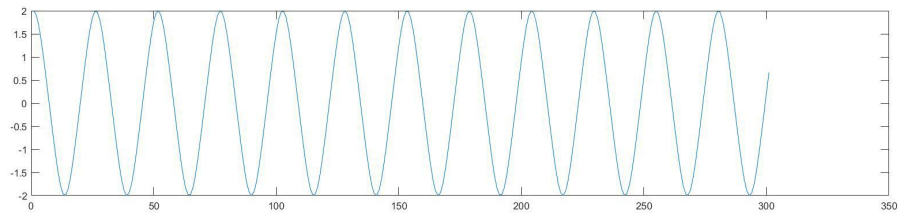
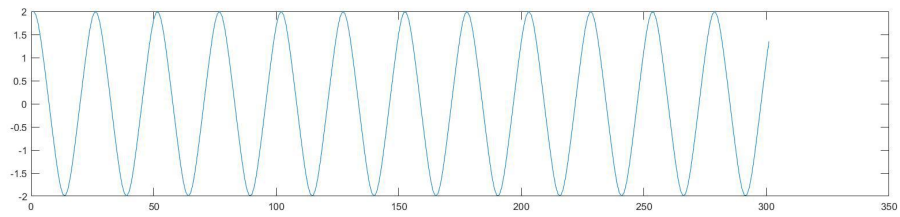
[z111, z222, z333]=solve(z11-b2*dt^2*z33==0, z22-y2*dt*z
33==0, z33+w0^2*z11+3*w0^2*z11*z1^2==-(z3+w0^2*z1+w0^2
*z1^3), z11, z22, z33);
    qe=qe+[z111; z222; z333];
    z1=qe(1);
    z2=qe(2);
    z3=qe(3);
end
    q2(:, i+1)=qe;
end
E2=1/2*max(q2(2, :))^2;
subplot(2, 1, 2)
plot(1:n+1, q2(1, :))

```

$E1=2.336107615270199e+02$

$E2=2.413743848261089e+02$

### 3.3



$$E1=2.336107615270199e+02$$

$$E2=2.413743848261089e+02$$

*Les deux méthodes sont presque même*

*oscillateur non linéaire à un degré de liberté***1.1.a**

$$\ddot{q} - \varepsilon \omega_0 \left[ 1 - \left( \frac{q}{u_0} \right)^2 \right] \dot{q} + \omega_0^2 q = 0$$

$$\begin{pmatrix} q \\ \dot{q} \end{pmatrix}' = \begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix} = \begin{pmatrix} \dot{q} \\ \varepsilon \omega_0 \left[ 1 - \left( \frac{q}{y_0} \right) \right] \dot{q} - \omega_0^2 q \end{pmatrix}$$

1.1. *b*

$$Q = \begin{pmatrix} q \\ \dot{q} \end{pmatrix}$$



## 1.2

$$f\left(\begin{pmatrix} q \\ \dot{q} \end{pmatrix}\right) = \begin{pmatrix} \dot{q} \\ \varepsilon\omega_0\left[1 - \left(\frac{q}{y_0}\right)\right]\dot{q} - \omega_0^2 q \end{pmatrix}$$

$$K1 = f(Q(:,i));$$

$$K2 = f(Q(:,i) + K1 * dt / 2);$$

$$K3 = f(Q(:,i) + K2 * dt / 2);$$

$$K4 = f(Q(:,i) + K3 * dt);$$

$$Q(:,i+1) = Q(:,i) + (K1 + 2 * K2 + 2 * K3 + K4) / 6 * dt;$$

## 1.3

```

function Y=f(X)
    e=4;
    w0=2*pi;
    y0=2;
    Y(1,1)=X(1,1);
    Y(2,1)=e*w0*(1-X(1,1)/y0)*X(2,1)-w0^2*X(1,1);
end

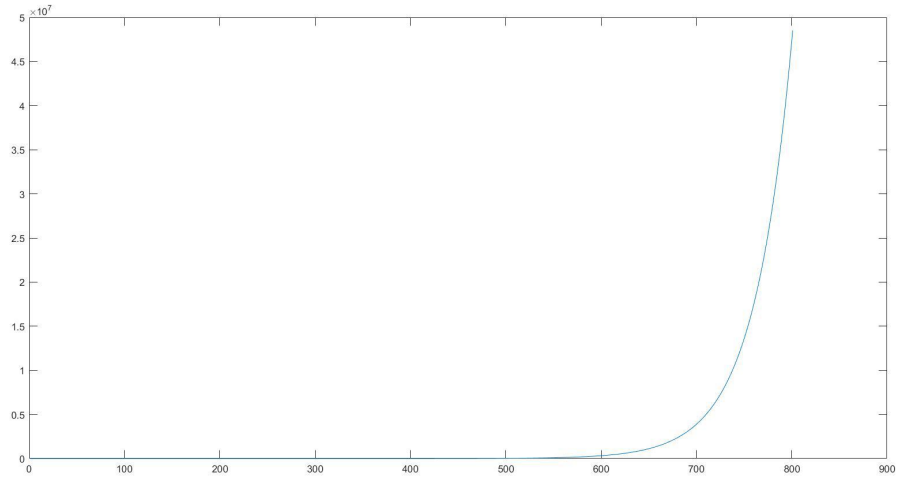
clear all;
q0=0.1;
dq0=0;
w0=2*pi;
y0=2;
T0=20;
dt=0.025;
n=fix(T0/dt);

%1.3
e=4;

Q(:,1)=[q0;dq0];
for i=1:n
    K1=f(Q(:,i));
    K2=f(Q(:,i)+K1*dt/2);
    K3=f(Q(:,i)+K2*dt/2);
    K4=f(Q(:,i)+K3*dt);
    Q(:,i+1)=Q(:,i)+(K1+2*K2+2*K3+K4)/6*dt;
end

plot(1:n+1,Q(1,:))

```



## 1.4

	$t=0s$	$t=0.025s$	$t=0.05s$	$t=0.125s$
$q$	0.100000000000000 0	0.102531512044271	0.105127109620845	0.11331484526152 5
$d$	0	-0.13637658473267 3	-0.38710166850606 1	-3.18278523572325
$q$				

## 2.1

$$q_{n+1} - \beta \Delta t^2 \ddot{q}_{n+1} = q_n + \Delta t \dot{q} + \Delta t^2 (0.5 - \beta) \ddot{q}_n$$

$$\dot{q}_{n+1} - \gamma \Delta t \ddot{q}_{n+1} = \dot{q}_n + \Delta t (1 - \gamma) \ddot{q}_n$$

$$\ddot{q}_{n+1} + \omega_0^2 q_{n+1} - \varepsilon \omega_0 \left[ 1 - \left( \frac{q_{n+1}}{y_0} \right) \right] \dot{q}_{n+1} = 0$$

## 2.2

```
clear all;
q0=0.1;
dq0=0;
w0=2*pi;
y0=2;
T0=20;
dt=0.025;
n=fix(T0/dt);
```

```
%1.3
```

```
e=4;
```

```
Q(:,1)=[q0;dq0];
for i=1:n
    K1=f(Q(:,i));
    K2=f(Q(:,i)+K1*dt/2);
    K3=f(Q(:,i)+K2*dt/2);
    K4=f(Q(:,i)+K3*dt);
    Q(:,i+1)=Q(:,i)+(K1+2*K2+2*K3+K4)/6*dt;
end
```

```
%plot(1:n+1,Q(1,:))
```

```
%2.3
```

```
ddq0=e*w0*(1-q0/y0)*dq0-w0^2*q0;
```

```
q(:,1)=[q0;dq0;ddq0];
```

```
y2=0.5;
```

```
b2=0;
```

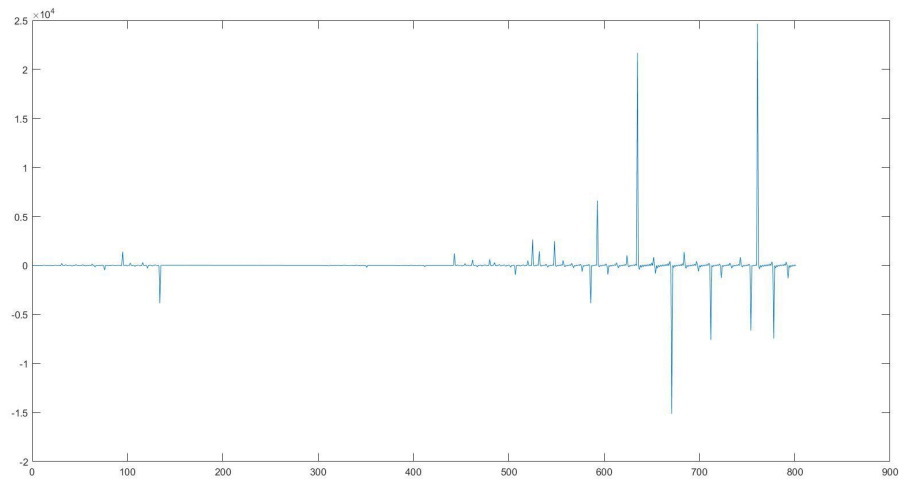
```
for i=1:n
    z1=q(1,i);
    z2=q(2,i);
    z3=q(3,i);
    syms z11 z22 z33;
```

```
[q(1,i+1),q(2,i+1),q(3,i+1)]=solve(z11-b2*dt^2*z33==z1+dt*z2+dt^2*(0.5-b2)*z3,z22-y2*dt*z33==z2+dt*(1-y2)*z3,z33+w0^2*z11-e*w0*[1-z11/y0]*z22,z11,z22,z33);
```

```
end
```

```
plot(1:n+1,q(1,:))
```

MecaNumTD2GUOZelingSY1924109Christian

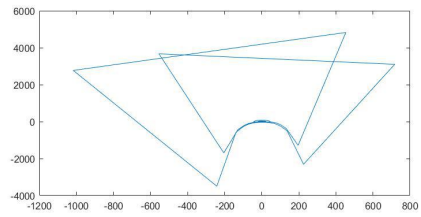
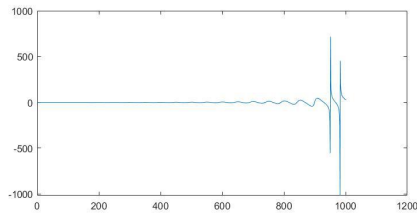
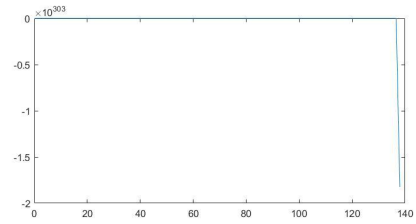
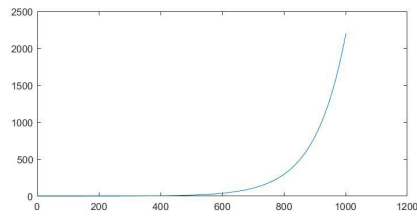


## 2.3

	$t=0s$	$t=0.025s$	$t=0.05s$	$t=0.125s$
$q$	0.10000000000000 00	0.09876629944986 38	0.09300730372183 62	-0.005827956109248 44
$dq$	0	-0.13985392556327 5	-0.39439928666078 9	-3.26938436633972
$dd$ $q$	-3.9478417604357 4	-7.24047228462629	-13.1231566031748	-82.1779502430733



### 3.1.1



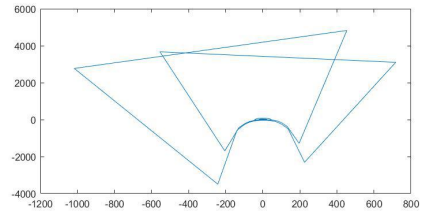
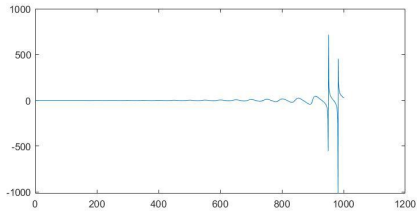
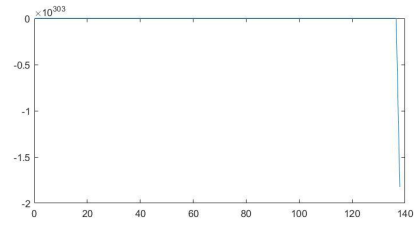
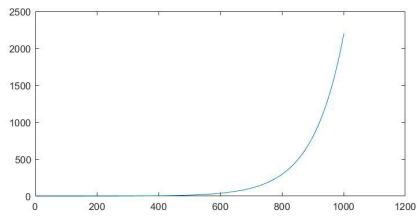
*Runge Kutta première ligne à gauche*

*Newmark deuxième ligne à gauche*

### 3.1.2

*Newmark diverge beaucoup moins vite*

### 3.1.3



### 3.2.1

```
clear all;
q0=0.1;
dq0=0;
w0=2*pi;
y0=2;
T0=20;
dt1=0.01;
dt=0.02;
n=fix(T0/dt);
```

```
%1.3
```

```
e=5;
```

```
Q(:,1)=[q0;dq0];
for i=1:n
    K1=f(Q(:,i));
    K2=f(Q(:,i)+K1*dt1/2);
    K3=f(Q(:,i)+K2*dt1/2);
    K4=f(Q(:,i)+K3*dt1);
    Q(:,i+1)=Q(:,i)+(K1+2*K2+2*K3+K4)/6*dt1;
end
subplot(2,2,1)
plot(1:n+1,Q(1,:))
subplot(2,2,2)
plot(Q(1,:),Q(2,)/w0)
```

```
%2.3
```

```
ddq0=e*w0*(1-q0/y0)*dq0-w0^2*q0;
```

```
q(:,1)=[q0;dq0;ddq0];
```

```
y2=0.5;
```

```
b2=0;
```

```
for i=1:n
```

```
    z1=q(1,i);
```

```
    z2=q(2,i);
```

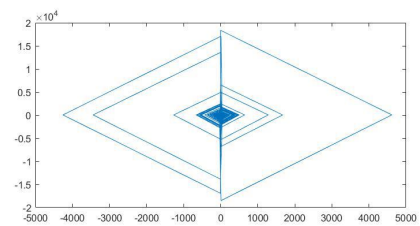
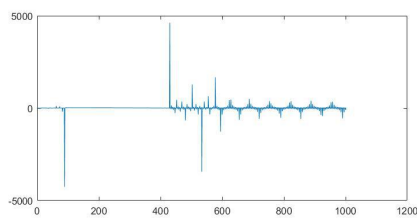
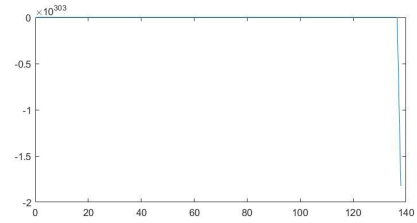
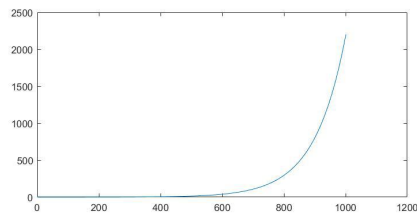
```
    z3=q(3,i);
```

```
    syms z11 z22 z33;
```

```
[q(1,i+1),q(2,i+1),q(3,i+1)]=solve(z11-b2*dt^2*z33==z1+dt*z2+dt^2*(0.5-b2)*z3,z22-y2*dt*z33==z2+dt*(1-y2)*z3,z33+w0^2*z11-e*w0*[1-z11/y0]*z22,z11,z22,z33);
```

end

```
subplot(2,2,3)
plot(1:n+1,q(1,:))
subplot(2,2,4)
plot(q(1,:),q(2,:)/w0)
```



*Runge Kutta première ligne à gauche*

*Newmark deuxième ligne à droite*

3.2.2

*Non, il y a des erreurs.*

3.2.3

$t < 0.01s$

### 3.2.4

