Etude d'un oscillateur linéaire amorti à un degré de liberté

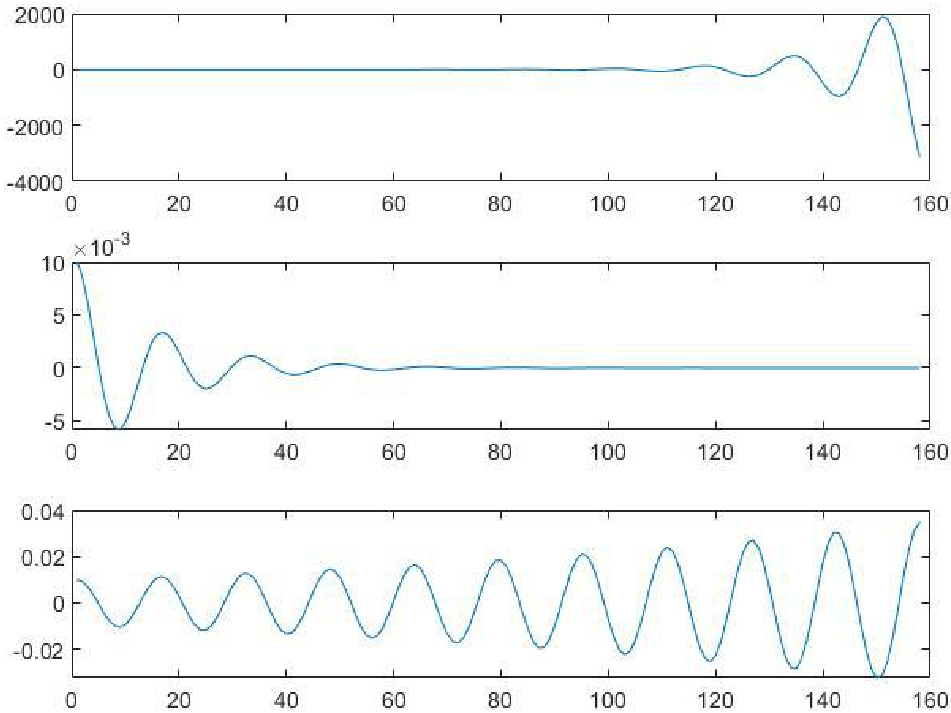
1.1.

```
clear all;
T0=1;
w0=2*pi/T0;
e=0.02;
m=1;
b=2*w0*m;
x0=0.01;
dx0=0;
v=2*e/w0;
F=0;
h=1;
dt=0.001*v*h;
n=fix(100*T0/dt);
W=w0*sqrt(1-e^2);

%1.1
U1(:,1)=[x0,dx0];
A1=[0 1;-w0^2 2*e*w0];
B1=[0; F/m];
X(1)=x0;
for i=1:n
    U1(:,i+1)=U1(:,i)+A1*U1(:,i)*dt+B1;

X(i+1)=exp(-e*w0*i*dt)*(x0*cos(W*i*dt)+(e*w0*x0+dx0)/
W*sin(W*i*dt));
end
subplot(3,1,1)
plot(1:n+1,U1(1,:),1:n+1,X)
```

a)

dt=2*e/w0*10

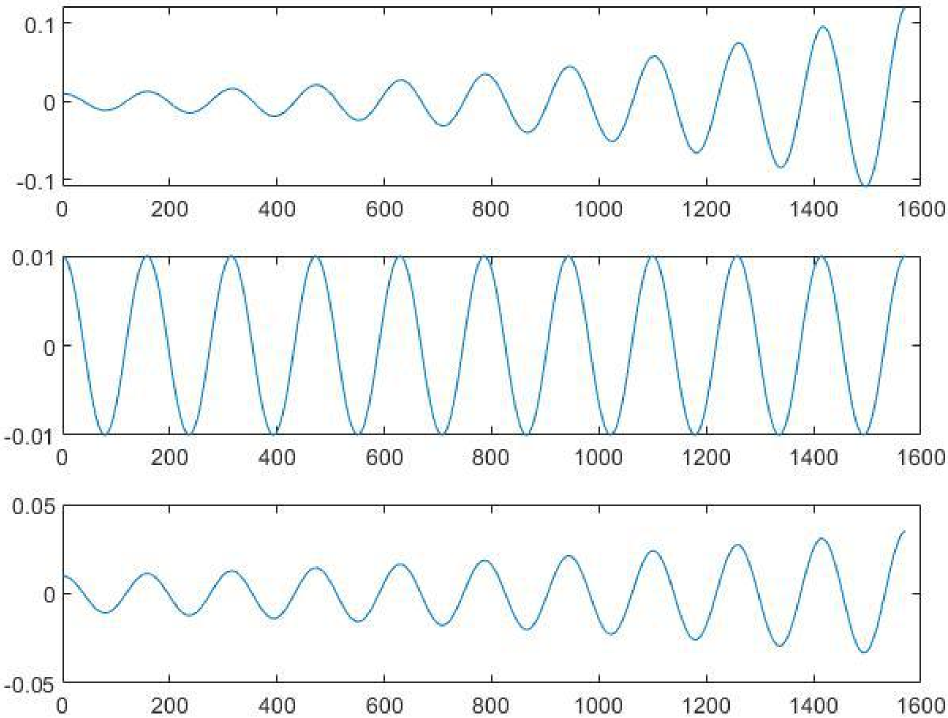
première figure



b)

dt=2*e/w0

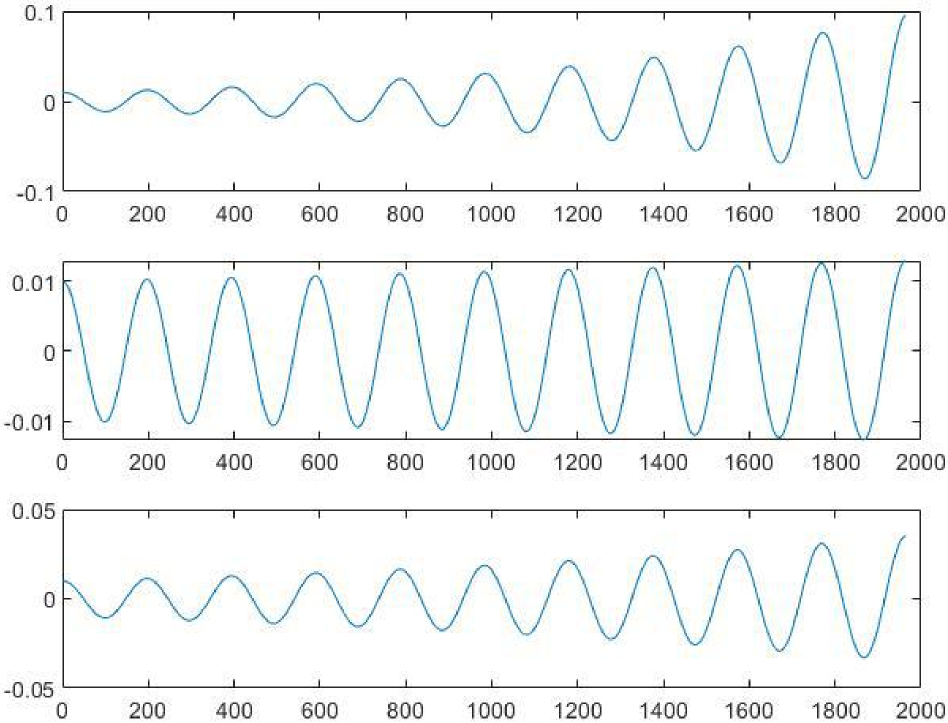
première figure



c)

dt=2*e/w0*0.8

prmière figure



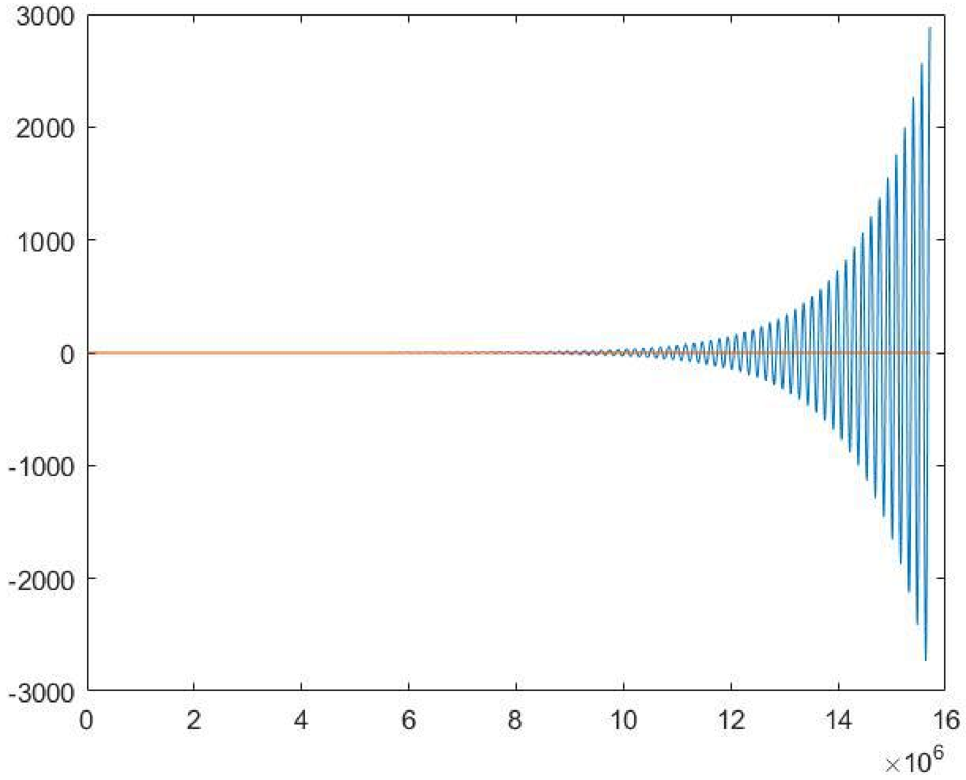
d)

la convergence.

S'il est converge, on peut calculer l'erreur entre le résultat infini et le résultat exacte.

$$\Delta t_c = \frac{2\varepsilon}{\omega_0}$$

Donc, à partir la valeur de 0.001, il est précis suffisante



1.2

```
clear all;
T0=1;
w0=2*pi/T0;
e=0.02;
m=1;
b=2*w0*m;
x0=0.01;
dx0=0;
v=2*e/w0;
F=0;
dt=v;
n=fix(10*T0/dt);

%1.1
U1(:,1)=[x0,dx0];
A1=[0 1;-w0^2 2*e*w0];
B1=[0; F/m];
for i=1:n
    U1(:,i+1)=U1(:,i)+A1*U1(:,i)*dt+B1;
end
subplot(3,1,1)
plot(1:n+1,U1(1,:))

%1.2
U2(:,1)=[x0,dx0];
A2=[0 1;-w0^2 2*e*w0];
B2=[0; F/m];
for i=1:n
    U2(:,i+1)=inv(eye(2)-A2*dt)*(U2(:,i)+B2);
end
subplot(3,1,2)
plot(1:n+1,U2(1,:))
```

$$\Delta t = \frac{2\varepsilon}{\omega_0} = \frac{0.04}{2\pi} = 0.0064\text{s}$$

1.3

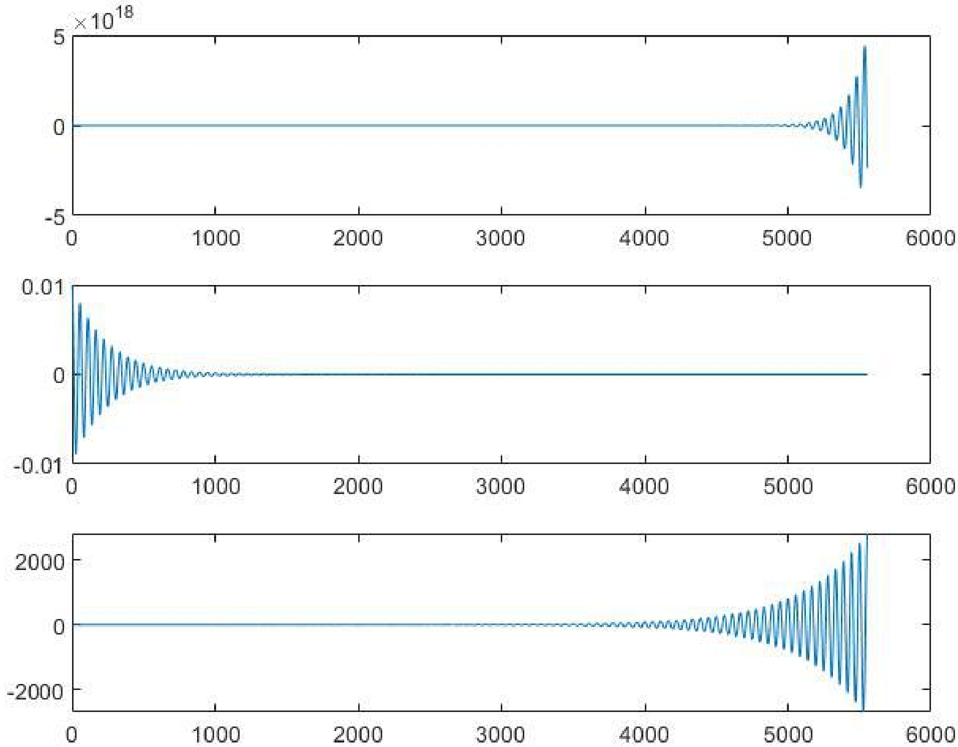
```
clear all;
T0=1;
w0=2*pi/T0;
e=0.02;
m=1;
b=2*w0*m;
x0=0.01;
dx0=0;
v=2*e/w0;
F=0;
h=1;
dt=0.001*v*h;
n=fix(100*T0/dt);
W=w0*sqrt(1-e^2);

%1.1
U1(:,1)=[x0,dx0];
A1=[0 1;-w0^2 2*e*w0];
B1=[0; F/m];
X(1)=x0;
for i=1:n
    U1(:,i+1)=U1(:,i)+A1*U1(:,i)*dt+B1;

X(i+1)=exp(-e*w0*i*dt)*(x0*cos(W*i*dt)+(e*w0*x0+dx0)/
W*sin(W*i*dt));
end
subplot(3,1,1)
plot(1:n+1,U1(1,:),1:n+1,X)

%1.2
U2(:,1)=[x0,dx0];
A2=[0 1;-w0^2 2*e*w0];
B2=[0; F/m];
for i=1:n
    U2(:,i+1)=inv(eye(2)-A2*dt)*(U2(:,i)+B2);
end
subplot(3,1,2)
plot(1:n+1,U2(1,:))

%1.3
U3(:,1)=[x0,dx0];
A3=[0 1;-w0^2 2*e*w0];
```

```
B3=[0; F/m];
for i=1:n
    K1=A3*U3(:,i);
    K2=A3*(U3(:,i)+K1*dt/2);
    K3=A3*(U3(:,i)+K2*dt/2);
    K4=A3*(U3(:,i)+K3*dt);
    U3(:,i+1)=U3(:,i)+dt*(K1+2*K2+2*K3+K4)/6;
end
subplot(3,1,3)
plot(1:n+1,U3(1,:))
```

1.3.a

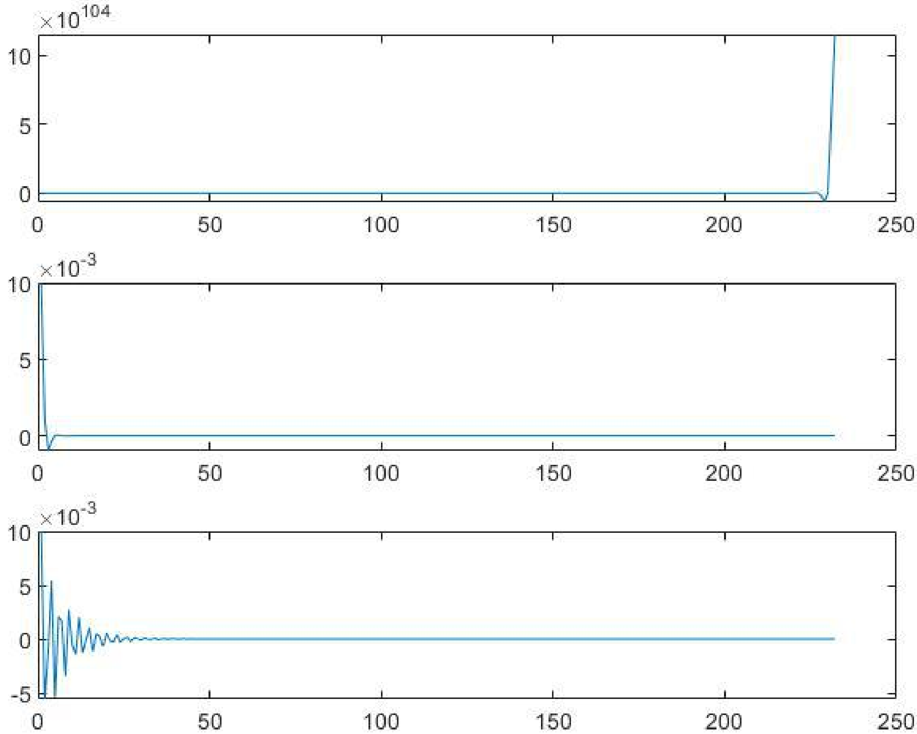
h=0.04

troisième figure



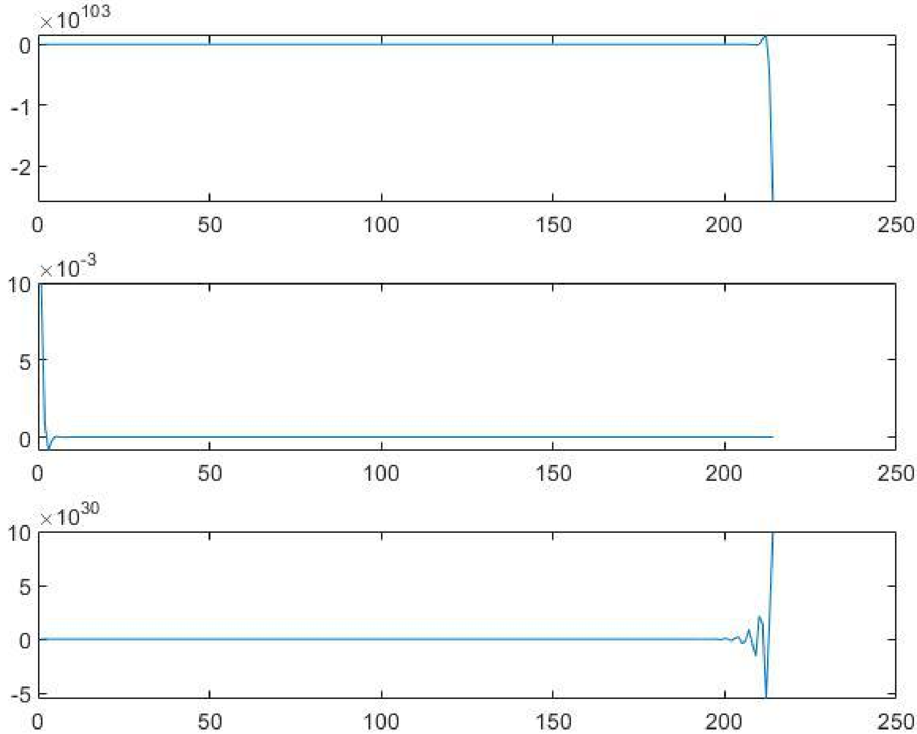
h=0.96

troisième figure



h=1.04

troisième figure



quand h augemente, la précision dimunue mais la stabilité augemente.

1.3.b

$h_{\max} = 0.015$

$h_{\min} = 0.014$

$h_c = 0.0141$

$\Delta t_c = 0.0064s$

Etude d'un double pendule avec l'hypothèse des petits mouvements

1.1

Matrice de l'amplification C1=inv(A1)*B1

$$A1 = \begin{pmatrix} 1 & 0 & -\beta\Delta t^2 \\ 0 & 1 & -\gamma\Delta t \\ \omega_0^2 & 0 & 1 \end{pmatrix}$$

$$B1 = \begin{pmatrix} 1 & \Delta t & \Delta t^2(0.5-\beta) \\ 0 & 1 & \Delta t(1-\gamma) \\ 0 & 0 & 0 \end{pmatrix}$$

et $\ddot{q} = -\omega_0^2 q$

Après avoir simplifié, on a

$$C11 = \begin{pmatrix} 1-\dfrac{\omega_0^2\Delta t^2}{2(1+\beta\omega_0^2\Delta t^2)} & \dfrac{\Delta t}{1+\beta\omega_0^2\Delta t^2} \\ -\omega_0^2\Delta t[1-\dfrac{\gamma\omega_0^2\Delta t^2}{2(1+\beta\omega_0^2\Delta t^2)}] & 1-\dfrac{\gamma\omega_0^2\Delta t^2}{1+\beta\omega_0^2\Delta t^2} \end{pmatrix}$$

1.2

quand dt=0.05s, les valeurs propres sont

0.950651977994553 + 0.310259273407006i

0.950651977994553 - 0.310259273407006i

Donc, les modules des deux valeurs propres sont 1.

On a $\varepsilon = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ dans l'équation (1)

Donc, le temps critique est 0s

1.3

$$\ddot{q}_0 + 0 \times \dot{q}_0 + \omega_0^2 q_0 = F_0 \sin \omega t \begin{pmatrix} a \\ \dfrac{a}{\sqrt{2}} \end{pmatrix}$$

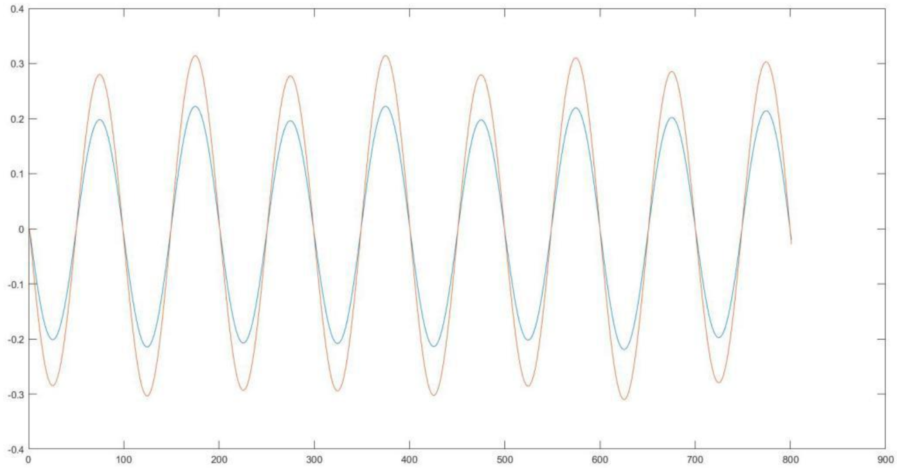
1.4

$$\begin{pmatrix} 1 & 0 & -\beta\Delta t^2 \\ 0 & 1 & -\gamma\Delta t \\ mga\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} & 0 & ma^2\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \end{pmatrix}\begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \\ \ddot{q}_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & \Delta t & \Delta t^2(0.5-\beta) \\ 0 & 1 & \Delta t(1-\gamma) \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} q_n \\ \dot{q}_n \\ \ddot{q}_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ F_0 \sin\omega t\begin{pmatrix} a \\ \dfrac{a}{\sqrt{2}} \end{pmatrix} \end{pmatrix}$$

1.5

```
clear all;
a=0.5
m=2;
g=9.81;
F0=20;
w=2*pi;
seita10=0;
seita20=0;
dseita10=-1.31519275;
dseita20=-1.85996342;
T0=8;
dt=0.05;
n=fix(T0/dt);
xm=[2 1;1 1];
xk=[2 0;0 1];

%1.5
b1=0;
y1=0.5;
A1=[1 0 -b1*dt^2;0 1 -y1*dt;w^2 0 1];
B1=[1 dt dt^2*(0.5-b1);0 1 dt*(1-y1);0 0 0];
C1=inv(A1)*B1;
X1=eig(C1);
C11=[1-(w^2*dt^2/2/(1+b1*w^2*dt^2))
dt/(1+b1*w^2*dt^2);-w^2*dt*(1-y1*w^2*dt^2/2/(1+b1*w^2
*dt^2)) 1-y1*w^2*dt^2/(1+b1*w^2*dt^2)];
X11=eig(C11);
Xm=real(X11(1))^2+imag(X11(1))^2;
q0=[seita10;seita20];
dq0=[dseita10;dseita20];
q(:,1)=[q0;dq0;-g/a*inv(xm)*xk*q0];

for i=1:n
   A111={eye(2) zeros(2) -b1*dt^2*eye(2);zeros(2)
eye(2) -y1*dt*eye(2);m*g*a*xk zeros(2) m*a^2*xm};
   A1111=cell2mat(A111);
   B111={eye(2) dt*eye(2) dt^2*(0.5-b1)*eye(2);zeros(2)
eye(2) dt*(1-y1)*eye(2);zeros(2) zeros(2) zeros(2)};
   B1111=cell2mat(B111);
   t=(i+1)*dt;
   C111={[0;0];[0;0];F0*sin(w*t)*[a;a/sqrt(2)]};
   C1111=cell2mat(C111);
```

```
    q(:,i+1)=inv(A1111)*(B1111*q(:,i)+C1111);
end

plot(1:(n+1),q(1,:),1:(n+1),q(2,:))
```



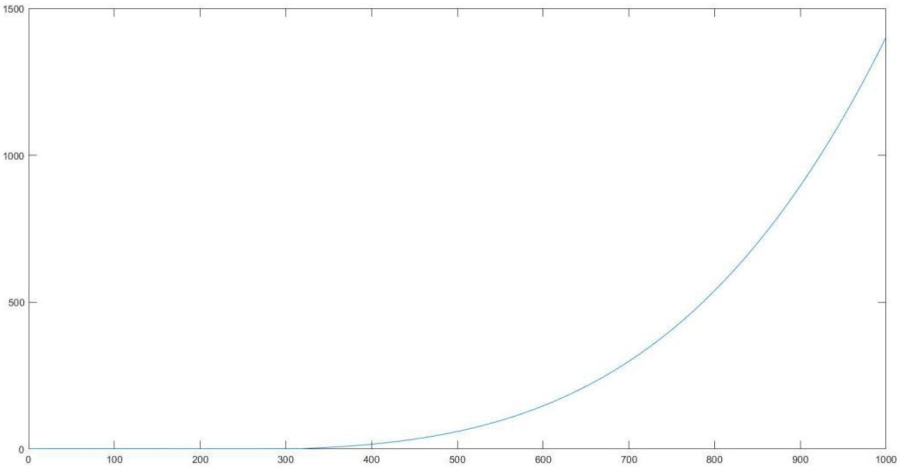
1.6

	t=0s	t=0.02s	t=0.04s	t=5s
q	0	-0.0263038550000000	-0.0519040678980187	0.0175332340203781
	0	-0.0371992684000000	-0.0734034365985639	0.0247957369250144
dq	-1.31519275000000	-1.29760169745047	-1.25248100588637	1.21777367165370
	-1.85996342000000	-1.83508591496410	-1.77127562117719	1.72219203993730
	0	1.75910525495337	2.75296390145598	-0.935696828549802
	0	2.48775050359035	3.89327887509984	-1.32327511714612

2.1

$$C21=\begin{pmatrix} 1-\dfrac{\omega_0^2\Delta t^2}{2(1+\beta\omega_0^2\Delta t^2)} & \dfrac{\Delta t}{1+\beta\omega_0^2\Delta t^2} \\ -\omega_0^2\Delta t[1-\dfrac{\gamma\omega_0^2\Delta t^2}{2(1+\beta\omega_0^2\Delta t^2)}] & 1-\dfrac{\gamma\omega_0^2\Delta t^2}{1+\beta\omega_0^2\Delta t^2} \end{pmatrix}$$

2.2



quand dt est supérieure que 0.3s, il diverge.

2.3

$$\ddot{q}_0 + 0 \times \dot{q}_0 + \omega_0^2 q_0 = F_0 \sin \omega t \begin{pmatrix} a \\ \dfrac{a}{\sqrt{2}} \end{pmatrix}$$

2.4

$$\begin{pmatrix} 1 & 0 & -\beta\Delta t^2 \\ 0 & 1 & -\gamma\Delta t \\ mga\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} & 0 & ma^2\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \end{pmatrix} \begin{pmatrix} q_{n+1} \\ \dot{q}_{n+1} \\ \ddot{q}_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & \Delta t & \Delta t^2(0.5-\beta) \\ 0 & 1 & \Delta t(1-\gamma) \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} q_n \\ \dot{q}_n \\ \ddot{q}_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ F_0 \sin\omega t \begin{pmatrix} a \\ \frac{a}{\sqrt{2}} \end{pmatrix} \end{pmatrix}$$

2.5

```
clear all;
a=0.5;
m=2;
g=9.81;
F0=20;
w=2*pi;
seita10=0;
seita20=0;
dseita10=-1.31519275;
dseita20=-1.85996342;
T0=8;
dt=0.02;
n=fix(T0/dt);
xm=[2 1;1 1];
xk=[2 0;0 1];

%1.5
b1=0;
y1=0.5;
A1=[1 0 -b1*dt^2;0 1 -y1*dt;w^2 0 1];
B1=[1 dt dt^2*(0.5-b1);0 1 dt*(1-y1);0 0 0];
C1=inv(A1)*B1;
X1=eig(C1);
C11=[1-(w^2*dt^2/2/(1+b1*w^2*dt^2))
dt/(1+b1*w^2*dt^2);-w^2*dt*(1-y1*w^2*dt^2/2/(1+b1*w^2
*dt^2)) 1-y1*w^2*dt^2/(1+b1*w^2*dt^2)];
X11=eig(C11);
Xm=real(X11(1))^2+imag(X11(1))^2;
q0=[seita10;seita20];
dq0=[dseita10;dseita20];
q(:,1)=[q0;dq0;-g/a*inv(xm)*xk*q0];

for i=1:n
    A111={eye(2) zeros(2) -b1*dt^2*eye(2);zeros(2)
eye(2) -y1*dt*eye(2);m*g*a*xk zeros(2) m*a^2*xm};
    A1111=cell2mat(A111);
    B111={eye(2) dt*eye(2) dt^2*(0.5-b1)*eye(2);zeros(2)
eye(2) dt*(1-y1)*eye(2);zeros(2) zeros(2) zeros(2)};
    B1111=cell2mat(B111);
    t=(i+1)*dt;
    C111={[0;0];[0;0];F0*sin(w*t)*[a;a/sqrt(2)]};
    C1111=cell2mat(C111);
```

```
    q(:,i+1)=inv(A1111)*(B1111*q(:,i)+C1111);
end

%plot(1:(n+1),q(1,:),1:(n+1),q(2,:))

%2
b2=0;
y2=0.5;
n2=1000;
for i=1:n2
    dt=1/n2*i;
    C21=[1-(w^2*dt^2/2/(1+b1*w^2*dt^2))
dt/(1+b1*w^2*dt^2);-w^2*dt*(1-y1*w^2*dt^2/2/(1+b1*w^2
*dt^2)) 1-y1*w^2*dt^2/(1+b1*w^2*dt^2)];
    X2(:,i)=eig(C21);
    X2(1,i)=real(X2(1,i))^2+imag(X2(1,i));
    X2(2,i)=real(X2(2,i))^2+imag(X2(2,i));
end
plot(1:n2,max(X2(1,:),X2(2,:)))

%2.5

q0=[seita10;seita20];
dq0=[dseita10;dseita20];
q2(:,1)=[q0;dq0;-g/a*inv(xm)*xk*q0];

for i=1:n
    A2={eye(2) zeros(2) -b2*dt^2*eye(2);zeros(2) eye(2)
-y2*dt*eye(2);m*g*a*xk zeros(2) m*a^2*xm};
    A22=cell2mat(A2);
    B2={eye(2) dt*eye(2) dt^2*(0.5-b2)*eye(2);zeros(2)
eye(2) dt*(1-y2)*eye(2);zeros(2) zeros(2) zeros(2)};
    B22=cell2mat(B2);
    t=(i+1)*dt;
    C2={[0;0];[0;0];F0*sin(w*t)*[a;a/sqrt(2)]};
    C22=cell2mat(C2);
    q2(:,i+1)=inv(A22)*(B22*q2(:,i)+C22);
end

plot(1:(n+1),q2(1,:),1:(n+1),q2(2,:))
```

2.6

	dt=0s	dt=0.02s	dt=0.04s	dt=0.5s
q	0	-1.31519275000000	12.4852957096000	-4.73778408252833e+34
	0	-1.85996342000000	17.6568742508000	6.70023850502457e+34
dq	-1.31519275000000	6.24264785480000	-57.9470769624041	1.53873958368167e+36
	-1.85996342000000	8.82843712540000	-81.9495333075439	-2.17610638819912e+36
ddq	0	15.1156812096000	-143.495130844008	3.17369326866994e+36
	0	21.3768010908000	-202.932741956688	-4.48828006335576e+36

oscillateur non linéaire à un degré de liberté (P26-P27)

1.1

$$q_{n+1} - \beta\Delta t^2 \ddot{q}_{n+1} = q_n + \Delta t\dot{q} + \Delta t^2 (0.5 - \beta)\ddot{q}_n$$

$$\dot{q}_{n+1} - \gamma\Delta t\ddot{q}_{n+1} = \dot{q}_n + \Delta t(1 - \gamma)\ddot{q}_n$$

$$\ddot{q}_{n+1} + \omega_0^2 q_{n+1} + \omega_0^2 q_{n+1}^3 = 0$$

1.2

```
clear all;
q0=2;
dq0=0;
w0=2*pi;
a=0.1;
T0=6;
dt=0.02;
n=fix(T0/dt);

%1
y1=0.5;
b1=0;
ddq0=-w0^2*q0-w0^2*q0^3;
q(:,1)=[q0;dq0;ddq0];

for i=1:n
    x1=q(1,i);
    x2=q(2,i);
    x3=q(3,i);
    syms x11 x22 x33;

[q(1,i+1),q(2,i+1),q(3,i+1)]=solve(x11-b1*dt^2*x33==x
1+dt*x2+dt^2*(0.5-b1)*x3,x22-y1*dt*x33==x2+dt*(1-y1)*
x3,x33+w0^2*x11+w0^2*x11^3==0,x11,x22,x33);
end

plot(1:n+1,q(1,:))
```

1.3

	t=0s	t=0.02s	t=0.04s	t=6s
q	2	1.92104316479129	1.69979862739727	1.35093027615946
dq	0	-7.50503431506814	-13.6721659549249	18.0473401015936
ddq	-394.784176043574	-355.719255463240	-260.993908522431	-150.665238190023

2.1

L'erreur entre la solution estimée et la solution exacte.

2.2

$$\ddot{q}^{*}_{j+1} = \ddot{q}^{*}_{j+1} + \Delta\ddot{q}_{j+1}$$

$$\Delta\ddot{q}_{j+1} + \omega_0^2 \Delta q_{j+1} + 3\omega_0^2 \Delta q_{j+1} q^{2*}_{j+1} = -(\ddot{q}^{*}_{j+1} + \omega_0^2 q^{*}_{j+1} + \omega_0^2 q^{3*}_{j+1})$$

2.3

```
clear all;
q0=2;
dq0=0;
w0=2*pi;
a=0.1;
T0=6;
dt=0.02;
n=fix(T0/dt);

%1.2
% y1=0.5;
% b1=0;
% ddq0=-w0^2*q0-w0^2*q0^3;
% q(:,1)=[q0;dq0;ddq0];
%
% for i=1:n
%     x1=q(1,i);
%     x2=q(2,i);
%     x3=q(3,i);
%     syms x11 x22 x33;
%
[q(1,i+1),q(2,i+1),q(3,i+1)]=solve(x11-b1*dt^2*x33==x
1+dt*x2+dt^2*(0.5-b1)*x3,x22-y1*dt*x33==x2+dt*(1-y1)*
x3,x33+w0^2*x11+w0^2*x11^3==0,x11,x22,x33);
% end

% plot(1:n+1,q(1,:))

%2.3
y2=0.5;
b2=0.25;
e=0.01;
ddq0=-w0^2*q0-w0^2*q0^3;
q2(:,1)=[q0;dq0;ddq0];
for i=1:n
   qe=[1 dt dt^2*(0.5-b2);0 1 dt*(1-y2);0 0 0] *q2(:,i);
       z1=qe(1);
       z2=qe(2);
       z3=qe(3);
   while (abs(z3+w0^2*z1+w0^2*z1^3)>e)
       syms z11 z22 z33;
```

```
[z111,z222,z333]=solve(z11-b2*dt^2*z33==0,z22-y2*dt*z
33==0,z33+w0^2*z11+3*w0^2*z11*z1^2==-(z3+w0^2*z1+w0^2
*z1^3),z11,z22,z33);
        qe=qe+[z111;z222;z333];
        z1=qe(1);
        z2=qe(2);
        z3=qe(3);
    end
    q2(:,i+1)=qe;
end
plot(1:n+1,q2(1,:))
```

2.4

	t=0s	t=0.02s	t=0.04s	t=6s
q	2	1.92477165802698	1.71199707168963	0.663959609884625
dq	0	-7.52283419730165	-13.7546244364337	21.4448995141960
ddq	-394.784176043574	-357.499243686591	-265.679780226614	-37.7663418997773

3.1

$$E = \frac{1}{2}mv^2 + \int_0^x F dx$$

3.2

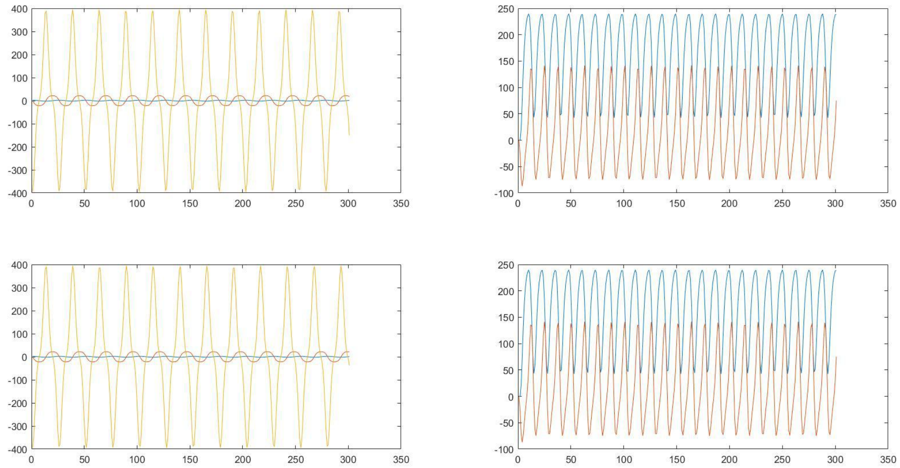
```
clear all;
q0=2;
dq0=0;
w0=2*pi;
a=0.1;
T0=6;
dt=0.02;
n=fix(T0/dt);

%1.2
y1=0.5;
b1=0;
ddq0=-w0^2*q0-w0^2*q0^3;
q(:,1)=[q0;dq0;ddq0];
E11(1)=dq0^2/2;
E111(1)=0;
for i=1:n
    x1=q(1,i);
    x2=q(2,i);
    x3=q(3,i);
    syms x11 x22 x33;

[q(1,i+1),q(2,i+1),q(3,i+1)]=solve(x11-b1*dt^2*x33==x
1+dt*x2+dt^2*(0.5-b1)*x3,x22-y1*dt*x33==x2+dt*(1-y1)*
x3,x33+w0^2*x11+w0^2*x11^3==0,x11,x22,x33);
    E111(i+1)=E111(i)+q(3,i+1)*(q(1,i)-q(1,i+1));
    E11(i+1)=E111(i+1)+q(2,i+1)^2/2;
end
E1=1/2*max(q(2,:))^2;
subplot(2,2,1)
plot(1:n+1,q(1,:))
subplot(2,2,2)
plot(1:n+1,E11,1:n+1,E111)

%2.3
y2=0.5;
b2=0.25;
e=0.01;
ddq0=-w0^2*q0-w0^2*q0^3;
q2(:,1)=[q0;dq0;ddq0];
E22(1)=dq0^2/2;
E222(1)=0;
```

```
for i=1:n
  qe=[1 dt dt^2*(0.5-b2);0 1 dt*(1-y2);0 0 0] *q2(:,i);
      z1=qe(1);
      z2=qe(2);
      z3=qe(3);
  while (abs(z3+w0^2*z1+w0^2*z1^3)>e)
      syms z11 z22 z33;

[z111,z222,z333]=solve(z11-b2*dt^2*z33==0,z22-y2*dt*z
33==0,z33+w0^2*z11+3*w0^2*z11*z1^2==-(z3+w0^2*z1+w0^2
*z1^3),z11,z22,z33);
      qe=qe+[z111;z222;z333];
      z1=qe(1);
      z2=qe(2);
      z3=qe(3);
  end
  q2(:,i+1)=qe;
  E222(i+1)=E222(i)+q(3,i+1)*(q(1,i)-q(1,i+1));
  E22(i+1)=E222(i+1)+q(2,i+1)^2/2;
end
E2=1/2*max(q2(2,:))^2;
subplot(2,2,3)
plot(1:n+1,q2(1,:))
subplot(2,2,4)
plot(1:n+1,E22,1:n+1,E222)
```

E1=2.336107615270199e+02

E2=2.413743848261089e+02

3.3



Les lignes bleues à droite representent l'énergies, explicite au-dessus et implicite au-dessous

E1=2.336107615270199e+02

E2=2.413743848261089e+02

Les deux méthodes sont presque même

oscillateur non linéaire à un degré de liberté (P28-P29)

1.1.a

$$\ddot{q} - \varepsilon\omega_0[1-(\frac{q}{u_0})^2]\dot{q} + \omega_0^2 q = 0$$

$$\begin{pmatrix} q \\ \dot{q} \end{pmatrix}' = \begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix} = \begin{pmatrix} \dot{q} \\ \varepsilon\omega_0[1-(\frac{q}{y_0})]\dot{q} - \omega_0^2 q \end{pmatrix}$$

1.1. b

$$Q = \begin{pmatrix} q \\ \dot{q} \end{pmatrix}$$

1.2

$$f\left(\begin{pmatrix} q \\ \dot{q} \end{pmatrix}\right) = \begin{pmatrix} \dot{q} \\ \varepsilon\omega_0[1-(\frac{q}{y_0})]\dot{q} - \omega_0^2 q \end{pmatrix}$$

K1=f(Q(:,i));

K2=f(Q(:,i)+K1*dt/2);

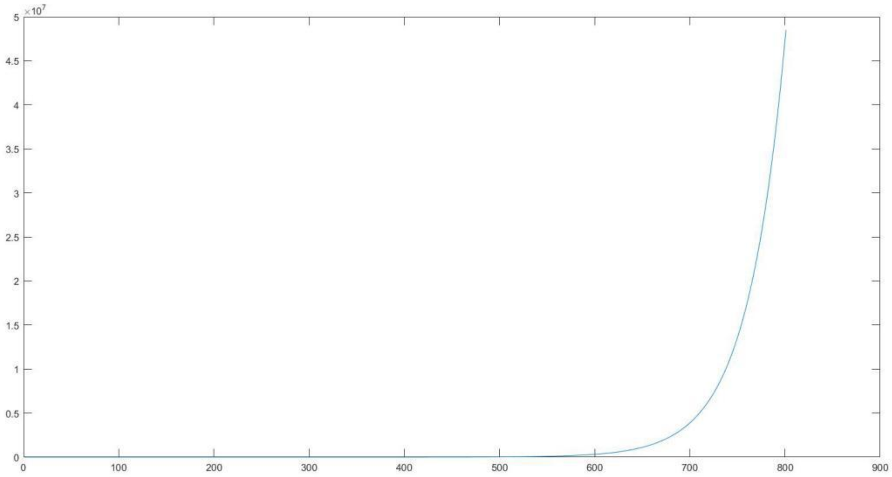
K3=f(Q(:,i)+K2*dt/2);

K4=f(Q(:,i)+K3*dt);

Q(:,i+1)=Q(:,i)+(K1+2*K2+2*K3+K4)/6*dt;

1.3

```
function Y=f(X)
    e=4;
    w0=2*pi;
    y0=2;
    Y(1,1)=X(1,1);
    Y(2,1)=e*w0*(1-X(1,1)/y0)*X(2,1)-w0^2*X(1,1);
end

clear all;
q0=0.1;
dq0=0;
w0=2*pi;
y0=2;
T0=20;
dt=0.025;
n=fix(T0/dt);

%1.3
e=4;


Q(:,1)=[q0;dq0];
for i=1:n
    K1=f(Q(:,i));
    K2=f(Q(:,i)+K1*dt/2);
    K3=f(Q(:,i)+K2*dt/2);
    K4=f(Q(:,i)+K3*dt);
    Q(:,i+1)=Q(:,i)+(K1+2*K2+2*K3+K4)/6*dt;
end

plot(1:n+1,Q(1,:))
```


×10^7
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
100
200
300
400
500
600
700
800
900

1.4

	t=0s	t=0.025s	t=0.05s	t=0.125s
q	0.100000000000000	0.102531512044271	0.105127109620845	0.113314845261525
dq	0	-0.136376584732673	-0.387101668506061	-3.18278523572325

2.1

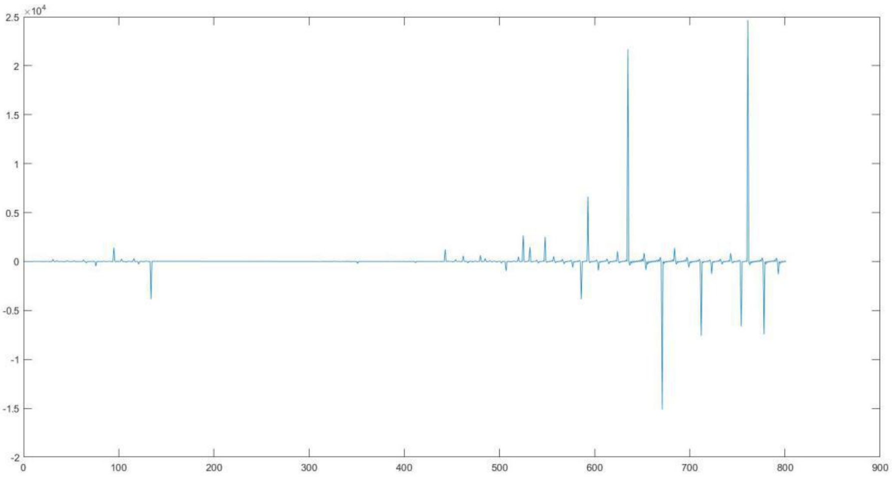
$$q_{n+1} - \beta \Delta t^2 \ddot{q}_{n+1} = q_n + \Delta t \dot{q} + \Delta t^2 (0.5 - \beta) \ddot{q}_n$$

$$\dot{q}_{n+1} - \gamma \Delta t \ddot{q}_{n+1} = \dot{q}_n + \Delta t (1 - \gamma) \ddot{q}_n$$

$$\ddot{q}_{n+1} + \omega_0^2 q_{n+1} - \varepsilon \omega_0 [1 - (\frac{q_{n+1}}{y_0})] \dot{q}_{n+1} = 0$$

2.2

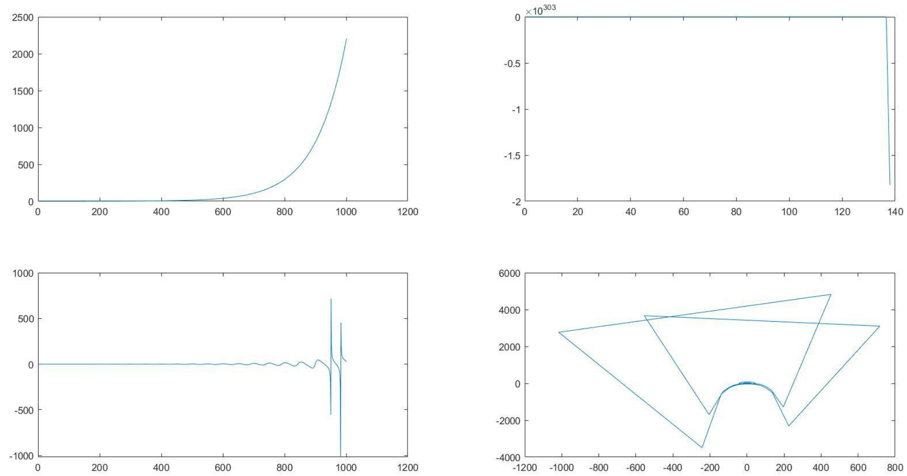
```
clear all;
q0=0.1;
dq0=0;
w0=2*pi;
y0=2;
T0=20;
dt=0.025;
n=fix(T0/dt);

%1.3
e=4;


Q(:,1)=[q0;dq0];
for i=1:n
    K1=f(Q(:,i));
    K2=f(Q(:,i)+K1*dt/2);
    K3=f(Q(:,i)+K2*dt/2);
    K4=f(Q(:,i)+K3*dt);
    Q(:,i+1)=Q(:,i)+(K1+2*K2+2*K3+K4)/6*dt;
end

%plot(1:n+1,Q(1,:))

%2.3
ddq0=e*w0*(1-q0/y0)*dq0-w0^2*q0;
q(:,1)=[q0;dq0;ddq0];
y2=0.5;
b2=0;
for i=1:n
   z1=q(1,i);
   z2=q(2,i);
   z3=q(3,i);
   syms z11 z22 z33;

[q(1,i+1),q(2,i+1),q(3,i+1)]=solve(z11-b2*dt^2*z33==z
1+dt*z2+dt^2*(0.5-b2)*z3,z22-y2*dt*z33==z2+dt*(1-y2)*
z3,z33+w0^2*z11-e*w0*[1-z11/y0]*z22,z11,z22,z33);

end

plot(1:n+1,q(1,:))
```


×10^4
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
100
200
300
400
500
600
700
800
900

2.3

	t=0s	t=0.025s	t=0.05s	t=0.125s
q	0.100000000000000	0.0987662994498638	0.0930073037218362	-0.00582795610924844
dq	0	-0.139853925563275	-0.394399286660789	-3.26938436633972
ddq	-3.94784176043574	-7.24047228462629	-13.1231566031748	-82.1779502430733

3.1.1



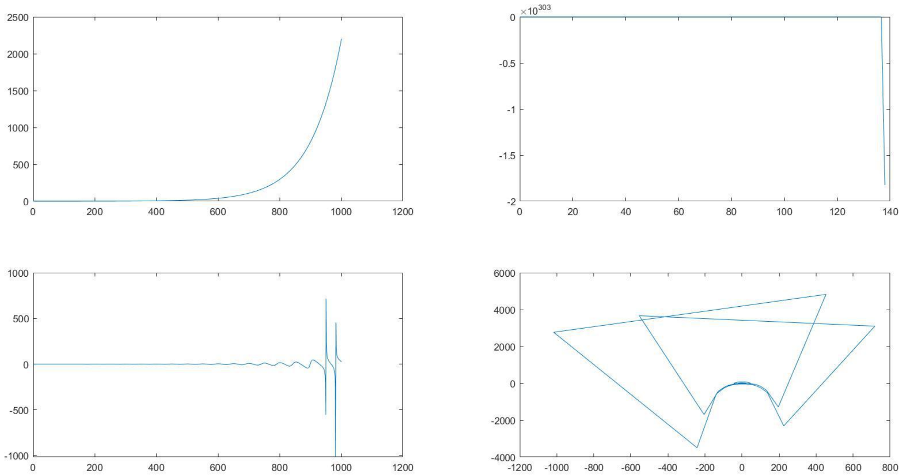
Runge Kutta prmière ligne à gauche

Newmark deuxième ligne à gauche

3.1.2

Newmark diverge beaucoup moins vite

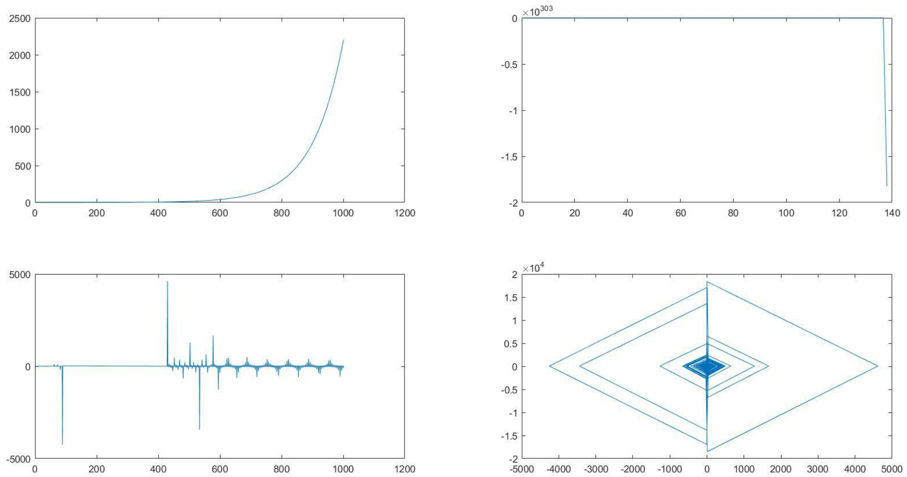
3.1.3


2500 2000 1500 1000 500 0
0 200 400 600 800 1000 1200
×10^303
0 -0.5 -1 -1.5 -2
0 20 40 60 80 100 120 140
1000 500 0 -500 -1000
0 200 400 600 800 1000 1200
6000 4000 2000 0 -2000 -4000
-1200 -1000 -800 -600 -400 -200 0 200 400 600 800

3.2.1

```
clear all;
q0=0.1;
dq0=0;
w0=2*pi;
y0=2;
T0=20;
dt1=0.01;
dt=0.02;
n=fix(T0/dt);

%1.3
e=5;


Q(:,1)=[q0;dq0];
for i=1:n
    K1=f(Q(:,i));
    K2=f(Q(:,i)+K1*dt1/2);
    K3=f(Q(:,i)+K2*dt1/2);
    K4=f(Q(:,i)+K3*dt1);
    Q(:,i+1)=Q(:,i)+(K1+2*K2+2*K3+K4)/6*dt1;
end
subplot(2,2,1)
plot(1:n+1,Q(1,:))
subplot(2,2,2)
plot(Q(1,:),Q(2,:)/w0)

%2.3
ddq0=e*w0*(1-q0/y0)*dq0-w0^2*q0;
q(:,1)=[q0;dq0;ddq0];
y2=0.5;
b2=0;
for i=1:n
   z1=q(1,i);
   z2=q(2,i);
   z3=q(3,i);
   syms z11 z22 z33;

[q(1,i+1),q(2,i+1),q(3,i+1)]=solve(z11-b2*dt^2*z33==z
1+dt*z2+dt^2*(0.5-b2)*z3,z22-y2*dt*z33==z2+dt*(1-y2)*
z3,z33+w0^2*z11-e*w0*[1-z11/y0]*z22,z11,z22,z33);
```

```
end

subplot(2,2,3)
plot(1:n+1,q(1,:))
subplot(2,2,4)
plot(q(1,:),q(2,:)/w0)
```



Runge Kutta première ligne à gauche

Newmark deuxième ligne à droite

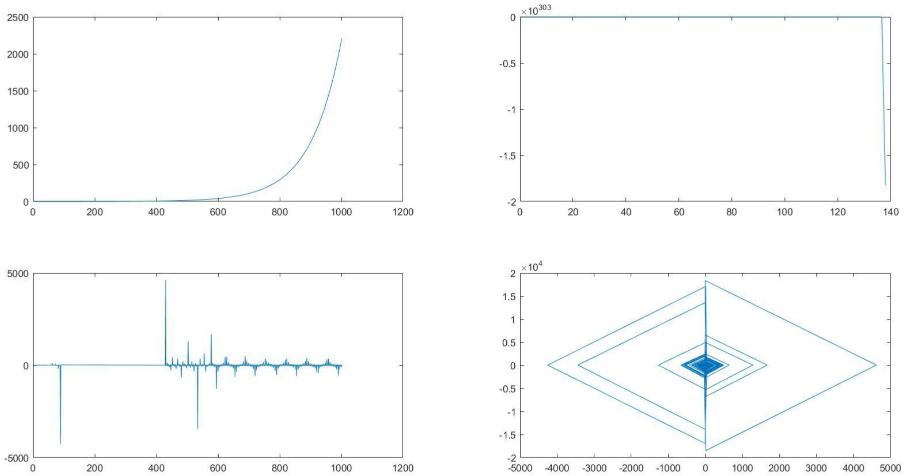
3.2.2

Non, il y a des erreurs.

3.2.3

t<0.01s

3.2.4


2500
2000
1500
1000
500
0
0 200 400 600 800 1000 1200
×10^303
0
-0.5
-1
-1.5
-2
0 20 40 60 80 100 120 140
5000
0
-5000
0 200 400 600 800 1000 1200
×10^4
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5000 -4000 -3000 -2000 -1000 0 1000 2000 3000 4000 5000