Exercice4

Oscillateur linéaire amorti à un degré de liberté

I.EULER explicite

```
%% Question
close all;
clc;
clear;

% initialisation
eps = 0.02;
T0 = 1;
w0 = 2*pi/T0;
% dt = 0.007;% question a
% dt = 2*eps/w0;% question b
dt = 0.8*2*eps/w0;% question c
% 2eps/w0 = 0.0064
te = 0:dt:10*T0;
[mp,np] = size(te);
x0 = 0.01;
dx0 = 0;
A = [1 dt;-dt*w0^2 1-2*eps*w0*dt];
U_e = [x0;dx0];
for ind = 1:np-1
    U_e(:,ind+1) = A * U_e(:,ind);
end
energe_e = 0.5*(U_e(2,:).^2 + w0^2*(U_e(1,:).^2));

figure();
plot(te,U_e(1,:),'-r','Linewidth',2);
figure();
plot(te,energe_e,'-r','Linewidth',2);
```

a) dt = 0.007

Figure 1

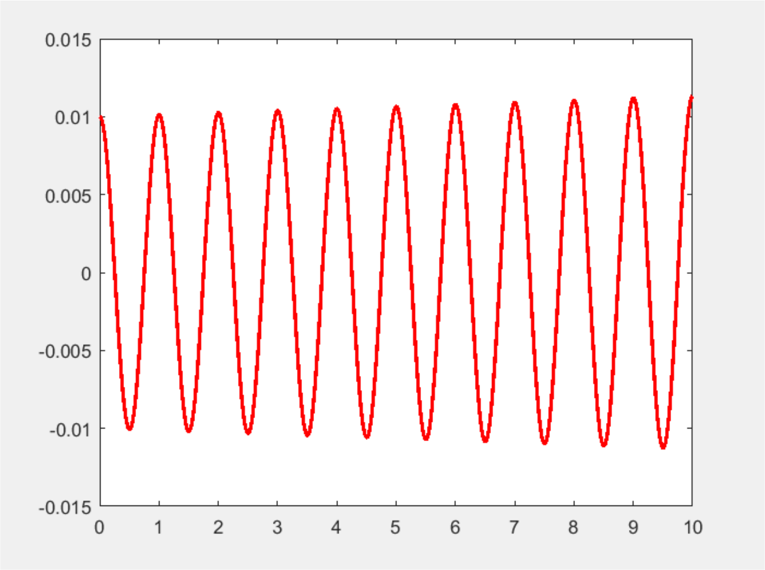
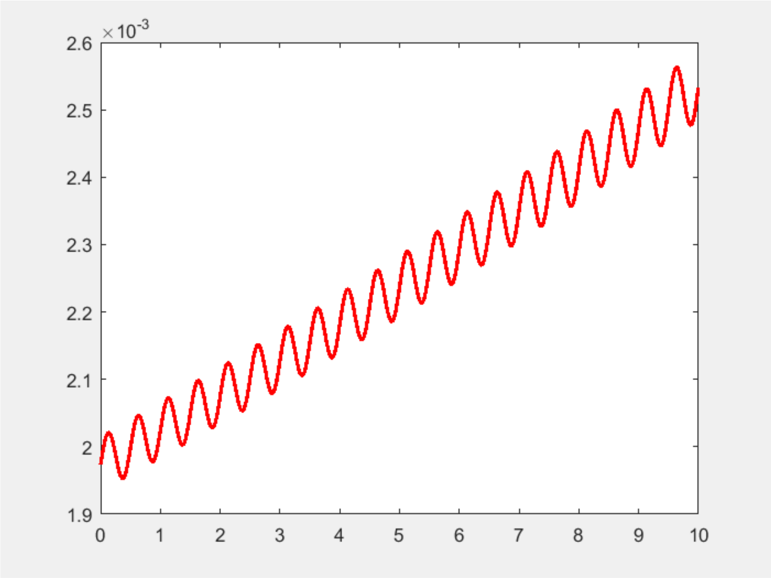


Figure 2



La solution explicite est plus grand que la solution exacte, et il est divergente. La solution est croissante.

b) dt = 2*eps/w0

Figure 1

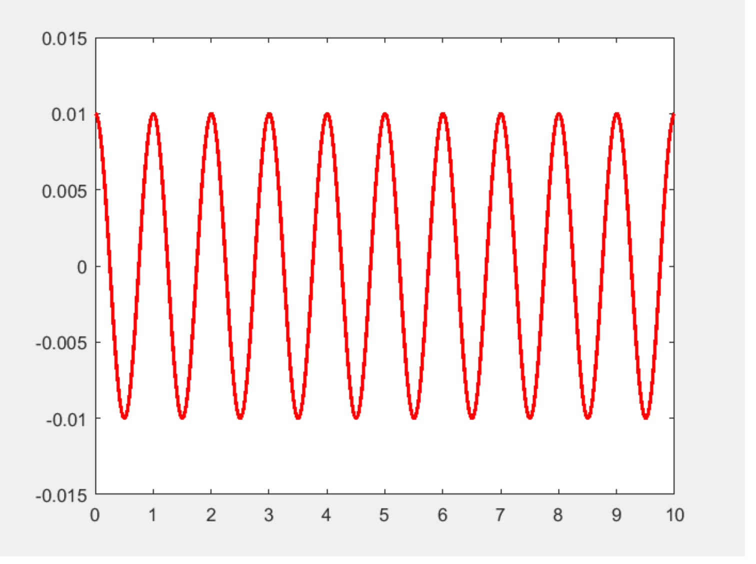
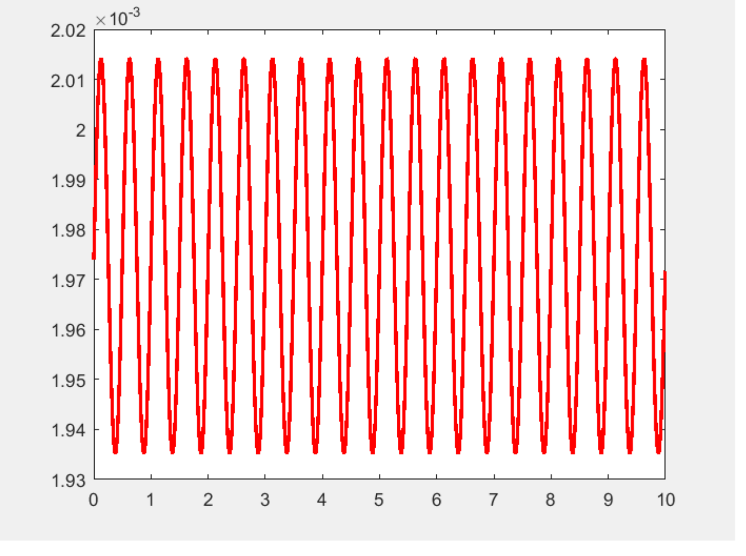


Figure 2



La solution explicite est plus grand que la solution exacte.

c) dt = 0.8*2*eps/w0

Figure 1

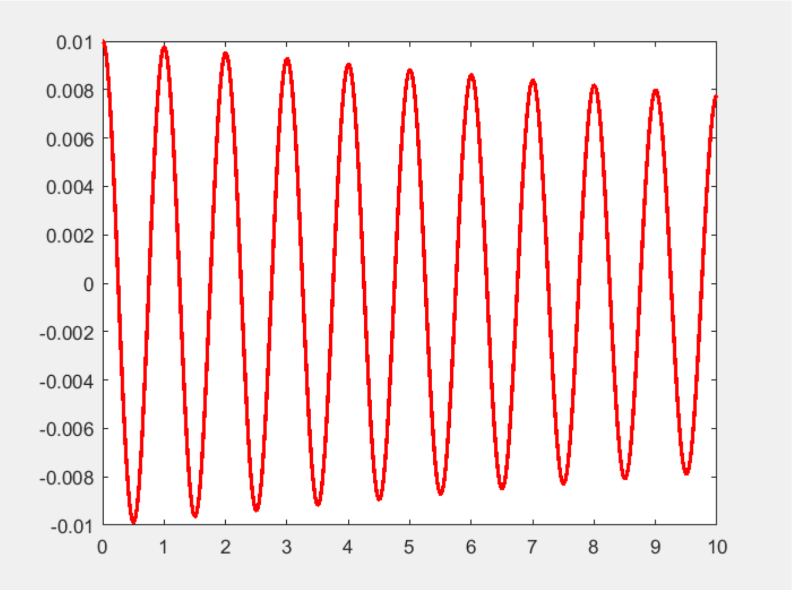
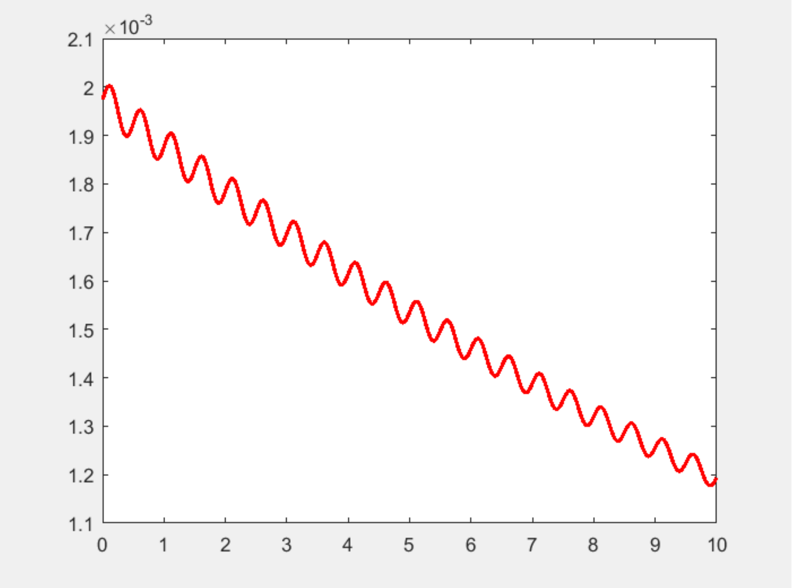


Figure 2



La solution explicite est plus grand que la solution exacte, et il est divergente. La solution est décroissant, mais il est toujours grand que la solution exacte.

d)

$$\begin{bmatrix} 1 & \Delta t \\ -{\omega_0}^2\Delta t & 1 - 2\varepsilon\omega_0\Delta t \end{bmatrix} = \mathrm{A}$$

$$\det(\lambda \mathrm{I} - \mathrm{A}) = 0$$

$$0 < \varepsilon < 1$$

$$\lambda = 1 - \varepsilon\omega_0\Delta t \pm i\omega_0\Delta t[1 - \varepsilon^2]^{\frac{1}{2}}$$

alors $|\lambda| < 1$

donc

$$\Delta t < \frac{2\varepsilon}{\omega_0}$$

- Quand $\frac{\Delta t}{\frac{2\varepsilon}{\omega_0}} \leq 0.05$, la solution calculée présente-t-elle une précision suffisante.

II.EULER implicite

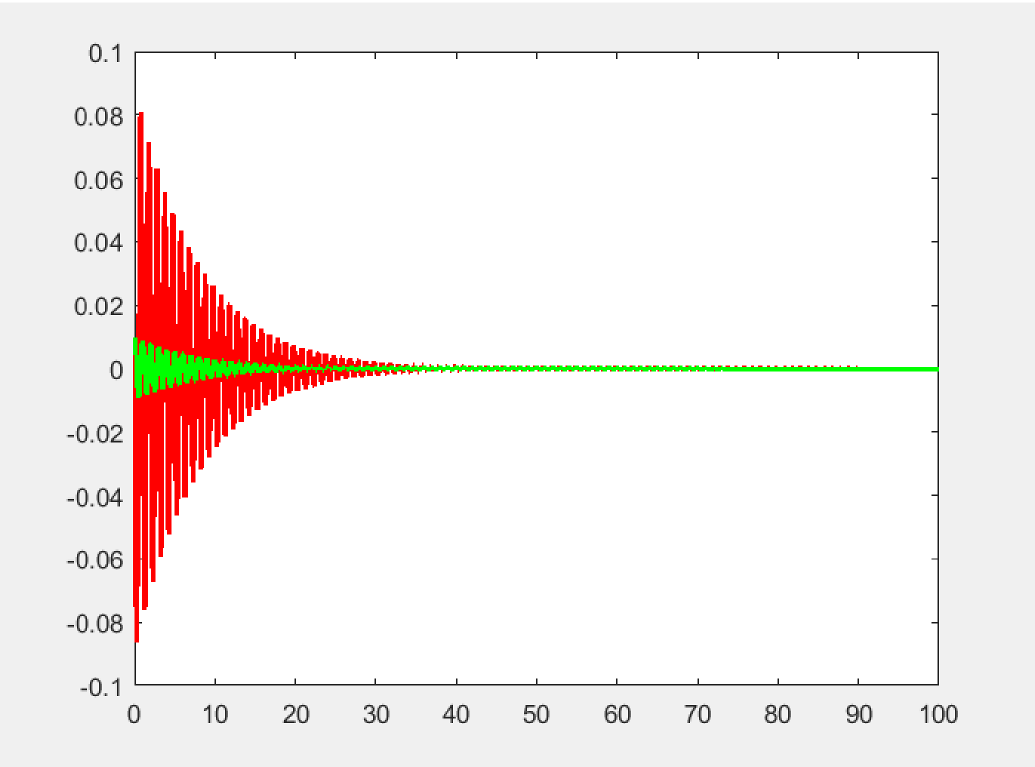
```
syms dt
B=[1 -dt; w0^2*dt 1+2*eps*w0*dt];
eig(B^(-1))

ans =
 (8388608*(8388608*pi*dt + dt*(70368744177664*pi^2 - 1736279168087509375)^(1/2) + 209715200))/(69451166723500375*dt^2 + 140737488355328*pi*dt + 1759218604441600)
 (8388608*(8388608*pi*dt - dt*(70368744177664*pi^2 - 1736279168087509375)^(1/2) + 209715200))/(69451166723500375*dt^2 + 140737488355328*pi*dt + 1759218604441600)
```

III.RUNGE KUTTA

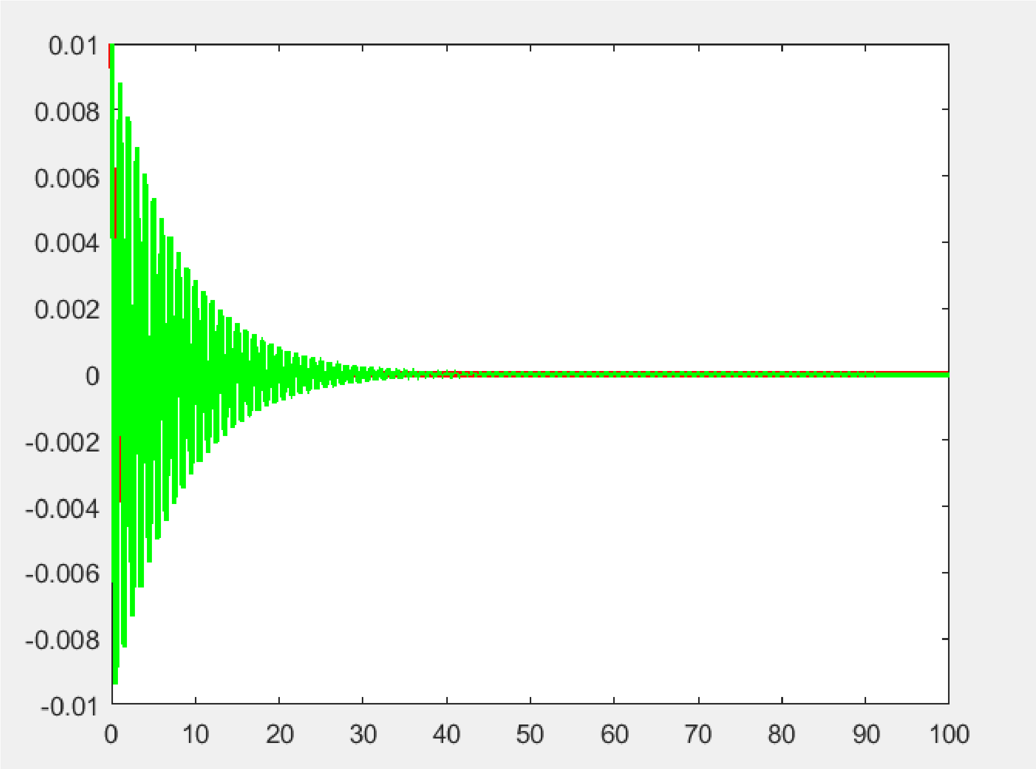
```
clear all
clc;
w0=2*pi;
eps=0.02;
T0=1;
x0=0.01;
dx0=0;
w0c = w0*w0;
h1 = 0.04;
dt1=h1*2*sqrt(2)/w0;
t = (0:dt1:100*T0)';
np = size(t,1);
x1 = zeros(np,1);
dx1 = zeros(np,1);
x1(1) = x0;
dx1(1) = dx0;
xj = [x0 ; dx0];
A = [-1 1;0 w0c*dt1];
B = [0 dt1;1 -(1+2*eps*w0*dt1)];
C = inv(B)*A;
for inc = 2 : np
k1 = C*xj;
k2 = C*(xj+k1*dt1/2);
k3 = C*(xj+k2*dt1/2);
```

```
k4 = C*(xj+k3*dt1);
K = (k1 + 2*k2 + 2*k3 + k4)/6;
xj = xj + K * dt1;
x1(inc) = xj(1);
dx1(inc) = xj(2);
end
plot(t,x1, '-r','Linewidth',2)

hold on

w0=2*pi;
eps=0.02;
w=w0*(1-eps^2)^(0.5);
dT=2*eps/w0;
T0=1;
x=0.01;
xd=0;
J=linspace(0,100*T0,100*T0/dT);
Y0=(exp(-eps*w0*J)).*(x*cos(w*J)+(eps*w0*x+xd)*(sin(w*J))/w);
plot(J,Y0,'-g','Linewidth',2);
```

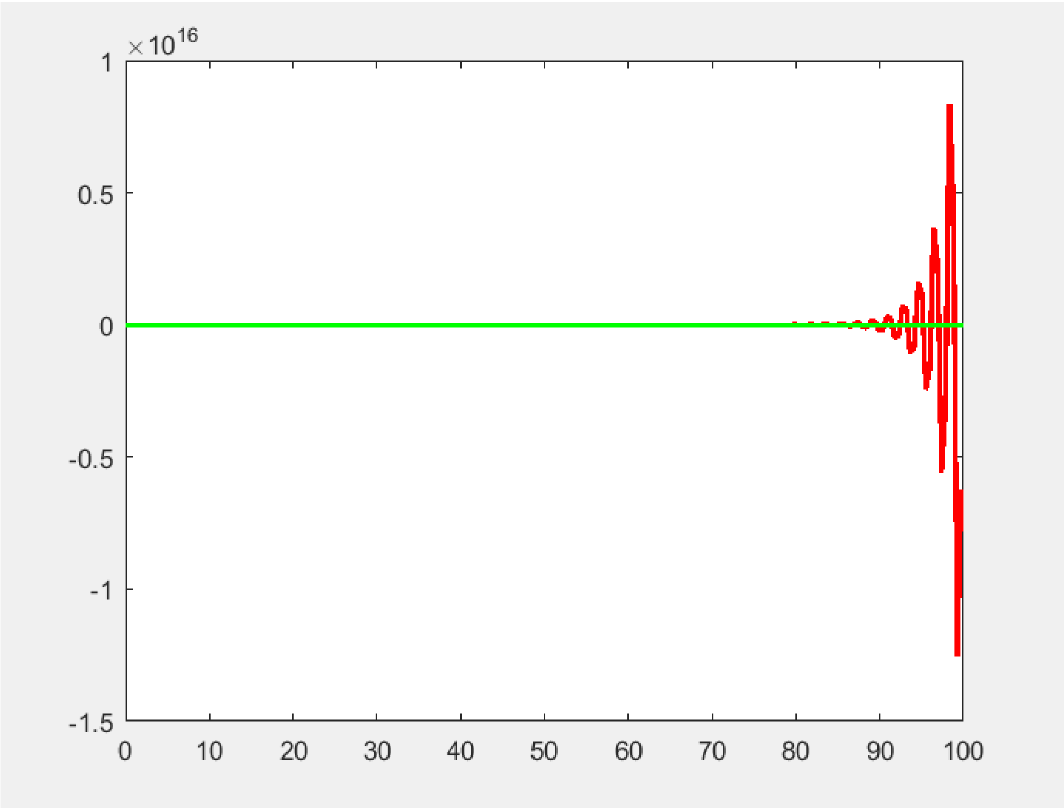
a) h=0.04



b) h=0.96



c) h=1.04



Quand h=0.04 et h=0.96, les résultats sont stable et h=1.04 n’est pas stable. Et h=0.04 est plus précise que h=0.96 et h=1.04.

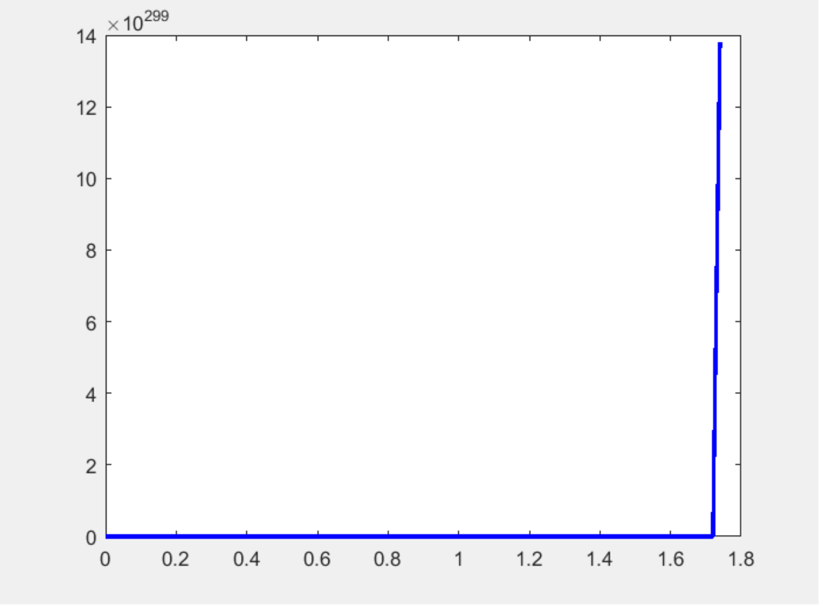
Double pendule avec l'hypothèse des petits mouvements

```
%% deux degre liberte
% initialisation
close all;
clc;
clear;
methode = 'explicite';
if methode == 'explicite'
    beta = 0;
    gamma = 0.5;
elseif methode == 'implicite'
    beta = 0.25;
    gamma = 0.5;
else
    return
end
m1 = 1;
m2 = 4;
k1 = 3;
k2 = 4;
k3 = 12;
M = [m1 0;0 m2];
K = [k1+k2 -k2;-k2 k2+k3];
dt = 0.02;
Tt = 6;
te = (0:dt:Tt);
[mp,np] = size(te);
A = [m1+beta*dt^2*(k1+k2) 0;0 m2+(k2+k3)*beta*dt^2];
tole = 1e-8;
q1(:,1) = [0.5;0;-1.5];
q2(:,1) = [0.5;0.2;-1.5];

for ind = 1:np-1
    q1_p = [q1(1,ind)+dt*q1(2,ind)+dt^2*(0.5-beta)*q1(3,ind);q1(2,ind)+dt*(1-gamma)*q1(3,ind);0];
    q2_p = [q2(1,ind)+dt*q2(2,ind)+dt^2*(0.5-beta)*q2(3,ind);q2(2,ind)+dt*(1-gamma)*q2(3,ind);0];
    for iter = 1:20;
        if
norm([m1*q1_p(3)+(k1+k2)*q1_p(1)-k2*q2_p(1);m2*q2_p(3)+(k2+k3)*q2_p(1)-k2*q1_p(1)]) >=
tole
            cor_ddq =
A\[m1*q1_p(3)+(k1+k2)*q1_p(1)-k2*q2_p(1);m2*q2_p(3)+(k2+k3)*q2_p(1)-k2*q1_p(1)];
            cor_dq = cor_ddq * gamma * dt;
            cor_q = beta * dt^2 * cor_ddq;
```

```
                q1_p = q1_p + [cor_q(1);cor_dq(1);cor_ddq(1)];
                q2_p = q2_p + [cor_q(2);cor_dq(2);cor_ddq(2)];
            end
        end
        q1(:,ind+1) = q1_p;
        q2(:,ind+1) = q2_p;
end

figure;
plot(te,q1(1,:),'-r','Linewidth',2);
plot(te,q2(1,:),'-b','Linewidth',2);
```

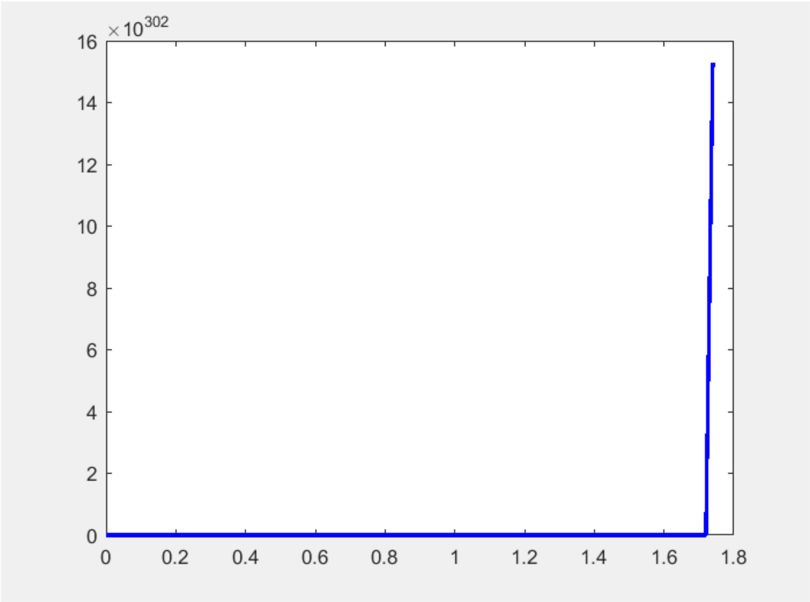
a) explicite

Figure



b) implicite

Figure



Oscillateur non linéaire à un degré de liberté

I. NEWMARK explicite

$$q_{j+1} = q_j + \Delta t\dot{q}_j + \Delta t^2(0.5-\beta)\ddot{q}_j + \Delta t^2\beta\ddot{q}_{j+1}$$

$$\dot{q}_{j+1} = \dot{q}_j + \Delta t(1-\gamma)\ddot{q}_j + \Delta t\gamma\ddot{q}_{j+1}$$

$$\ddot{q}_{j+1} = -{\omega_0}^2 q_{j+1}$$

Donc

$$\begin{bmatrix} 1 & 0 & -\beta\Delta t^2 \\ 0 & 1 & -\gamma\Delta t \\ {\omega_0}^2 & 0 & 1 \end{bmatrix}\begin{bmatrix} q_{j+1} \\ \dot{q}_{j+1} \\ \ddot{q}_{j+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & \Delta t^2(0.5-\beta) \\ 0 & 1 & \Delta t(1-\gamma) \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} q_j \\ \dot{q}_j \\ \ddot{q}_j \end{bmatrix}$$

$$\ddot{q}_{j+1} = -{\omega_0}^2 q_{j+1}$$

$$\ddot{q}_j = -{\omega_0}^2 q_j$$

On a $\begin{bmatrix} 1+\beta\Delta t^2{\omega_0}^2 & 0 \\ \gamma\Delta t{\omega_0}^2 & 1 \end{bmatrix}\begin{bmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{bmatrix} = \begin{bmatrix} 1-\Delta t^2(0.5-\beta){\omega_0}^2 & \Delta t \\ (\gamma-1)\Delta t{\omega_0}^2 & 1 \end{bmatrix}\begin{bmatrix} q_j \\ \dot{q}_j \end{bmatrix}$

Comme $B\begin{bmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{bmatrix} = C\begin{bmatrix} q_j \\ \dot{q}_j \end{bmatrix}$

Donc $\begin{bmatrix} q_{j+1} \\ \dot{q}_{j+1} \end{bmatrix} = (B^{-1}\times C)\begin{bmatrix} q_j \\ \dot{q}_j \end{bmatrix}$

matrice d'amplification

$$A = (B^{-1}\times C)$$

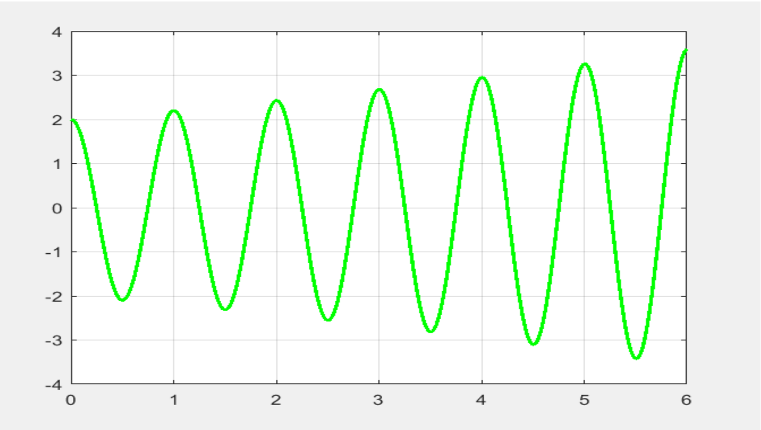
$$= \begin{bmatrix} 1 - \frac{{\omega_0}^2 \Delta t^2}{2(1+\beta\Delta t^2 {\omega_0}^2)} & \frac{\Delta t}{1+\beta\Delta t^2 {\omega_0}^2} \\ -{\omega_0}^2 \Delta t[1 - \frac{\gamma {\omega_0}^2 \Delta t^2}{2(1+\beta\Delta t^2 {\omega_0}^2)}] & 1 - \frac{\gamma {\omega_0}^2 \Delta t^2}{2(1+\beta\Delta t^2 {\omega_0}^2)} \end{bmatrix}$$

Car $\beta = 0, \gamma = 0.5$

On a

$$A = \begin{bmatrix} 1 - \frac{{\omega_0}^2 \Delta t^2}{2} & \Delta t \\ -{\omega_0}^2 \Delta t[1 - \frac{0.5{\omega_0}^2 \Delta t^2}{2}] & 1 - \frac{0.5{\omega_0}^2 \Delta t^2}{2} \end{bmatrix}$$

```
clear all
clc;
dt1 = 0.02;
T0 = 6;
w0 = 2*pi;
q0 = 2;
dq0 = 0;
a = 0.1;
w0c = w0*w0;
t1 = (0:dt1:T0)';
np1 = size(t1,1);
q1 = zeros(np1,1);
dq1 = zeros(np1,1);
q1(1) = q0;
dq1(1) = dq0;
for inc = 2 : np1
q1(inc) = q1(inc-1)*(1-0.5*w0c*dt1*dt1) + dt1*dq1(inc-1);
dq1(inc) = q1(inc-1)*w0c*dt1*(0.25*w0c*dt1*dt1-1) + dq1(inc-1)*(1-0.25*w0c*dt1*dt1);
end
plot (t1,q1, '-g','Linewidth',2)
grid on
```



3 On choisit Δt = 0.02 s. Les valeurs numériques de q(t) pour les valeurs de t égales à 0s , Δt , 2Δt et T_0 est

$$q_{t=0} = 2$$
$$q_{t=\Delta t} = 1.9842$$
$$q_{t=2\Delta t} = 1.9371$$
$$q_{t=T_0} = 3.5927$$

II. NEWMARK implicite

On doit chercher à minimiser la correction entre la valeur exact et la valeur estimée.

Calcul de la correction :

$$f\left(\ddot{q}_{j+1}^{*} + \Delta \ddot{q}_{j+1}, q_{j+1}^{*} + \Delta q_{j+1}\right) = 0$$

$$\ddot{q}_{j+1}^{*} + \Delta \ddot{q}_{j+1} + {\omega_0}^2\left(q_{j+1}^{*} + \Delta q_{j+1}\right)\left(1 + a\left(q_{j+1}^{*} + \Delta q_{j+1}\right)^2\right) = 0$$

$$f\left(\ddot{q}_{j+1}^{*} + \Delta \ddot{q}_{j+1}, q_{j+1}^{*} + \Delta q_{j+1}\right) = 0 = f\left(\ddot{q}_{j+1}^{*}, q_{j+1}^{*}\right) + \frac{\partial f}{\partial q_{j+1}^{*}}\Delta q_{j+1} + \frac{\partial f}{\partial \ddot{q}_{j+1}^{*}}\Delta \ddot{q}_{j+1}$$

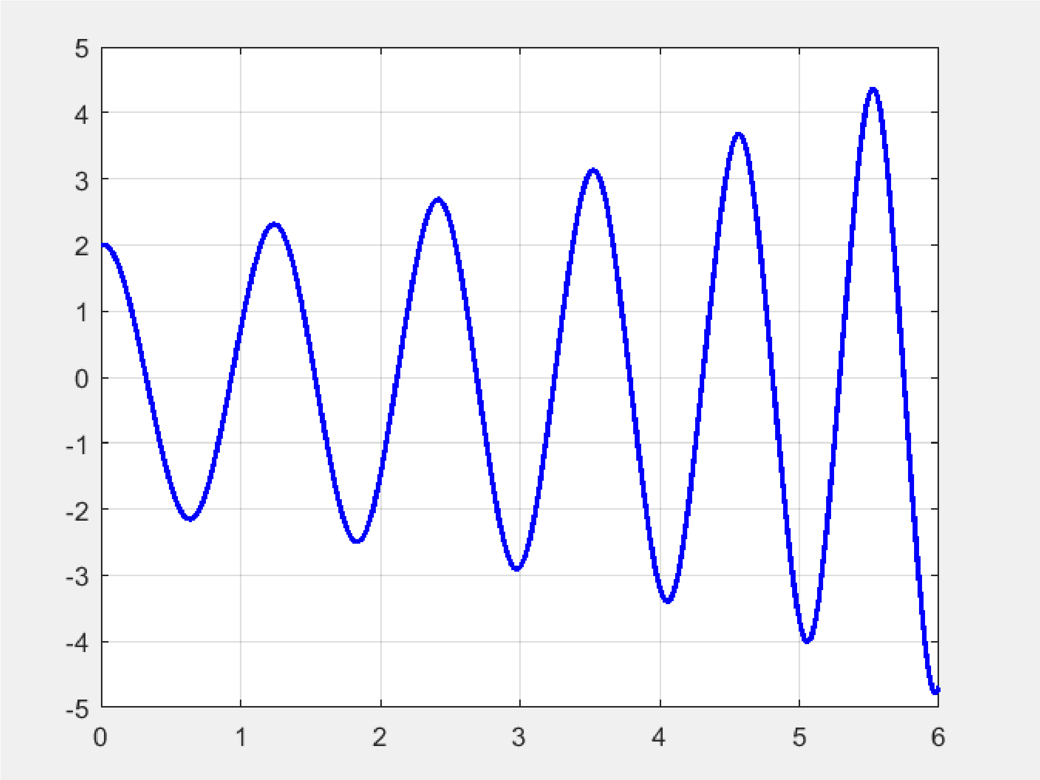
avec $\Delta q_{j+1} = \beta \Delta t^2 \Delta \ddot{q}_{j+1}$

on obtient que

$$\Delta \ddot{q}_{j+1} = -\frac{f\left(\ddot{q}_{j+1}^{*}, \dot{q}_{j+1}^{*}, q_{j+1}^{*}\right)}{\frac{\partial f}{\partial \ddot{q}_{j+1}^{*}} + 0.25\frac{\partial f}{\partial q_{j+1}^{*}}\Delta t^2}$$

```
clear all
clc;
dt2 = 0.02;
T0 = 6 ;
q0 = 2;
dq0=0 ;
a = 0.1;
w0 = 2*pi;
w0c = w0*w0;
t2 = (0:dt2:T0)';
np2 = size(t2,1);
q2 = zeros(np2,1);
dq2 = zeros(np2,1);
q2(1) = q0;
dq2(1) = dq0;
ddq2(1) = 0;
for inc = 2 : np2
q2(inc) = q2(inc-1)+dt2*dq2(inc-1)+0.25*dt2*dt2*ddq2(inc-1);
dq2(inc) = dq2(inc-1)+0.5*dt2*ddq2(inc-1);
ddq2(inc) = -w0c*q2(inc)*(1+a*q2(inc)*q2(inc));
```

```
end

plot(t2,q2,'b-','Linewidth',2)
grid on
```



4

$$q_{t=0} = 2$$
$$q_{t=\Delta t} = 1.9889$$
$$q_{t=2\Delta t} = 1.9559$$

$$q_{t=T_0} = -4.6870$$

III. Energie mécanique

L'énergie mécanique

$$E = \frac{1}{2}m\dot{q}^2 + \frac{1}{2}kq^2 - kq(1 + aq^2)$$

$$E = m(-\frac{k}{m}aq^3 + \frac{k}{m}\frac{q^2}{2} - \frac{k}{m}q + \frac{1}{2}q^2)$$

Car $\frac{k}{m} = \omega_0$

Donc

$$E = m(-\omega_0 aq^3 + \omega_0\frac{q^2}{2} - \omega_0 q + \frac{1}{2}q^2)$$

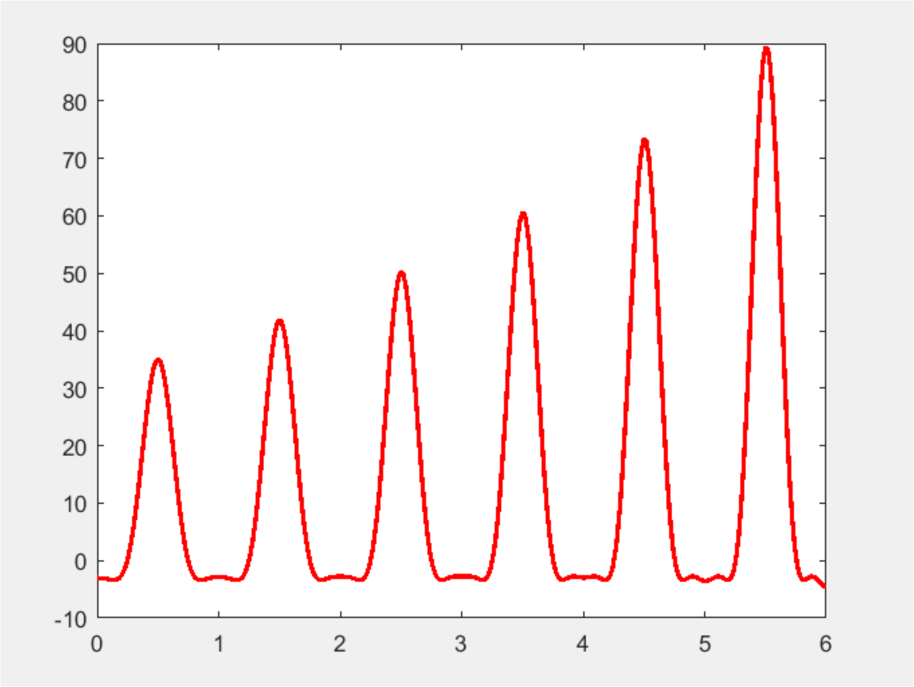
On pose que $E^* = \frac{E}{m}$

Alors

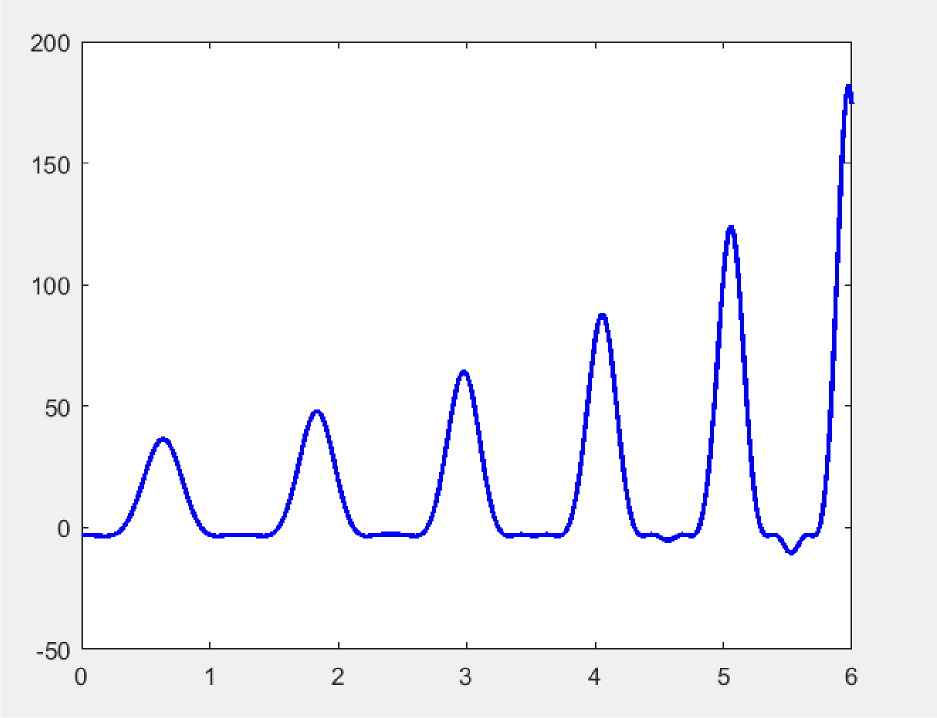
$$E^* = -\omega_0 a q^3 + \omega_0 \frac{q^2}{2} - \omega_0 q + \frac{1}{2} q^2$$

```
energ1 = -w0*a*(q1).^3+w0*0.5*(q1).^2-w0*q1+0.5*(q1).^2;
plot(t1,energ1,'r-','Linewidth',2)
```

a) Solution explicite :



b) Solution implicite :



Les résultats sont croissant, les énergie mécanique du système de oscillateur non linéaire à un degré de liberté devient de plus en plus grand. Et le résultat implicite augmente plus rapidement que la solution explicite.